Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.2.3-g-sec-^p-a+b-sec-^m-c+d-sec-^n

Nasser M. Abbasi

July 17, 2021 Compiled on July 17, 2021 at 6:11pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [286]. This is test number [122].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (286)	% 0.00 (0)
Mathematica	% 95.45 (273)	% 4.55 (13)
Maple	% 91.61 (262)	% 8.39 (24)
Maxima	% 58.04 (166)	% 41.96 (120)
Fricas	% 82.52 (236)	% 17.48 (50)
Sympy	% 0.35 (1)	% 99.65 (285)
Giac	% 28.67 (82)	% 71.33 (204)
Mupad	% 66.78 (191)	% 33.22 (95)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

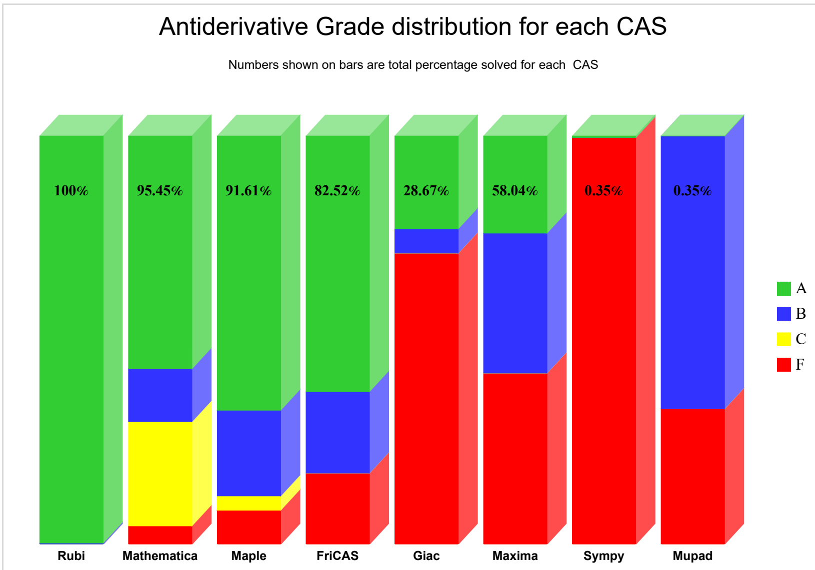
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

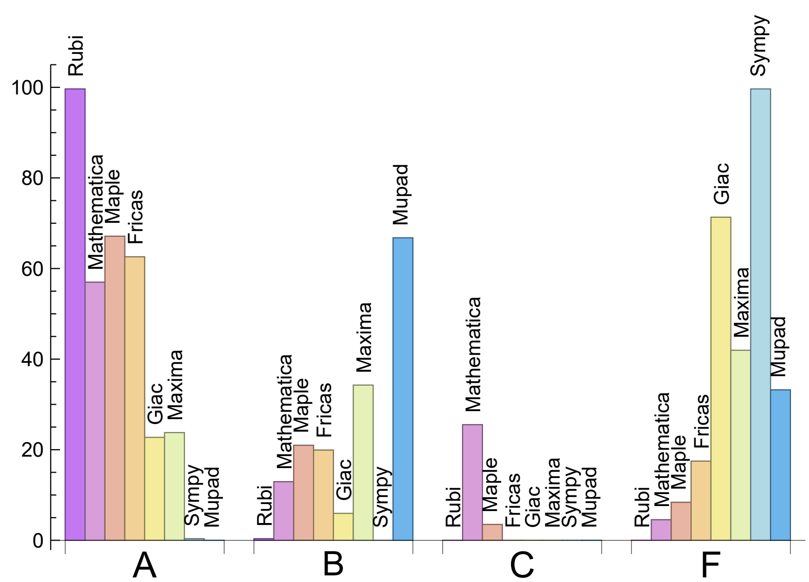
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.65	0.35	0.00	0.00
Mathematica	56.99	12.94	25.52	4.55
Maple	67.13	20.98	3.50	8.39
Maxima	23.78	34.27	0.00	41.96
Fricas	62.59	19.93	0.00	17.48
Sympy	0.35	0.00	0.00	99.65
Giac	22.73	5.94	0.00	71.33
Mupad	0.00	66.78	0.00	33.22

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	13	100.00 %	0.00 %	0.00 %
Maple	24	87.50 %	0.00 %	12.50 %
Maxima	120	49.17 %	15.83 %	35.00 %
Fricas	50	68.00 %	32.00 %	0.00 %
Sympy	285	83.51 %	16.49 %	0.00 %
Giac	204	23.53 %	0.98 %	75.49 %
Mupad	95	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

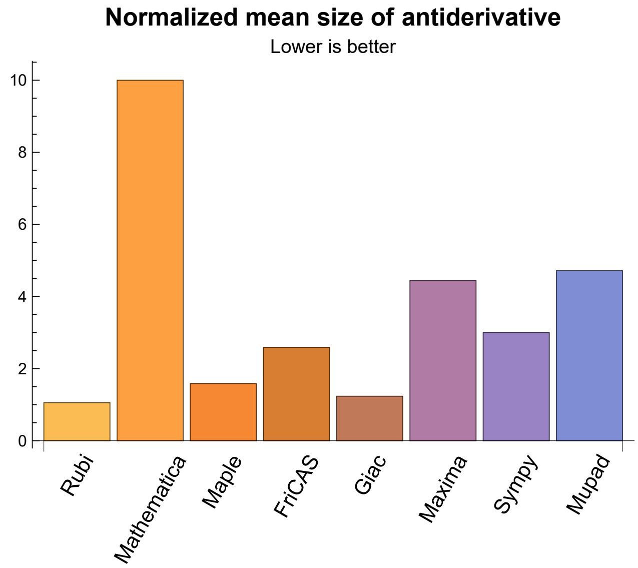
1.3 Performance

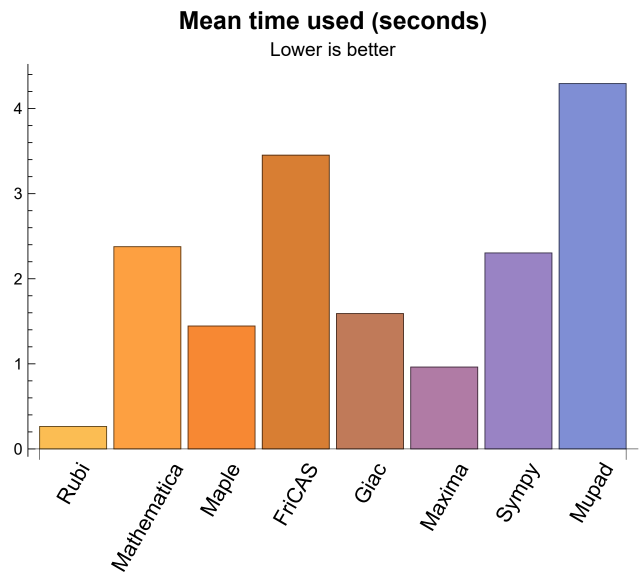
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	135.98	1.05	121.00	1.00
Mathematica	2.38	1402.67	10.00	154.00	1.39
Maple	1.44	214.45	1.58	147.50	1.33
Maxima	0.96	453.47	4.44	219.00	2.21
Fricas	3.45	354.14	2.59	172.00	1.72
Sympy	2.30	51.00	3.00	51.00	3.00
Giac	1.59	185.72	1.23	89.50	0.85
Mupad	4.29	803.47	4.72	158.00	1.46

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {77, 162, 163, 164, 174, 176, 178, 180, 182, 216, 224, 232, 240, 265, 269, 275, 277}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

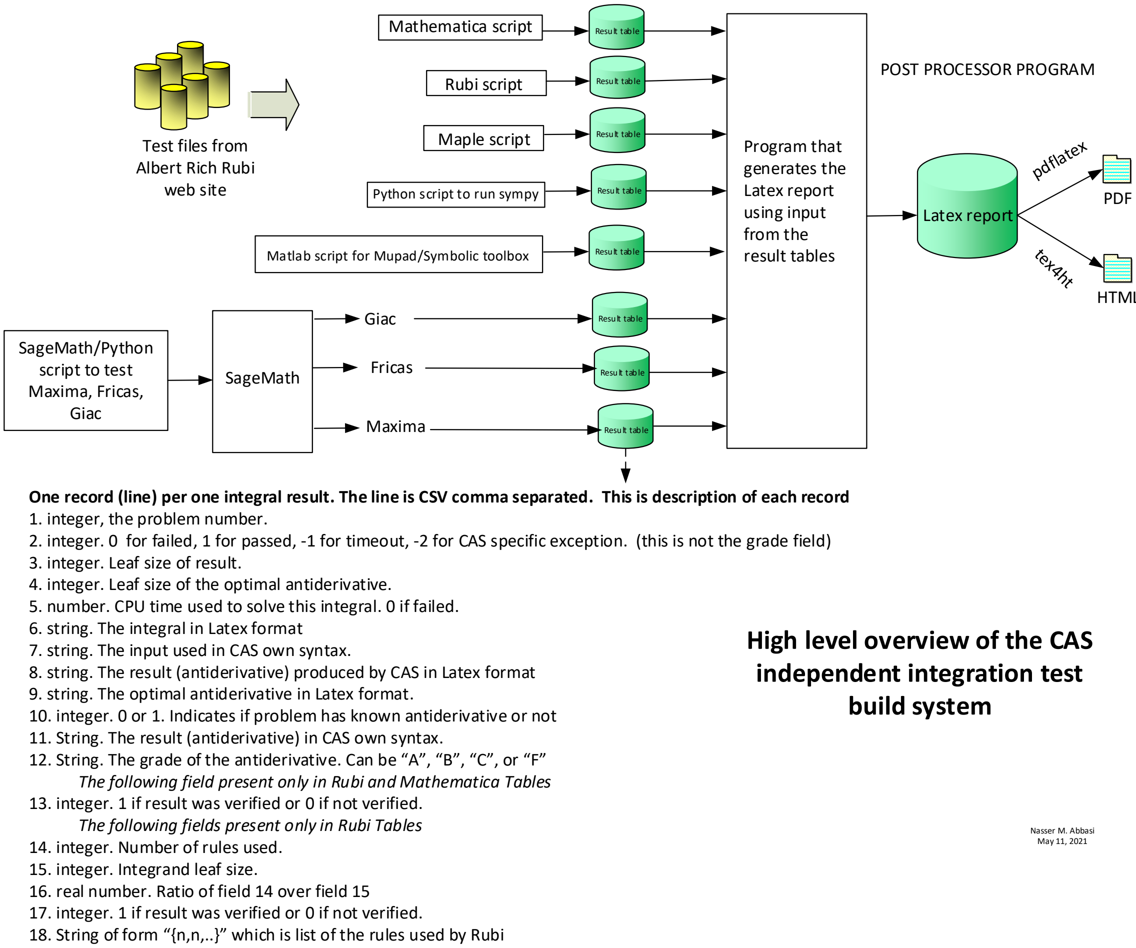
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

B grade: { 197 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 130, 131, 132, 141, 146, 147, 152, 153, 168, 169, 170, 173, 175, 178, 179, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 200, 201, 202, 203, 209, 217, 218, 221, 226, 227, 229, 230, 234, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 266, 267, 268, 271, 275, 276, 279, 281, 282, 283, 286 }

B grade: { 1, 2, 3, 15, 27, 28, 34, 35, 36, 42, 43, 44, 54, 61, 62, 107, 126, 171, 172, 180, 181, 195, 196, 204, 210, 211, 212, 213, 219, 220, 225, 228, 252, 253, 258, 273, 285 }

C grade: { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 158, 162, 163, 164, 165, 174, 176, 183, 197, 198, 199, 205, 206, 207, 208, 214, 215, 216, 222, 223, 224, 231, 232, 233, 239, 240, 265, 269, 270, 272, 277, 280, 284 }

F grade: { 151, 154, 155, 156, 157, 159, 160, 161, 166, 167, 177, 274, 278 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 136, 137, 138, 139, 141, 142, 143, 144, 148, 149, 150, 168, 169, 171, 172, 173, 181, 182, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 208, 209, 213, 214, 215, 216, 220, 221, 222, 223, 224, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 255, 256, 257, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 279, 282, 285, 286 }

B grade: { 69, 70, 77, 78, 91, 92, 98, 99, 106, 107, 110, 118, 128, 129, 130, 135, 140, 145, 146, 147, 170, 178, 179, 180, 183, 184, 189, 197, 198, 205, 206, 207, 210, 211, 212, 217, 218, 219, 225, 226, 227, 228, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 258, 259, 260, 261, 281 }

C grade: { 267, 270, 271, 272, 274, 277, 278, 280, 283, 284 }

F grade: { 38, 48, 60, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177 }

2.1.4 Maxima

A grade: { 2, 3, 4, 7, 8, 9, 14, 26, 38, 39, 40, 41, 47, 48, 49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 93, 94, 95, 100, 101, 109, 110, 116, 118, 126, 129, 135, 136, 140, 141, 145, 147, 156, 157, 162, 163, 164, 168, 173, 183, 185, 186, 187, 188, 195, 196, 203, 221, 229, 230, 244, 245, 246, 247, 285, 286 }

B grade: { 1, 5, 6, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 89, 96, 102, 103, 107, 108, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 130, 131, 132, 133, 134, 137, 138, 139, 142, 143, 146, 148, 150, 158, 169, 170, 171, 172, 178, 180, 181, 182, 193, 194, 202, 204, 210, 211, 212, 213, 217, 218, 219, 220, 225, 226, 227, 228 }

C grade: { }

F grade: { 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 144, 149, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 179, 184, 189, 190, 191, 192, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 214, 215, 216, 222, 223, 224, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 113, 114, 115, 120, 121, 123, 124, 125, 132, 136, 138, 143, 144, 148, 149, 150, 156, 157, 158, 162, 163, 164, 168, 169, 171, 172, 173, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 202, 203, 204, 205, 210, 211, 213, 214, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 262, 286 }

B grade: { 17, 30, 56, 69, 74, 82, 107, 108, 111, 112, 116, 119, 122, 126, 130, 131, 137, 141, 142, 146, 147, 170, 183, 184, 190, 191, 192, 198, 199, 200, 201, 206, 207, 208, 209, 212, 215, 216, 219, 222, 223, 224, 231, 232, 233, 234, 238, 250, 251, 252, 253, 254, 257, 260, 261, 263, 285 }

C grade: { }

F grade: { 110, 117, 118, 127, 128, 129, 133, 134, 135, 139, 140, 145, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.1.6 Sympy

A grade: { 170 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

2.1.7 Giac

A grade: { 6, 7, 8, 9, 17, 18, 19, 20, 30, 31, 32, 33, 38, 39, 40, 41, 46, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 93, 94, 95, 100, 101, 102, 170, 173, 199, 208, 214, 215, 216, 221, 229, 230, 249, 262, 285, 286 }

B grade: { 190, 191, 192, 200, 201, 209, 222, 223, 224, 231, 232, 233, 250, 251, 256, 257, 263 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 34, 35, 36, 37, 42, 43, 44, 45, 52, 53, 54, 55, 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 217, 218, 219, 220, 225, 226, 227, 228, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 252, 253, 254, 255, 258, 259, 260, 261, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 107, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 141, 146, 147, 157, 162, 163, 164, 168, 169, 170, 171, 172, 173, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 285, 286 }

C grade: { }

F grade: { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	499	130	215	131	0	0	176
normalized size	1	1.00	4.75	1.24	2.05	1.25	0.00	0.00	1.68
time (sec)	N/A	0.195	1.697	1.725	0.650	0.460	0.000	0.000	6.644

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	887	107	133	117	0	0	146
normalized size	1	1.00	10.31	1.24	1.55	1.36	0.00	0.00	1.70
time (sec)	N/A	0.157	6.483	1.306	0.418	0.473	0.000	0.000	5.197

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	313	84	108	103	0	0	114
normalized size	1	1.00	5.13	1.38	1.77	1.69	0.00	0.00	1.87
time (sec)	N/A	0.104	0.722	1.169	0.588	0.446	0.000	0.000	3.810

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	42	68	67	0	0	77
normalized size	1	1.00	1.00	1.11	1.79	1.76	0.00	0.00	2.03
time (sec)	N/A	0.050	0.028	0.662	0.460	0.445	0.000	0.000	2.238

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	77	63	101	66	0	0	31
normalized size	1	1.00	1.83	1.50	2.40	1.57	0.00	0.00	0.74
time (sec)	N/A	0.054	0.072	0.867	1.172	0.444	0.000	0.000	1.846

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	50	21	97	51	0	21	20
normalized size	1	1.00	1.39	0.58	2.69	1.42	0.00	0.58	0.56
time (sec)	N/A	0.050	0.252	0.793	0.598	0.428	0.000	1.577	2.060

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	87	37	117	78	0	39	35
normalized size	1	1.00	1.14	0.49	1.54	1.03	0.00	0.51	0.46
time (sec)	N/A	0.098	0.407	0.720	0.338	0.438	0.000	0.505	1.710

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	50	177	104	0	54	61
normalized size	1	1.00	0.97	0.43	1.53	0.90	0.00	0.47	0.53
time (sec)	N/A	0.150	0.464	0.759	0.458	0.437	0.000	0.354	1.746

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	63	197	128	0	69	106
normalized size	1	1.00	0.88	0.40	1.25	0.81	0.00	0.44	0.67
time (sec)	N/A	0.210	0.365	0.773	0.696	0.423	0.000	0.469	1.843

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	102	192	368	177	0	0	251
normalized size	1	1.00	0.60	1.12	2.15	1.04	0.00	0.00	1.47
time (sec)	N/A	0.274	1.852	1.936	0.337	0.460	0.000	0.000	5.760

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	91	167	321	161	0	0	219
normalized size	1	1.00	0.61	1.11	2.14	1.07	0.00	0.00	1.46
time (sec)	N/A	0.241	1.396	1.658	0.564	0.489	0.000	0.000	5.608

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	142	227	145	0	0	187
normalized size	1	1.00	0.87	1.51	2.41	1.54	0.00	0.00	1.99
time (sec)	N/A	0.149	0.750	1.508	0.405	0.476	0.000	0.000	6.503

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	75	150	99	0	0	155
normalized size	1	1.00	0.70	1.03	2.05	1.36	0.00	0.00	2.12
time (sec)	N/A	0.108	0.152	0.874	0.540	0.450	0.000	0.000	5.213

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	84	108	103	0	0	113
normalized size	1	1.00	0.74	1.38	1.77	1.69	0.00	0.00	1.85
time (sec)	N/A	0.103	0.107	1.104	0.379	0.464	0.000	0.000	3.786

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	220	116	225	108	0	0	77
normalized size	1	1.00	2.97	1.57	3.04	1.46	0.00	0.00	1.04
time (sec)	N/A	0.099	1.865	0.640	0.371	0.445	0.000	0.000	1.907

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	109	91	201	128	0	0	63
normalized size	1	1.00	1.22	1.02	2.26	1.44	0.00	0.00	0.71
time (sec)	N/A	0.128	0.088	0.789	0.352	0.471	0.000	0.000	1.779

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	189	83	0	23	22
normalized size	1	1.00	0.66	0.61	4.97	2.18	0.00	0.61	0.58
time (sec)	N/A	0.075	0.123	0.806	0.351	0.437	0.000	2.378	1.643

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	115	39	270	114	0	43	37
normalized size	1	1.00	1.44	0.49	3.38	1.42	0.00	0.54	0.46
time (sec)	N/A	0.150	0.440	0.801	0.351	0.467	0.000	0.390	1.621

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	141	52	269	140	0	60	67
normalized size	1	1.00	1.17	0.43	2.22	1.16	0.00	0.50	0.55
time (sec)	N/A	0.230	0.435	0.831	0.361	0.432	0.000	0.458	1.631

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	167	65	389	168	0	77	108
normalized size	1	1.00	1.02	0.40	2.39	1.03	0.00	0.47	0.66
time (sec)	N/A	0.315	0.675	0.899	0.355	0.453	0.000	0.488	1.691

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	122	242	443	209	0	0	316
normalized size	1	1.00	0.54	1.07	1.95	0.92	0.00	0.00	1.39
time (sec)	N/A	0.335	3.394	1.932	0.348	0.522	0.000	0.000	5.579

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	111	217	408	193	0	0	284
normalized size	1	1.00	0.54	1.05	1.98	0.94	0.00	0.00	1.38
time (sec)	N/A	0.302	2.540	1.911	0.349	0.500	0.000	0.000	5.518

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	192	368	177	0	0	252
normalized size	1	1.00	0.84	1.59	3.04	1.46	0.00	0.00	2.08
time (sec)	N/A	0.178	1.638	1.949	0.357	0.510	0.000	0.000	5.707

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	100	244	115	0	0	220
normalized size	1	1.00	0.60	1.00	2.44	1.15	0.00	0.00	2.20
time (sec)	N/A	0.131	0.250	1.141	0.346	0.475	0.000	0.000	5.560

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	142	227	145	0	0	188
normalized size	1	1.00	0.86	1.51	2.41	1.54	0.00	0.00	2.00
time (sec)	N/A	0.148	0.807	1.559	0.349	0.490	0.000	0.000	6.265

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	107	133	117	0	0	146
normalized size	1	1.00	0.81	1.24	1.55	1.36	0.00	0.00	1.70
time (sec)	N/A	0.156	0.600	1.118	0.339	0.487	0.000	0.000	4.792

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	287	166	387	125	0	0	105
normalized size	1	1.00	2.87	1.66	3.87	1.25	0.00	0.00	1.05
time (sec)	N/A	0.129	2.643	0.698	0.345	0.487	0.000	0.000	3.172

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	402	140	349	165	0	0	93
normalized size	1	1.00	3.38	1.18	2.93	1.39	0.00	0.00	0.78
time (sec)	N/A	0.183	3.453	0.688	0.357	0.488	0.000	0.000	2.043

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	139	113	309	176	0	0	78
normalized size	1	1.00	1.05	0.86	2.34	1.33	0.00	0.00	0.59
time (sec)	N/A	0.214	0.115	0.917	0.363	0.488	0.000	0.000	1.778

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	356	111	0	23	22
normalized size	1	1.00	0.66	0.61	9.37	2.92	0.00	0.61	0.58
time (sec)	N/A	0.072	0.155	0.923	0.371	0.454	0.000	0.418	1.811

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	141	39	357	140	0	43	37
normalized size	1	1.00	1.76	0.49	4.46	1.75	0.00	0.54	0.46
time (sec)	N/A	0.152	0.403	0.953	0.368	0.452	0.000	0.885	1.621

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	167	52	518	168	0	60	67
normalized size	1	1.00	1.38	0.43	4.28	1.39	0.00	0.50	0.55
time (sec)	N/A	0.230	0.700	0.960	0.380	0.500	0.000	1.501	1.859

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	193	65	517	194	0	77	108
normalized size	1	1.00	1.19	0.40	3.19	1.20	0.00	0.48	0.67
time (sec)	N/A	0.317	0.592	0.962	0.388	0.459	0.000	0.714	1.716

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	1036	212	591	153	0	0	112
normalized size	1	1.00	8.56	1.75	4.88	1.26	0.00	0.00	0.93
time (sec)	N/A	0.156	6.423	0.746	0.347	0.519	0.000	0.000	1.854

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	287	164	386	140	0	0	96
normalized size	1	1.00	2.87	1.64	3.86	1.40	0.00	0.00	0.96
time (sec)	N/A	0.130	2.652	0.705	0.346	0.459	0.000	0.000	1.660

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	220	116	224	119	0	0	77
normalized size	1	1.00	2.97	1.57	3.03	1.61	0.00	0.00	1.04
time (sec)	N/A	0.103	1.657	0.705	0.347	0.491	0.000	0.000	1.636

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	77	61	101	70	0	0	31
normalized size	1	1.00	1.88	1.49	2.46	1.71	0.00	0.00	0.76
time (sec)	N/A	0.053	0.064	0.776	0.331	0.455	0.000	0.000	1.577

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	18	18	0	19	18
normalized size	1	1.00	1.00	0.00	1.12	1.12	0.00	1.19	1.12
time (sec)	N/A	0.089	0.027	180.000	0.324	0.442	0.000	0.418	1.556

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	48	77	50	0	59	50
normalized size	1	1.00	1.37	0.81	1.31	0.85	0.00	1.00	0.85
time (sec)	N/A	0.137	0.482	0.784	0.339	0.431	0.000	0.310	1.642

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	107	61	97	74	0	73	63
normalized size	1	1.00	1.37	0.78	1.24	0.95	0.00	0.94	0.81
time (sec)	N/A	0.179	0.850	1.029	0.340	0.442	0.000	0.360	1.722

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	145	74	117	102	0	87	83
normalized size	1	1.00	1.21	0.62	0.98	0.85	0.00	0.72	0.69
time (sec)	N/A	0.229	0.902	0.964	0.338	0.439	0.000	0.351	2.131

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	380	234	765	210	0	0	170
normalized size	1	1.00	2.32	1.43	4.66	1.28	0.00	0.00	1.04
time (sec)	N/A	0.247	1.198	0.795	0.353	0.472	0.000	0.000	1.734

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	349	186	531	197	0	0	136
normalized size	1	1.00	2.33	1.24	3.54	1.31	0.00	0.00	0.91
time (sec)	N/A	0.215	1.970	0.743	0.354	0.464	0.000	0.000	1.681

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	485	136	341	178	0	0	104
normalized size	1	1.00	4.08	1.14	2.87	1.50	0.00	0.00	0.87
time (sec)	N/A	0.186	4.322	0.678	0.342	0.492	0.000	0.000	1.649

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	109	89	196	138	0	0	46
normalized size	1	1.00	1.24	1.01	2.23	1.57	0.00	0.00	0.52
time (sec)	N/A	0.131	0.111	0.823	0.329	0.627	0.000	0.000	1.612

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	94	53	0	21	20
normalized size	1	1.00	0.64	0.58	2.61	1.47	0.00	0.58	0.56
time (sec)	N/A	0.048	0.088	0.800	0.332	0.448	0.000	0.298	1.567

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	83	48	76	49	0	72	61
normalized size	1	1.00	1.41	0.81	1.29	0.83	0.00	1.22	1.03
time (sec)	N/A	0.136	0.573	0.645	0.334	0.470	0.000	0.509	1.618

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	31	50	0	33	28
normalized size	1	1.00	0.87	0.00	0.82	1.32	0.00	0.87	0.74
time (sec)	N/A	0.098	0.052	180.000	0.335	0.448	0.000	0.320	1.565

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	147	76	121	109	0	101	76
normalized size	1	1.00	1.84	0.95	1.51	1.36	0.00	1.26	0.95
time (sec)	N/A	0.146	0.964	0.857	0.339	0.447	0.000	0.356	2.009

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	179	87	140	120	0	115	89
normalized size	1	1.00	1.83	0.89	1.43	1.22	0.00	1.17	0.91
time (sec)	N/A	0.189	0.930	0.845	0.341	0.453	0.000	0.440	2.668

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	211	102	161	163	0	129	102
normalized size	1	1.00	1.50	0.72	1.14	1.16	0.00	0.91	0.72
time (sec)	N/A	0.246	1.432	0.979	0.342	0.461	0.000	1.383	4.237

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	406	256	935	263	0	0	193
normalized size	1	1.00	1.89	1.19	4.35	1.22	0.00	0.00	0.90
time (sec)	N/A	0.336	2.166	0.822	0.383	0.484	0.000	0.000	1.653

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	380	208	680	250	0	0	159
normalized size	1	1.00	1.97	1.08	3.52	1.30	0.00	0.00	0.82
time (sec)	N/A	0.309	1.408	0.800	0.369	0.473	0.000	0.000	1.642

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	826	160	470	231	0	0	126
normalized size	1	1.00	5.04	0.98	2.87	1.41	0.00	0.00	0.77
time (sec)	N/A	0.275	6.322	0.685	0.358	0.482	0.000	0.000	1.629

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	139	111	304	192	0	0	61
normalized size	1	1.00	1.06	0.85	2.32	1.47	0.00	0.00	0.47
time (sec)	N/A	0.215	0.122	0.830	0.350	0.483	0.000	0.000	1.646

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	185	82	0	23	22
normalized size	1	1.00	0.66	0.61	4.87	2.16	0.00	0.61	0.58
time (sec)	N/A	0.074	0.126	0.836	0.337	0.421	0.000	1.798	1.593

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	87	37	115	79	0	39	35
normalized size	1	1.00	1.14	0.49	1.51	1.04	0.00	0.51	0.46
time (sec)	N/A	0.099	0.345	0.800	0.339	0.434	0.000	0.398	1.576

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	109	61	95	76	0	91	74
normalized size	1	1.00	1.40	0.78	1.22	0.97	0.00	1.17	0.95
time (sec)	N/A	0.181	0.821	0.705	0.331	0.471	0.000	0.370	1.624

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	147	76	120	109	0	108	111
normalized size	1	1.00	1.84	0.95	1.50	1.36	0.00	1.35	1.39
time (sec)	N/A	0.142	0.825	0.868	0.340	0.439	0.000	0.391	1.708

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	0	41	76	0	44	38
normalized size	1	1.00	0.85	0.00	0.69	1.29	0.00	0.75	0.64
time (sec)	N/A	0.109	0.075	180.000	0.377	0.433	0.000	0.696	1.628

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	211	102	159	163	0	135	129
normalized size	1	1.00	2.13	1.03	1.61	1.65	0.00	1.36	1.30
time (sec)	N/A	0.149	1.406	0.926	0.350	0.424	0.000	1.005	2.536

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	257	115	181	190	0	150	109
normalized size	1	1.00	2.14	0.96	1.51	1.58	0.00	1.25	0.91
time (sec)	N/A	0.204	1.607	0.972	0.338	0.454	0.000	0.553	2.882

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	289	128	200	217	0	164	120
normalized size	1	1.00	1.78	0.79	1.23	1.34	0.00	1.01	0.74
time (sec)	N/A	0.260	2.320	0.998	0.339	0.450	0.000	0.529	3.107

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	86	83	0	119	0	111	483
normalized size	1	1.00	0.53	0.51	0.00	0.73	0.00	0.68	2.96
time (sec)	N/A	0.278	0.910	1.223	0.000	0.456	0.000	3.363	9.131

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	76	73	0	105	0	86	384
normalized size	1	1.00	0.62	0.60	0.00	0.86	0.00	0.70	3.15
time (sec)	N/A	0.200	0.486	1.177	0.000	0.448	0.000	3.409	6.004

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	63	0	82	0	58	120
normalized size	1	1.00	0.79	0.78	0.00	1.01	0.00	0.72	1.48
time (sec)	N/A	0.128	0.275	1.211	0.000	0.426	0.000	2.543	5.364

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	51	53	0	65	0	32	87
normalized size	1	1.00	1.31	1.36	0.00	1.67	0.00	0.82	2.23
time (sec)	N/A	0.059	0.184	1.340	0.000	0.458	0.000	2.393	2.749

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	132	85	0	272	0	0	-1
normalized size	1	1.00	1.71	1.10	0.00	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.644	1.595	0.000	0.522	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	246	164	0	342	0	0	-1
normalized size	1	1.00	3.24	2.16	0.00	4.50	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.146	1.375	0.000	0.535	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	309	308	0	405	0	0	-1
normalized size	1	1.00	2.73	2.73	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.155	1.125	1.422	0.000	0.514	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	88	85	0	147	0	113	606
normalized size	1	1.00	0.51	0.50	0.00	0.86	0.00	0.66	3.54
time (sec)	N/A	0.448	1.641	1.654	0.000	0.453	0.000	4.139	14.397

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	78	75	0	131	0	88	503
normalized size	1	1.00	0.61	0.59	0.00	1.02	0.00	0.69	3.93
time (sec)	N/A	0.327	1.242	1.639	0.000	0.436	0.000	2.737	7.990

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	0	105	0	60	384
normalized size	1	1.00	0.78	0.76	0.00	1.24	0.00	0.71	4.52
time (sec)	N/A	0.208	0.782	1.576	0.000	0.442	0.000	3.403	6.139

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	0	84	0	34	93
normalized size	1	1.00	1.34	1.34	0.00	2.05	0.00	0.83	2.27
time (sec)	N/A	0.097	0.441	1.805	0.000	0.446	0.000	2.825	5.725

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	123	173	145	0	343	0	0	-1
normalized size	1	1.05	1.48	1.24	0.00	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.929	1.751	0.000	0.533	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	124	184	144	0	372	0	0	-1
normalized size	1	1.10	1.63	1.27	0.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.228	1.530	1.701	0.000	0.518	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	130	359	230	0	429	0	0	-1
normalized size	1	1.11	3.07	1.97	0.00	3.67	0.00	0.00	-0.01
time (sec)	N/A	0.238	2.630	1.898	0.000	0.557	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	398	402	0	517	0	0	-1
normalized size	1	1.00	2.43	2.45	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.282	5.897	2.463	0.000	0.582	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	88	85	0	163	0	113	710
normalized size	1	1.00	0.51	0.50	0.00	0.95	0.00	0.66	4.15
time (sec)	N/A	0.444	2.540	1.955	0.000	0.472	0.000	4.903	14.671

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	78	75	0	147	0	88	607
normalized size	1	1.00	0.61	0.59	0.00	1.15	0.00	0.69	4.74
time (sec)	N/A	0.323	1.562	1.835	0.000	0.430	0.000	3.834	13.711

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	0	119	0	60	471
normalized size	1	1.00	0.78	0.76	0.00	1.40	0.00	0.71	5.54
time (sec)	N/A	0.209	1.111	1.805	0.000	0.432	0.000	3.134	9.194

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	0	97	0	34	375
normalized size	1	1.00	1.34	1.34	0.00	2.37	0.00	0.83	9.15
time (sec)	N/A	0.096	0.709	1.980	0.000	0.423	0.000	3.023	5.616

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	185	206	0	377	0	0	-1
normalized size	1	1.00	1.13	1.26	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.325	1.535	1.908	0.000	0.512	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	324	157	0	432	0	0	-1
normalized size	1	1.00	1.93	0.93	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.336	3.099	1.731	0.000	0.540	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	263	206	0	441	0	0	-1
normalized size	1	1.00	1.51	1.18	0.00	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.352	4.574	1.884	0.000	0.581	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	86	83	163	88	0	112	164
normalized size	1	1.00	0.61	0.58	1.15	0.62	0.00	0.79	1.15
time (sec)	N/A	0.234	0.738	1.759	0.691	0.445	0.000	2.665	6.331

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	74	73	137	77	0	87	125
normalized size	1	1.00	0.69	0.68	1.27	0.71	0.00	0.81	1.16
time (sec)	N/A	0.191	0.435	1.886	0.638	0.430	0.000	2.507	4.229

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	63	110	50	0	62	77
normalized size	1	1.00	0.75	0.88	1.53	0.69	0.00	0.86	1.07
time (sec)	N/A	0.153	0.245	1.851	0.858	0.428	0.000	2.730	2.367

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	43	84	45	0	0	40
normalized size	1	1.00	0.74	1.10	2.15	1.15	0.00	0.00	1.03
time (sec)	N/A	0.107	0.137	2.123	0.747	0.441	0.000	0.000	1.770

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	155	107	0	269	0	0	-1
normalized size	1	1.00	1.74	1.20	0.00	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.479	2.161	0.000	0.512	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	183	266	0	329	0	0	-1
normalized size	1	1.00	1.50	2.18	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.211	1.417	2.135	0.000	0.512	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	306	471	0	401	0	0	-1
normalized size	1	1.00	1.96	3.02	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.255	2.933	2.317	0.000	0.552	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	84	85	188	103	0	125	188
normalized size	1	1.00	0.54	0.55	1.21	0.66	0.00	0.81	1.21
time (sec)	N/A	0.317	0.940	1.633	0.495	0.430	0.000	3.378	6.017

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	68	75	163	82	0	102	136
normalized size	1	1.00	0.55	0.61	1.33	0.67	0.00	0.83	1.11
time (sec)	N/A	0.274	0.394	1.667	0.615	0.451	0.000	2.768	5.542

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	60	53	110	72	0	62	134
normalized size	1	1.00	0.67	0.60	1.24	0.81	0.00	0.70	1.51
time (sec)	N/A	0.219	0.260	1.658	0.674	0.449	0.000	3.681	5.233

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	53	109	60	0	0	94
normalized size	1	1.00	1.34	1.29	2.66	1.46	0.00	0.00	2.29
time (sec)	N/A	0.100	0.153	1.848	0.717	0.495	0.000	0.000	5.295

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	259	131	0	331	0	0	-1
normalized size	1	1.00	1.88	0.95	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.271	2.080	1.825	0.000	0.535	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	365	320	0	369	0	0	-1
normalized size	1	1.00	2.16	1.89	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.343	1.392	1.990	0.000	0.533	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	434	551	0	487	0	0	-1
normalized size	1	1.00	2.14	2.71	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.389	2.334	2.027	0.000	0.572	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	78	85	214	109	0	130	492
normalized size	1	1.00	0.46	0.50	1.27	0.64	0.00	0.77	2.91
time (sec)	N/A	0.402	0.633	1.812	0.439	0.430	0.000	4.465	10.238

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	74	65	189	104	0	93	456
normalized size	1	1.00	0.55	0.48	1.40	0.77	0.00	0.69	3.38
time (sec)	N/A	0.348	0.346	1.820	0.442	0.446	0.000	3.154	7.312

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	60	63	163	88	0	60	446
normalized size	1	1.00	0.68	0.72	1.85	1.00	0.00	0.68	5.07
time (sec)	N/A	0.222	0.319	1.815	0.436	0.452	0.000	3.489	7.719

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	136	74	0	0	441
normalized size	1	1.00	1.34	1.34	3.32	1.80	0.00	0.00	10.76
time (sec)	N/A	0.102	0.154	1.906	0.440	0.446	0.000	0.000	7.590

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	225	155	0	401	0	0	-1
normalized size	1	1.00	1.24	0.86	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.394	1.533	2.054	0.000	0.538	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	398	370	0	483	0	0	-1
normalized size	1	1.00	1.88	1.75	0.00	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.477	6.467	2.016	0.000	0.533	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	468	631	0	461	0	0	-1
normalized size	1	1.00	1.90	2.57	0.00	1.87	0.00	0.00	-0.00
time (sec)	N/A	0.532	6.663	2.198	0.000	0.616	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	82	638	93	0	0	136
normalized size	1	1.00	2.02	1.91	14.84	2.16	0.00	0.00	3.16
time (sec)	N/A	0.133	0.466	1.965	0.832	0.427	0.000	0.000	3.800

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	73	72	298	78	0	0	78
normalized size	1	1.00	1.70	1.67	6.93	1.81	0.00	0.00	1.81
time (sec)	N/A	0.133	0.306	1.976	0.779	0.422	0.000	0.000	2.637

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	56	62	55	56	0	0	47
normalized size	1	1.00	1.37	1.51	1.34	1.37	0.00	0.00	1.15
time (sec)	N/A	0.120	0.170	1.949	0.848	0.451	0.000	0.000	1.942

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	99	141	92	0	0	0	-1
normalized size	1	1.00	1.94	2.76	1.80	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.125	0.839	2.019	0.899	0.555	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	60	514	79	0	0	118
normalized size	1	1.00	1.48	1.43	12.24	1.88	0.00	0.00	2.81
time (sec)	N/A	0.139	0.247	2.050	1.327	0.462	0.000	0.000	2.996

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	69	70	758	106	0	0	203
normalized size	1	1.00	1.60	1.63	17.63	2.47	0.00	0.00	4.72
time (sec)	N/A	0.137	0.390	2.117	1.224	0.437	0.000	0.000	6.610

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	108	103	1680	112	0	0	294
normalized size	1	1.00	1.21	1.16	18.88	1.26	0.00	0.00	3.30
time (sec)	N/A	0.275	1.059	1.955	0.907	0.472	0.000	0.000	6.149

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	97	93	1105	112	0	0	195
normalized size	1	1.00	1.09	1.04	12.42	1.26	0.00	0.00	2.19
time (sec)	N/A	0.276	0.586	2.441	0.856	0.449	0.000	0.000	5.499

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	83	550	82	0	0	108
normalized size	1	1.00	0.88	0.93	6.18	0.92	0.00	0.00	1.21
time (sec)	N/A	0.274	0.427	2.172	0.993	0.432	0.000	0.000	3.145

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	73	73	56	76	0	0	76
normalized size	1	1.00	1.70	1.70	1.30	1.77	0.00	0.00	1.77
time (sec)	N/A	0.130	0.277	2.072	0.773	0.430	0.000	0.000	2.587

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	174	161	275	0	0	0	-1
normalized size	1	1.00	1.83	1.69	2.89	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	1.416	2.061	1.026	0.672	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	134	246	122	0	0	0	-1
normalized size	1	1.00	1.35	2.48	1.23	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.695	1.894	0.742	0.857	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	63	73	533	95	0	0	165
normalized size	1	1.00	1.50	1.74	12.69	2.26	0.00	0.00	3.93
time (sec)	N/A	0.149	0.482	1.936	0.976	0.442	0.000	0.000	4.830

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	80	83	1559	133	0	0	273
normalized size	1	1.00	0.91	0.94	17.72	1.51	0.00	0.00	3.10
time (sec)	N/A	0.301	0.556	2.182	2.074	0.419	0.000	0.000	7.031

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	93	2608	158	0	0	340
normalized size	1	1.00	0.98	1.01	28.35	1.72	0.00	0.00	3.70
time (sec)	N/A	0.285	0.828	2.130	7.889	0.454	0.000	0.000	7.620

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	100	103	3906	184	0	0	407
normalized size	1	1.00	1.09	1.12	42.46	2.00	0.00	0.00	4.42
time (sec)	N/A	0.287	1.219	2.174	38.317	0.428	0.000	0.000	7.680

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	113	105	2454	152	0	0	307
normalized size	1	1.00	0.84	0.78	18.31	1.13	0.00	0.00	2.29
time (sec)	N/A	0.421	1.298	2.427	0.606	0.476	0.000	0.000	6.234

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	92	95	1526	106	0	0	215
normalized size	1	1.00	0.69	0.71	11.39	0.79	0.00	0.00	1.60
time (sec)	N/A	0.424	0.779	2.320	0.586	0.464	0.000	0.000	5.609

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	85	1106	112	0	0	195
normalized size	1	1.00	1.08	0.96	12.43	1.26	0.00	0.00	2.19
time (sec)	N/A	0.275	0.594	2.184	0.560	0.437	0.000	0.000	5.442

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	88	75	58	93	0	0	136
normalized size	1	1.00	2.05	1.74	1.35	2.16	0.00	0.00	3.16
time (sec)	N/A	0.128	0.468	2.299	0.457	0.444	0.000	0.000	3.610

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	328	189	737	0	0	0	-1
normalized size	1	1.00	2.33	1.34	5.23	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.405	6.604	2.470	0.592	0.770	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	188	289	2035	0	0	0	-1
normalized size	1	1.00	1.30	1.99	14.03	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.420	1.496	2.196	0.738	0.970	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	182	364	169	0	0	0	-1
normalized size	1	1.00	1.26	2.51	1.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.431	1.343	2.293	0.453	1.279	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	76	75	1815	126	0	0	199
normalized size	1	1.00	1.81	1.79	43.21	3.00	0.00	0.00	4.74
time (sec)	N/A	0.146	0.560	1.956	0.583	0.441	0.000	0.000	6.040

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	85	2719	166	0	0	350
normalized size	1	1.00	1.05	0.97	30.90	1.89	0.00	0.00	3.98
time (sec)	N/A	0.298	0.827	1.990	4.434	0.447	0.000	0.000	6.785

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	102	95	4108	194	0	0	419
normalized size	1	1.00	0.77	0.71	30.89	1.46	0.00	0.00	3.15
time (sec)	N/A	0.456	1.238	2.034	23.290	0.436	0.000	0.000	7.169

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	141	165	737	0	0	0	-1
normalized size	1	1.00	1.01	1.19	5.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.409	1.484	2.062	0.603	0.690	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	173	149	276	0	0	0	-1
normalized size	1	1.00	1.84	1.59	2.94	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	1.834	1.893	0.585	0.639	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	140	116	64	0	0	0	-1
normalized size	1	1.00	2.80	2.32	1.28	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.122	0.428	1.985	0.454	0.609	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	94	85	44	204	0	0	-1
normalized size	1	1.00	2.00	1.81	0.94	4.34	0.00	0.00	-0.02
time (sec)	N/A	0.135	0.788	1.899	0.554	0.565	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	131	406	382	0	0	-1
normalized size	1	1.00	0.83	1.38	4.27	4.02	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.897	1.981	0.592	0.549	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	91	170	1201	456	0	0	-1
normalized size	1	1.00	0.65	1.21	8.58	3.26	0.00	0.00	-0.01
time (sec)	N/A	0.430	0.829	2.284	0.659	0.551	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	183	235	2035	0	0	0	-1
normalized size	1	1.00	1.29	1.65	14.33	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.424	1.047	1.873	0.749	0.928	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	132	193	99	0	0	0	-1
normalized size	1	1.00	1.39	2.03	1.04	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	1.064	1.931	0.468	0.782	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	73	54	78	0	0	50
normalized size	1	1.00	1.00	1.74	1.29	1.86	0.00	0.00	1.19
time (sec)	N/A	0.134	0.194	2.069	0.466	0.445	0.000	0.000	2.548

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	157	123	397	380	0	0	-1
normalized size	1	1.00	1.65	1.29	4.18	4.00	0.00	0.00	-0.01
time (sec)	N/A	0.279	1.401	2.104	0.578	0.555	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	69	133	567	402	0	0	-1
normalized size	1	1.00	0.66	1.28	5.45	3.87	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.796	2.045	0.584	0.595	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	122	211	0	544	0	0	-1
normalized size	1	1.00	0.84	1.45	0.00	3.73	0.00	0.00	-0.01
time (sec)	N/A	0.341	1.584	2.166	0.000	0.594	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	178	281	133	0	0	0	-1
normalized size	1	1.00	1.23	1.94	0.92	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.436	1.395	2.075	0.468	1.096	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	68	75	98	95	0	0	119
normalized size	1	1.00	1.62	1.79	2.33	2.26	0.00	0.00	2.83
time (sec)	N/A	0.147	0.313	2.059	0.472	0.439	0.000	0.000	3.512

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	83	58	104	0	0	120
normalized size	1	1.00	1.65	1.93	1.35	2.42	0.00	0.00	2.79
time (sec)	N/A	0.134	0.257	2.169	0.460	0.452	0.000	0.000	3.248

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	91	164	1191	456	0	0	-1
normalized size	1	1.00	0.65	1.17	8.51	3.26	0.00	0.00	-0.01
time (sec)	N/A	0.424	1.040	2.261	0.640	0.561	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	204	0	536	0	0	-1
normalized size	1	1.00	0.89	1.40	0.00	3.67	0.00	0.00	-0.01
time (sec)	N/A	0.344	1.392	2.211	0.000	0.582	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	84	173	1659	482	0	0	-1
normalized size	1	1.00	0.52	1.08	10.37	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.202	1.379	2.569	0.822	0.575	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.347	2.956	0.000	0.452	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.248	2.340	0.000	0.415	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.158	1.759	0.000	0.445	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.569	1.677	0.000	0.460	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	2.591	1.974	0.000	0.452	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	0	0	228	191	0	0	-1
normalized size	1	1.00	0.00	0.00	1.42	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.380	15.646	1.766	0.477	0.448	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	A	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	0	171	112	0	0	154
normalized size	1	1.00	0.00	0.00	1.71	1.12	0.00	0.00	1.54
time (sec)	N/A	0.222	38.436	1.634	0.465	0.437	0.000	0.000	3.592

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	163	0	114	70	0	0	-1
normalized size	1	1.00	3.54	0.00	2.48	1.52	0.00	0.00	-0.02
time (sec)	N/A	0.102	18.895	1.861	0.462	0.473	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.653	1.676	0.000	0.476	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.713	1.588	0.000	0.455	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	2.142	1.657	0.000	0.503	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	321	0	156	120	0	0	290
normalized size	1	1.00	1.90	0.00	0.92	0.71	0.00	0.00	1.72
time (sec)	N/A	0.374	9.085	3.883	0.661	0.526	0.000	0.000	10.646

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	250	0	107	93	0	0	145
normalized size	1	1.00	2.40	0.00	1.03	0.89	0.00	0.00	1.39
time (sec)	N/A	0.225	3.050	3.806	0.834	0.457	0.000	0.000	8.002

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	208	0	62	72	0	0	105
normalized size	1	1.00	4.43	0.00	1.32	1.53	0.00	0.00	2.23
time (sec)	N/A	0.101	1.178	3.817	0.597	0.487	0.000	0.000	2.909

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	257	0	0	0	0	0	-1
normalized size	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.420	2.547	0.000	0.439	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	2.262	3.685	0.000	0.466	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	2.708	3.990	0.000	0.448	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	68	130	172	131	0	0	175
normalized size	1	1.00	0.65	1.24	1.64	1.25	0.00	0.00	1.67
time (sec)	N/A	0.193	0.299	1.400	0.459	0.431	0.000	0.000	6.320

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	57	107	160	117	0	0	146
normalized size	1	1.00	0.66	1.24	1.86	1.36	0.00	0.00	1.70
time (sec)	N/A	0.156	0.179	1.369	0.476	0.464	0.000	0.000	4.923

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	36	36	35	51	16	15
normalized size	1	1.00	1.00	2.12	2.12	2.06	3.00	0.94	0.88
time (sec)	N/A	0.071	0.018	0.937	0.555	0.426	2.303	1.389	1.704

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	154	104	194	105	0	0	71
normalized size	1	1.00	2.75	1.86	3.46	1.88	0.00	0.00	1.27
time (sec)	N/A	0.108	0.619	0.583	0.413	0.438	0.000	0.000	1.745

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	335	81	144	123	0	0	44
normalized size	1	1.00	4.79	1.16	2.06	1.76	0.00	0.00	0.63
time (sec)	N/A	0.161	0.456	0.712	0.607	0.466	0.000	0.000	1.686

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	43	37	115	78	0	39	35
normalized size	1	1.00	0.50	0.43	1.34	0.91	0.00	0.45	0.41
time (sec)	N/A	0.163	0.173	0.757	0.512	0.404	0.000	0.817	1.700

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	7087	0	0	0	0	0	-1
normalized size	1	1.00	50.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	30.614	3.000	0.000	0.438	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.334	2.615	0.000	0.444	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	3396	0	0	0	0	0	-1
normalized size	1	1.00	18.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	16.808	4.055	0.000	0.486	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	13.329	2.516	0.000	0.462	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	143	162	236	979	340	0	0	-1
normalized size	1	1.38	1.56	2.27	9.41	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.278	1.534	2.293	1.224	0.583	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	116	73	204	0	260	0	0	-1
normalized size	1	1.43	0.90	2.52	0.00	3.21	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.471	1.684	0.000	0.519	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	213	724	317	1310	462	0	0	-1
normalized size	1	1.52	5.17	2.26	9.36	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.281	6.465	2.876	1.058	0.507	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	150	236	152	536	330	0	0	-1
normalized size	1	1.29	2.03	1.31	4.62	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.303	2.982	2.698	1.094	0.447	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	242	328	294	1400	569	0	0	-1
normalized size	1	1.35	1.83	1.64	7.82	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.358	2.285	2.954	1.216	0.601	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	94	142	56	255	0	0	-1
normalized size	1	1.00	2.04	3.09	1.22	5.54	0.00	0.00	-0.02
time (sec)	N/A	0.255	0.777	1.977	1.090	0.551	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	98	414	0	357	0	0	-1
normalized size	1	1.00	1.51	6.37	0.00	5.49	0.00	0.00	-0.02
time (sec)	N/A	0.159	0.264	1.664	0.000	0.694	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	153	431	379	281	0	0	361
normalized size	1	1.00	0.65	1.83	1.61	1.19	0.00	0.00	1.53
time (sec)	N/A	0.437	1.742	1.614	0.382	0.442	0.000	0.000	5.508

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	103	290	266	211	0	0	255
normalized size	1	1.00	0.60	1.70	1.56	1.23	0.00	0.00	1.49
time (sec)	N/A	0.295	0.681	1.329	0.620	0.475	0.000	0.000	5.337

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	75	174	165	150	0	0	196
normalized size	1	1.00	0.69	1.61	1.53	1.39	0.00	0.00	1.81
time (sec)	N/A	0.165	0.432	1.111	0.470	0.480	0.000	0.000	4.761

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	88	96	0	0	111
normalized size	1	1.00	1.34	1.54	1.57	1.71	0.00	0.00	1.98
time (sec)	N/A	0.072	0.030	0.961	0.385	0.460	0.000	0.000	2.572

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	107	135	0	255	0	0	195
normalized size	1	1.00	1.55	1.96	0.00	3.70	0.00	0.00	2.83
time (sec)	N/A	0.144	0.191	0.703	0.000	0.534	0.000	0.000	2.159

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	105	0	357	0	143	85
normalized size	1	1.00	0.95	1.33	0.00	4.52	0.00	1.81	1.08
time (sec)	N/A	0.139	0.229	0.717	0.000	0.469	0.000	0.343	1.914

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	167	178	0	736	0	274	171
normalized size	1	1.00	1.27	1.36	0.00	5.62	0.00	2.09	1.31
time (sec)	N/A	0.270	1.243	0.745	0.000	0.517	0.000	0.419	3.776

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	247	271	0	1278	0	468	321
normalized size	1	1.00	1.31	1.43	0.00	6.76	0.00	2.48	1.70
time (sec)	N/A	0.454	3.353	0.779	0.000	0.537	0.000	0.386	5.283

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	371	460	602	683	387	0	0	484
normalized size	1	1.13	1.41	1.84	2.09	1.18	0.00	0.00	1.48
time (sec)	N/A	0.424	2.034	1.964	0.475	0.466	0.000	0.000	5.354

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	277	326	420	469	294	0	0	394
normalized size	1	1.14	1.35	1.74	1.94	1.21	0.00	0.00	1.63
time (sec)	N/A	0.343	1.395	1.624	0.407	0.454	0.000	0.000	5.485

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	234	479	268	324	209	0	0	237
normalized size	1	1.33	2.72	1.52	1.84	1.19	0.00	0.00	1.35
time (sec)	N/A	0.262	0.998	1.420	0.628	0.450	0.000	0.000	5.464

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	481	141	167	138	0	0	161
normalized size	1	1.00	4.67	1.37	1.62	1.34	0.00	0.00	1.56
time (sec)	N/A	0.117	6.362	1.156	0.531	0.422	0.000	0.000	4.525

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	B	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	208	329	291	0	398	0	0	529
normalized size	1	2.19	3.46	3.06	0.00	4.19	0.00	0.00	5.57
time (sec)	N/A	0.245	2.019	0.622	0.000	0.634	0.000	0.000	2.587

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	231	312	330	0	567	0	0	2563
normalized size	1	1.97	2.67	2.82	0.00	4.85	0.00	0.00	21.91
time (sec)	N/A	0.257	1.505	0.821	0.000	0.673	0.000	0.000	4.790

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	184	249	167	0	622	0	220	158
normalized size	1	1.42	1.92	1.28	0.00	4.78	0.00	1.69	1.22
time (sec)	N/A	0.197	1.245	0.772	0.000	0.505	0.000	3.136	3.574

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	268	211	228	0	1234	0	420	286
normalized size	1	1.26	0.99	1.07	0.00	5.79	0.00	1.97	1.34
time (sec)	N/A	0.283	4.780	0.826	0.000	0.548	0.000	1.106	5.092

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	330	322	352	0	1908	0	739	438
normalized size	1	1.20	1.17	1.28	0.00	6.91	0.00	2.68	1.59
time (sec)	N/A	0.557	9.059	0.921	0.000	0.589	0.000	0.650	5.237

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	333	380	523	701	337	0	0	411
normalized size	1	1.16	1.32	1.82	2.43	1.17	0.00	0.00	1.43
time (sec)	N/A	0.427	2.694	1.950	0.577	0.459	0.000	0.000	5.238

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	273	433	342	459	245	0	0	287
normalized size	1	1.06	1.68	1.33	1.79	0.95	0.00	0.00	1.12
time (sec)	N/A	0.300	2.692	1.721	0.433	0.470	0.000	0.000	5.518

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	273	188	262	161	0	0	203
normalized size	1	1.00	2.18	1.50	2.10	1.29	0.00	0.00	1.62
time (sec)	N/A	0.153	1.368	1.390	0.417	0.434	0.000	0.000	5.307

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	257	419	491	0	532	0	0	1902
normalized size	1	1.68	2.74	3.21	0.00	3.48	0.00	0.00	12.43
time (sec)	N/A	0.325	2.414	0.666	0.000	0.893	0.000	0.000	2.892

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	274	455	548	0	859	0	0	3135
normalized size	1	1.70	2.83	3.40	0.00	5.34	0.00	0.00	19.47
time (sec)	N/A	0.355	4.214	0.695	0.000	0.924	0.000	0.000	5.274

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	301	393	768	0	1176	0	0	4131
normalized size	1	1.60	2.09	4.09	0.00	6.26	0.00	0.00	21.97
time (sec)	N/A	0.394	3.667	0.872	0.000	0.950	0.000	0.000	8.500

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	227	398	227	0	1012	0	320	264
normalized size	1	1.28	2.24	1.28	0.00	5.69	0.00	1.80	1.48
time (sec)	N/A	0.223	3.577	0.960	0.000	0.558	0.000	0.525	4.985

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	327	274	303	0	1714	0	626	385
normalized size	1	1.23	1.03	1.14	0.00	6.44	0.00	2.35	1.45
time (sec)	N/A	0.337	9.191	1.005	0.000	0.582	0.000	1.702	5.081

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	236	1243	596	596	297	0	0	211
normalized size	1	1.29	6.79	3.26	3.26	1.62	0.00	0.00	1.15
time (sec)	N/A	0.335	6.459	0.672	0.462	0.477	0.000	0.000	2.451

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	171	275	371	388	216	0	0	139
normalized size	1	1.46	2.35	3.17	3.32	1.85	0.00	0.00	1.19
time (sec)	N/A	0.254	2.658	0.680	0.529	0.449	0.000	0.000	1.938

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	125	237	196	223	155	0	0	85
normalized size	1	1.84	3.49	2.88	3.28	2.28	0.00	0.00	1.25
time (sec)	N/A	0.146	1.735	0.639	0.445	0.444	0.000	0.000	1.824

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	109	78	99	74	0	0	41
normalized size	1	1.00	2.53	1.81	2.30	1.72	0.00	0.00	0.95
time (sec)	N/A	0.085	0.266	0.744	0.382	0.477	0.000	0.000	1.727

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	134	160	74	0	353	0	114	110
normalized size	1	1.61	1.93	0.89	0.00	4.25	0.00	1.37	1.33
time (sec)	N/A	0.155	0.668	0.686	0.000	0.484	0.000	0.520	1.947

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	196	286	146	0	691	0	228	187
normalized size	1	1.35	1.97	1.01	0.00	4.77	0.00	1.57	1.29
time (sec)	N/A	0.253	3.248	0.787	0.000	0.485	0.000	0.301	2.036

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	268	1422	221	0	1331	0	374	379
normalized size	1	1.29	6.87	1.07	0.00	6.43	0.00	1.81	1.83
time (sec)	N/A	0.400	6.823	1.006	0.000	0.774	0.000	0.716	2.975

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	315	446	766	772	456	0	0	268
normalized size	1	1.22	1.73	2.97	2.99	1.77	0.00	0.00	1.04
time (sec)	N/A	0.438	4.028	1.023	0.364	0.502	0.000	0.000	1.973

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	249	310	514	536	361	0	0	193
normalized size	1	1.29	1.61	2.66	2.78	1.87	0.00	0.00	1.00
time (sec)	N/A	0.320	2.827	0.840	0.365	0.461	0.000	0.000	1.912

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	193	294	316	342	268	0	0	136
normalized size	1	1.45	2.21	2.38	2.57	2.02	0.00	0.00	1.02
time (sec)	N/A	0.233	1.703	0.679	0.350	0.471	0.000	0.000	1.880

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	149	181	170	195	155	0	0	89
normalized size	1	1.67	2.03	1.91	2.19	1.74	0.00	0.00	1.00
time (sec)	N/A	0.154	0.785	0.767	0.342	0.457	0.000	0.000	1.771

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	93	58	0	64	45
normalized size	1	1.00	1.17	0.92	1.43	0.89	0.00	0.98	0.69
time (sec)	N/A	0.087	0.222	0.724	0.349	0.450	0.000	0.288	1.714

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	183	209	122	0	598	0	258	168
normalized size	1	1.42	1.62	0.95	0.00	4.64	0.00	2.00	1.30
time (sec)	N/A	0.241	1.649	0.742	0.000	0.481	0.000	0.591	1.915

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	260	376	203	0	1242	0	490	314
normalized size	1	1.23	1.78	0.96	0.00	5.89	0.00	2.32	1.49
time (sec)	N/A	0.373	3.962	0.796	0.000	0.530	0.000	0.392	2.181

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	346	2220	280	0	2030	0	776	505
normalized size	1	1.22	7.82	0.99	0.00	7.15	0.00	2.73	1.78
time (sec)	N/A	0.558	7.347	0.954	0.000	0.609	0.000	0.530	2.282

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	405	1338	956	946	620	0	0	327
normalized size	1	1.12	3.69	2.63	2.61	1.71	0.00	0.00	0.90
time (sec)	N/A	0.539	6.719	0.835	0.393	0.501	0.000	0.000	1.908

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	329	439	679	689	502	0	0	252
normalized size	1	1.15	1.53	2.37	2.40	1.75	0.00	0.00	0.88
time (sec)	N/A	0.416	2.231	0.792	0.371	0.481	0.000	0.000	1.865

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	265	292	454	475	385	0	0	195
normalized size	1	1.29	1.42	2.21	2.32	1.88	0.00	0.00	0.95
time (sec)	N/A	0.280	2.393	0.707	0.365	0.458	0.000	0.000	1.816

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	193	295	286	307	248	0	0	147
normalized size	1	1.45	2.22	2.15	2.31	1.86	0.00	0.00	1.11
time (sec)	N/A	0.201	1.480	0.835	0.344	0.498	0.000	0.000	1.799

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	180	128	184	113	0	137	79
normalized size	1	1.00	1.57	1.11	1.60	0.98	0.00	1.19	0.69
time (sec)	N/A	0.163	0.480	0.834	0.346	0.421	0.000	0.349	1.833

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	115	93	0	80	66
normalized size	1	1.00	1.32	0.63	1.13	0.91	0.00	0.78	0.65
time (sec)	N/A	0.115	0.340	0.731	0.334	0.446	0.000	1.391	1.738

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	235	345	203	0	1001	0	489	228
normalized size	1	1.30	1.91	1.12	0.00	5.53	0.00	2.70	1.26
time (sec)	N/A	0.323	3.025	0.873	0.000	0.512	0.000	1.074	1.950

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	325	1772	284	0	1693	0	951	464
normalized size	1	1.13	6.15	0.99	0.00	5.88	0.00	3.30	1.61
time (sec)	N/A	0.486	7.094	0.936	0.000	0.539	0.000	0.966	2.120

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	414	1096	365	0	2677	0	1419	655
normalized size	1	1.12	2.98	0.99	0.00	7.27	0.00	3.86	1.78
time (sec)	N/A	0.706	7.828	0.970	0.000	0.616	0.000	0.616	2.359

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	102	301	0	307	0	0	-1
normalized size	1	1.00	1.67	4.93	0.00	5.03	0.00	0.00	-0.02
time (sec)	N/A	0.155	0.234	2.194	0.000	0.674	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	187	503	0	1048	0	0	-1
normalized size	1	1.00	1.34	3.59	0.00	7.49	0.00	0.00	-0.01
time (sec)	N/A	0.462	17.020	1.993	0.000	0.775	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	107	170	0	246	0	0	-1
normalized size	1	1.00	1.37	2.18	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.221	1.952	0.000	0.541	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	171	403	0	1100	0	0	-1
normalized size	1	1.00	1.21	2.86	0.00	7.80	0.00	0.00	-0.01
time (sec)	N/A	0.529	0.278	2.033	0.000	0.812	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	94	433	0	343	0	0	-1
normalized size	1	1.00	1.54	7.10	0.00	5.62	0.00	0.00	-0.02
time (sec)	N/A	0.136	0.226	1.498	0.000	0.675	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	427	566	0	1126	0	0	-1
normalized size	1	1.00	2.87	3.80	0.00	7.56	0.00	0.00	-0.01
time (sec)	N/A	0.570	1.369	2.168	0.000	7.934	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	229015	520	0	963	0	0	-1
normalized size	1	1.00	1877.17	4.26	0.00	7.89	0.00	0.00	-0.01
time (sec)	N/A	0.301	33.712	1.477	0.000	0.780	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	141	520	0	1041	0	0	-1
normalized size	1	1.00	1.14	4.19	0.00	8.40	0.00	0.00	-0.01
time (sec)	N/A	0.368	0.408	1.426	0.000	0.870	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	198	472	0	1103	0	0	-1
normalized size	1	1.00	1.19	2.83	0.00	6.60	0.00	0.00	-0.01
time (sec)	N/A	0.583	0.395	2.219	0.000	1.262	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	155	725	0	1597	0	0	-1
normalized size	1	1.00	0.67	3.14	0.00	6.91	0.00	0.00	-0.00
time (sec)	N/A	0.816	0.382	2.198	0.000	89.926	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	201	431	379	281	0	0	555
normalized size	1	1.00	0.80	1.72	1.52	1.12	0.00	0.00	2.22
time (sec)	N/A	0.501	4.479	1.688	0.921	0.469	0.000	0.000	5.553

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	143	290	266	211	0	0	395
normalized size	1	1.00	0.79	1.61	1.48	1.17	0.00	0.00	2.19
time (sec)	N/A	0.356	1.110	1.372	0.336	0.455	0.000	0.000	5.494

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	174	165	150	0	0	227
normalized size	1	1.00	0.77	1.51	1.43	1.30	0.00	0.00	1.97
time (sec)	N/A	0.185	0.608	1.120	0.333	0.474	0.000	0.000	5.208

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	88	96	0	0	104
normalized size	1	1.00	1.23	1.41	1.44	1.57	0.00	0.00	1.70
time (sec)	N/A	0.075	0.025	0.917	0.326	0.464	0.000	0.000	2.789

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	316	0	0	573
normalized size	1	1.00	1.47	1.78	0.00	4.16	0.00	0.00	7.54
time (sec)	N/A	0.129	0.212	0.591	0.000	1.080	0.000	0.000	2.730

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	132	0	389	0	179	106
normalized size	1	1.00	0.98	1.33	0.00	3.93	0.00	1.81	1.07
time (sec)	N/A	0.144	0.387	0.657	0.000	0.492	0.000	0.664	2.122

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	172	236	0	752	0	418	250
normalized size	1	1.00	1.04	1.42	0.00	4.53	0.00	2.52	1.51
time (sec)	N/A	0.301	0.981	0.594	0.000	0.544	0.000	1.949	4.987

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	405	376	0	1238	0	726	439
normalized size	1	1.00	1.71	1.59	0.00	5.22	0.00	3.06	1.85
time (sec)	N/A	0.512	1.082	0.760	0.000	0.566	0.000	0.400	6.395

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	580	1066	0	1093	0	0	9987
normalized size	1	1.00	2.35	4.32	0.00	4.43	0.00	0.00	40.43
time (sec)	N/A	0.445	4.695	0.898	0.000	172.776	0.000	0.000	11.315

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	389	593	0	779	0	0	6730
normalized size	1	1.00	2.29	3.49	0.00	4.58	0.00	0.00	39.59
time (sec)	N/A	0.350	1.474	0.671	0.000	37.602	0.000	0.000	9.661

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	135	288	0	518	0	0	3559
normalized size	1	1.00	1.31	2.80	0.00	5.03	0.00	0.00	34.55
time (sec)	N/A	0.297	0.827	0.643	0.000	5.651	0.000	0.000	7.316

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	309	0	0	571
normalized size	1	1.00	1.47	1.78	0.00	4.07	0.00	0.00	7.51
time (sec)	N/A	0.127	0.183	0.660	0.000	0.968	0.000	0.000	2.810

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	108	0	1040	0	526	2665
normalized size	1	1.00	0.98	0.89	0.00	8.60	0.00	4.35	22.02
time (sec)	N/A	0.275	0.254	0.632	0.000	2.953	0.000	1.449	4.358

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	229	208	0	2863	0	340	20827
normalized size	1	1.00	1.22	1.11	0.00	15.31	0.00	1.82	111.37
time (sec)	N/A	0.612	0.720	0.713	0.000	167.792	0.000	2.504	15.423

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	1137	1870	0	0	0	0	17256
normalized size	1	1.00	3.00	4.93	0.00	0.00	0.00	0.00	45.53
time (sec)	N/A	0.668	6.547	0.765	0.000	0.000	0.000	0.000	16.949

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	511	1249	0	0	0	0	12483
normalized size	1	1.00	1.72	4.21	0.00	0.00	0.00	0.00	42.03
time (sec)	N/A	0.527	4.120	0.677	0.000	0.000	0.000	0.000	14.374

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	362	790	0	1326	0	0	7958
normalized size	1	1.00	1.59	3.46	0.00	5.82	0.00	0.00	34.90
time (sec)	N/A	0.467	1.804	0.601	0.000	96.704	0.000	0.000	11.297

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	180	486	0	798	0	0	4926
normalized size	1	1.00	0.91	2.45	0.00	4.03	0.00	0.00	24.88
time (sec)	N/A	0.367	0.721	0.844	0.000	9.541	0.000	0.000	9.749

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	394	0	180	106
normalized size	1	1.00	0.97	1.32	0.00	3.94	0.00	1.80	1.06
time (sec)	N/A	0.136	0.398	0.628	0.000	0.507	0.000	1.574	2.199

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	176	210	0	2852	0	339	20827
normalized size	1	1.00	0.95	1.13	0.00	15.33	0.00	1.82	111.97
time (sec)	N/A	0.605	1.058	0.719	0.000	108.508	0.000	0.409	15.563

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	183	355	0	0	0	0	-1
normalized size	1	1.00	0.86	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	3.848	1.671	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	44216	351	0	0	0	0	-1
normalized size	1	1.00	225.59	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	32.591	2.396	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	233	219	0	0	0	0	-1
normalized size	1	1.00	1.21	1.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	3.597	2.087	0.000	1.731	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	177	0	0	0	0	-1
normalized size	1	1.00	1.60	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	1.791	2.251	0.000	0.824	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	176	170	0	0	0	0	-1
normalized size	1	1.00	1.41	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.466	2.066	0.000	0.921	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	39039	291	0	0	0	0	-1
normalized size	1	1.00	98.58	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	32.477	2.166	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	223	465	0	0	0	0	-1
normalized size	1	1.00	1.31	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.849	4.016	2.235	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	236	0	0	0	0	-1
normalized size	1	1.00	1.00	2.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.433	0.233	2.154	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	222	479	0	0	0	0	-1
normalized size	1	1.00	1.32	2.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.070	3.892	2.233	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	264	153	0	0	0	0	-1
normalized size	1	1.00	2.78	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	5.752	1.967	0.000	0.945	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	0	292	0	0	0	0	-1
normalized size	1	1.00	0.00	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.922	20.052	2.220	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	375	225	0	0	0	0	-1
normalized size	1	1.00	1.79	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	14.702	2.008	0.000	0.693	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	156	224	0	0	0	0	-1
normalized size	1	1.00	0.73	1.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	5.315	1.897	0.000	0.879	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	1019	222	0	0	0	0	-1
normalized size	1	1.00	4.45	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	8.458	2.308	0.000	0.689	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	0	355	0	0	0	0	-1
normalized size	1	1.00	0.00	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.889	15.551	2.497	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	183	355	0	0	0	0	-1
normalized size	1	1.00	0.86	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.396	1.621	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	223	465	0	0	0	0	-1
normalized size	1	1.00	1.31	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.843	3.985	2.031	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	187	238	0	0	0	0	-1
normalized size	1	1.00	1.83	2.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	4.210	1.806	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	165	236	0	0	0	0	-1
normalized size	1	1.00	0.79	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	3.061	1.786	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	238	0	0	0	0	-1
normalized size	1	1.00	1.00	2.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.424	0.286	2.082	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	246	346	0	0	0	0	-1
normalized size	1	1.00	1.48	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.856	4.122	2.135	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	151	49	68	121	0	50	47
normalized size	1	1.00	2.25	0.73	1.01	1.81	0.00	0.75	0.70
time (sec)	N/A	0.298	0.965	1.037	0.334	1.183	0.000	3.487	1.934

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	175	62	88	146	0	64	60
normalized size	1	1.00	1.97	0.70	0.99	1.64	0.00	0.72	0.67
time (sec)	N/A	0.336	1.119	1.197	0.341	0.891	0.000	10.614	2.429

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [258] had the largest ratio of [.3226]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	7	1.00	30	0.233
2	A	9	6	1.00	30	0.200
3	A	6	5	1.00	30	0.167
4	A	3	3	1.00	28	0.107
5	A	2	2	1.00	30	0.067
6	A	1	1	1.00	30	0.033
7	A	2	2	1.00	30	0.067
8	A	3	2	1.00	30	0.067
9	A	4	2	1.00	30	0.067
10	A	14	7	1.00	32	0.219
11	A	11	6	1.00	32	0.188
12	A	7	5	1.00	32	0.156
13	A	4	3	1.00	32	0.094
14	A	6	5	1.00	30	0.167
15	A	5	5	1.00	32	0.156
16	A	3	2	1.00	32	0.062
17	A	1	1	1.00	32	0.031
18	A	2	2	1.00	32	0.062
19	A	3	2	1.00	32	0.062
20	A	4	2	1.00	32	0.062
21	A	16	7	1.00	32	0.219
22	A	13	6	1.00	32	0.188
23	A	8	5	1.00	32	0.156
24	A	5	3	1.00	32	0.094
25	A	7	5	1.00	32	0.156

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	9	6	1.00	30	0.200
27	A	6	6	1.00	32	0.188
28	A	6	5	1.00	32	0.156
29	A	4	2	1.00	32	0.062
30	A	1	1	1.00	32	0.031
31	A	2	2	1.00	32	0.062
32	A	3	2	1.00	32	0.062
33	A	4	2	1.00	32	0.062
34	A	10	6	1.00	32	0.188
35	A	6	6	1.00	32	0.188
36	A	5	5	1.00	32	0.156
37	A	2	2	1.00	30	0.067
38	A	3	3	1.00	32	0.094
39	A	6	4	1.00	32	0.125
40	A	10	6	1.00	32	0.188
41	A	13	7	1.00	32	0.219
42	A	11	6	1.00	32	0.188
43	A	7	6	1.00	32	0.188
44	A	6	5	1.00	32	0.156
45	A	3	2	1.00	32	0.062
46	A	1	1	1.00	30	0.033
47	A	6	4	1.00	32	0.125
48	A	3	2	1.00	32	0.062
49	A	7	5	1.00	32	0.156
50	A	10	6	1.00	32	0.188
51	A	13	7	1.00	32	0.219

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	12	6	1.00	32	0.188
53	A	8	6	1.00	32	0.188
54	A	7	5	1.00	32	0.156
55	A	4	2	1.00	32	0.062
56	A	1	1	1.00	32	0.031
57	A	2	2	1.00	30	0.067
58	A	10	6	1.00	32	0.188
59	A	7	5	1.00	32	0.156
60	A	4	3	1.00	32	0.094
61	A	7	5	1.00	32	0.156
62	A	10	6	1.00	32	0.188
63	A	13	7	1.00	32	0.219
64	A	4	2	1.00	32	0.062
65	A	3	2	1.00	32	0.062
66	A	2	2	1.00	32	0.062
67	A	1	1	1.00	32	0.031
68	A	3	3	1.00	32	0.094
69	A	3	3	1.00	32	0.094
70	A	4	4	1.00	32	0.125
71	A	4	2	1.00	34	0.059
72	A	3	2	1.00	34	0.059
73	A	2	2	1.00	34	0.059
74	A	1	1	1.00	34	0.029
75	A	4	3	1.05	34	0.088
76	A	4	4	1.10	34	0.118
77	A	4	3	1.11	34	0.088

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	5	4	1.00	34	0.118
79	A	4	2	1.00	34	0.059
80	A	3	2	1.00	34	0.059
81	A	2	2	1.00	34	0.059
82	A	1	1	1.00	34	0.029
83	A	5	3	1.00	34	0.088
84	A	5	4	1.00	34	0.118
85	A	5	4	1.00	34	0.118
86	A	4	3	1.00	34	0.088
87	A	3	3	1.00	34	0.088
88	A	2	2	1.00	34	0.059
89	A	1	1	1.00	34	0.029
90	A	3	3	1.00	34	0.088
91	A	4	4	1.00	34	0.118
92	A	5	4	1.00	34	0.118
93	A	4	3	1.00	34	0.088
94	A	3	2	1.00	34	0.059
95	A	2	2	1.00	34	0.059
96	A	1	1	1.00	34	0.029
97	A	4	3	1.00	34	0.088
98	A	5	4	1.00	34	0.118
99	A	6	4	1.00	34	0.118
100	A	4	2	1.00	34	0.059
101	A	3	2	1.00	34	0.059
102	A	2	2	1.00	34	0.059
103	A	1	1	1.00	34	0.029

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	5	3	1.00	34	0.088
105	A	6	4	1.00	34	0.118
106	A	7	4	1.00	34	0.118
107	A	1	1	1.00	36	0.028
108	A	1	1	1.00	36	0.028
109	A	1	1	1.00	36	0.028
110	A	1	1	1.00	36	0.028
111	A	1	1	1.00	36	0.028
112	A	1	1	1.00	36	0.028
113	A	2	2	1.00	36	0.056
114	A	2	2	1.00	36	0.056
115	A	2	2	1.00	36	0.056
116	A	1	1	1.00	36	0.028
117	A	2	2	1.00	36	0.056
118	A	2	2	1.00	36	0.056
119	A	1	1	1.00	36	0.028
120	A	2	2	1.00	36	0.056
121	A	2	2	1.00	36	0.056
122	A	2	2	1.00	36	0.056
123	A	3	2	1.00	36	0.056
124	A	3	2	1.00	36	0.056
125	A	2	2	1.00	36	0.056
126	A	1	1	1.00	36	0.028
127	A	3	2	1.00	36	0.056
128	A	3	3	1.00	36	0.083
129	A	3	2	1.00	36	0.056

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	1	1	1.00	36	0.028
131	A	2	2	1.00	36	0.056
132	A	3	2	1.00	36	0.056
133	A	3	2	1.00	36	0.056
134	A	2	2	1.00	36	0.056
135	A	1	1	1.00	36	0.028
136	A	2	2	1.00	36	0.056
137	A	3	3	1.00	36	0.083
138	A	4	3	1.00	36	0.083
139	A	3	3	1.00	36	0.083
140	A	2	2	1.00	36	0.056
141	A	1	1	1.00	36	0.028
142	A	3	3	1.00	36	0.083
143	A	3	3	1.00	36	0.083
144	A	4	4	1.00	36	0.111
145	A	3	2	1.00	36	0.056
146	A	1	1	1.00	36	0.028
147	A	1	1	1.00	36	0.028
148	A	4	3	1.00	36	0.083
149	A	4	4	1.00	36	0.111
150	A	4	3	1.00	36	0.083
151	A	3	3	1.00	32	0.094
152	A	3	3	1.00	32	0.094
153	A	3	3	1.00	30	0.100
154	A	3	3	1.00	32	0.094
155	A	3	3	1.00	32	0.094

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	3	2	1.00	34	0.059
157	A	2	2	1.00	34	0.059
158	A	1	1	1.00	34	0.029
159	A	2	2	1.00	34	0.059
160	A	2	2	1.00	34	0.059
161	A	2	2	1.00	34	0.059
162	A	3	2	1.00	36	0.056
163	A	2	2	1.00	36	0.056
164	A	1	1	1.00	36	0.028
165	A	3	3	1.00	34	0.088
166	A	3	3	1.00	36	0.083
167	A	3	3	1.00	36	0.083
168	A	10	7	1.00	32	0.219
169	A	7	6	1.00	32	0.188
170	A	3	3	1.00	30	0.100
171	A	5	5	1.00	32	0.156
172	A	4	4	1.00	32	0.125
173	A	3	3	1.00	32	0.094
174	A	5	3	1.00	34	0.088
175	A	2	2	1.00	32	0.062
176	A	6	4	1.00	34	0.118
177	A	7	4	1.00	34	0.118
178	A	5	5	1.38	40	0.125
179	A	4	4	1.43	36	0.111
180	A	8	8	1.52	38	0.210
181	A	4	4	1.29	40	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	8	8	1.35	40	0.200
183	A	3	3	1.00	38	0.079
184	A	2	2	1.00	34	0.059
185	A	8	6	1.00	29	0.207
186	A	7	6	1.00	29	0.207
187	A	6	6	1.00	29	0.207
188	A	5	5	1.00	27	0.185
189	A	5	5	1.00	29	0.172
190	A	5	5	1.00	29	0.172
191	A	6	5	1.00	29	0.172
192	A	7	5	1.00	29	0.172
193	A	9	8	1.13	31	0.258
194	A	8	7	1.14	31	0.226
195	A	8	7	1.33	31	0.226
196	A	6	6	1.00	29	0.207
197	B	8	8	2.19	31	0.258
198	A	8	8	1.97	31	0.258
199	A	5	4	1.42	31	0.129
200	A	6	5	1.26	31	0.161
201	A	8	6	1.20	31	0.194
202	A	9	7	1.16	31	0.226
203	A	9	7	1.06	31	0.226
204	A	10	6	1.00	29	0.207
205	A	9	9	1.68	31	0.290
206	A	9	9	1.70	31	0.290
207	A	9	9	1.60	31	0.290

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	6	4	1.28	31	0.129
209	A	7	5	1.23	31	0.161
210	A	7	7	1.29	31	0.226
211	A	6	6	1.46	31	0.194
212	A	6	6	1.84	31	0.194
213	A	3	3	1.00	29	0.103
214	A	4	4	1.61	31	0.129
215	A	6	6	1.35	31	0.194
216	A	7	7	1.29	31	0.226
217	A	8	8	1.22	31	0.258
218	A	7	7	1.29	31	0.226
219	A	6	6	1.45	31	0.194
220	A	6	6	1.67	31	0.194
221	A	2	2	1.00	29	0.069
222	A	6	6	1.42	31	0.194
223	A	7	6	1.23	31	0.194
224	A	8	7	1.22	31	0.226
225	A	9	8	1.12	31	0.258
226	A	8	7	1.15	31	0.226
227	A	7	7	1.29	31	0.226
228	A	6	6	1.45	31	0.194
229	A	4	4	1.00	31	0.129
230	A	3	3	1.00	29	0.103
231	A	7	6	1.30	31	0.194
232	A	8	6	1.13	31	0.194
233	A	9	7	1.12	31	0.226

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
234	A	2	2	1.00	35	0.057
235	A	5	5	1.00	35	0.143
236	A	2	2	1.00	35	0.057
237	A	5	5	1.00	37	0.135
238	A	2	2	1.00	33	0.061
239	A	5	4	1.00	39	0.103
240	A	5	5	1.00	33	0.152
241	A	5	5	1.00	35	0.143
242	A	5	4	1.00	39	0.103
243	A	8	6	1.00	39	0.154
244	A	8	6	1.00	29	0.207
245	A	7	6	1.00	29	0.207
246	A	6	6	1.00	29	0.207
247	A	5	5	1.00	27	0.185
248	A	5	5	1.00	29	0.172
249	A	5	5	1.00	29	0.172
250	A	6	5	1.00	29	0.172
251	A	7	5	1.00	29	0.172
252	A	12	8	1.00	31	0.258
253	A	10	8	1.00	31	0.258
254	A	8	7	1.00	31	0.226
255	A	5	5	1.00	29	0.172
256	A	6	4	1.00	31	0.129
257	A	7	5	1.00	31	0.161
258	A	16	10	1.00	31	0.323
259	A	14	10	1.00	31	0.323

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	12	9	1.00	31	0.290
261	A	10	7	1.00	31	0.226
262	A	5	5	1.00	29	0.172
263	A	7	5	1.00	31	0.161
264	A	3	3	1.00	33	0.091
265	A	1	1	1.00	35	0.029
266	A	1	1	1.00	35	0.029
267	A	1	1	1.00	35	0.029
268	A	1	1	1.00	35	0.029
269	A	3	3	1.00	37	0.081
270	A	7	5	1.00	39	0.128
271	A	3	3	1.00	39	0.077
272	A	8	6	1.00	39	0.154
273	A	1	1	1.00	33	0.030
274	A	11	11	1.00	39	0.282
275	A	3	3	1.00	33	0.091
276	A	3	3	1.00	35	0.086
277	A	7	7	1.00	39	0.180
278	A	11	11	1.00	39	0.282
279	A	3	3	1.00	33	0.091
280	A	7	5	1.00	39	0.128
281	A	1	1	1.00	33	0.030
282	A	3	3	1.00	35	0.086
283	A	3	3	1.00	39	0.077
284	A	7	5	1.00	39	0.128
285	A	4	2	1.00	28	0.071

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	A	4	2	1.00	28	0.071

Chapter 3

Listing of integrals

3.1 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=105

$$\frac{ac^4\tan^5(e+fx)}{5f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^4\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{ac^4\tan(e+fx\cdots}{8f\cdots}$$

```
[Out] 7/8*a*c^4*arctanh(sin(f*x+e))/f-1/8*a*c^4*sec(f*x+e)*tan(f*x+e)/f-3/4*a*c^4
*sec(f*x+e)^3*tan(f*x+e)/f+4/3*a*c^4*tan(f*x+e)^3/f+1/5*a*c^4*tan(f*x+e)^5/
f
```

Rubi [A] time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3958, 2611, 3770, 2607, 30, 3768, 14}

$$\frac{ac^4\tan^5(e+fx)}{5f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^4\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{ac^4\tan(e+fx\cdots}{8f\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*ArcTanh[Sin[e + f*x]])/(8*f) - (a*c^4*Sec[e + f*x]*Tan[e + f*x])/(
8*f) - (3*a*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (4*a*c^4*Tan[e + f*x]^
3)/(3*f) + (a*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = -\left((ac)\int\left(c^3\sec(e+fx)\tan^2(e+fx)-3c^3\sec^2(e+\right.\right.$$
$$= -\left(\left(ac^4\right)\int\sec(e+fx)\tan^2(e+fx)\,dx\right)+\left(ac^4\right)\int \text{se}$$
$$= -\frac{ac^4\sec(e+fx)\tan(e+fx)}{2f}-\frac{3ac^4\sec^3(e+fx)\tan}{4f}$$
$$= \frac{ac^4\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^4\sec(e+fx)\tan(e+fx}{8f}$$
$$= \frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e+f}{8f}$$

Mathematica [B] time = 1.70, size = 499, normalized size = 4.75

$$-\frac{ac^4\sec(e)\sec^5(e+fx)\left(-1920\sin(2e+fx)+780\sin(e+2fx)+780\sin(3e+2fx)+640\sin(2e+3fx)-72\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/3840*(a*c^4*Sec[e]*Sec[e + f*x]^5*(525*Cos[2*e + 3*f*x]*Log[Cos[(e + f*x
)/2] - Sin[(e + f*x)/2]] + 525*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] + 105*Cos[4*e + 5*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] + 105*Cos[6*e + 5*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 1050*C
os[f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]]) + 1050*Cos[2*e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 525*Cos[2*e + 3*f*x]*
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 525*Cos[4*e + 3*f*x]*Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] - 105*Cos[4*e + 5*f*x]*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]] - 105*Cos[6*e + 5*f*x]*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]] + 800*Sin[f*x] - 1920*Sin[2*e + f*x] + 780*Sin[e + 2*f*x] + 780*Sin
[3*e + 2*f*x] + 640*Sin[2*e + 3*f*x] - 720*Sin[4*e + 3*f*x] + 30*Sin[3*e +
4*f*x] + 30*Sin[5*e + 4*f*x] + 272*Sin[4*e + 5*f*x]))/f
```

fricas [A] time = 0.46, size = 131, normalized size = 1.25

$$\frac{105\,ac^4\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-105\,ac^4\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\left(136\,ac^4\cos\left(f\right.\right.}{240\,f\cos\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(105*a*c^4*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 105*a*c^4*cos(f*x +
 e)^5*log(-sin(f*x + e) + 1) - 2*(136*a*c^4*cos(f*x + e)^4 + 15*a*c^4*cos(f
*x + e)^3 - 112*a*c^4*cos(f*x + e)^2 + 90*a*c^4*cos(f*x + e) - 24*a*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-7*a*c^4/16*ln(abs(tan((f*x+exp(1))/2)-1))+7*a*c^4/16*ln(abs(t
an((f*x+exp(1))/2)+1))-(105*tan((f*x+exp(1))/2)^9*a*c^4+790*tan((f*x+exp(1)
)/2)^7*a*c^4-896*tan((f*x+exp(1))/2)^5*a*c^4+490*tan((f*x+exp(1))/2)^3*a*c^
4-105*tan((f*x+exp(1))/2)*a*c^4)*1/120/(tan((f*x+exp(1))/2)^2-1)^5)
```

maple [A] time = 1.72, size = 130, normalized size = 1.24

$$-\frac{3a\,c^4\left(\sec^3\left(fx+e\right)\right)\tan\left(fx+e\right)}{4f}-\frac{a\,c^4\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{7a\,c^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}-\frac{17a\,c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -3/4*a*c^4*sec(f*x+e)^3*tan(f*x+e)/f-1/8*a*c^4*sec(f*x+e)*tan(f*x+e)/f+7/8/
f*a*c^4*ln(sec(f*x+e)+tan(f*x+e))-17/15/f*a*c^4*tan(f*x+e)+14/15/f*a*c^4*ta
n(f*x+e)*sec(f*x+e)^2+1/5/f*a*c^4*tan(f*x+e)*sec(f*x+e)^4
```

maxima [B] time = 0.65, size = 215, normalized size = 2.05

$$\frac{16\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)ac^4+160\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ac^4+45\,ac^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a*c^4 +
160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^4 + 45*a*c^4*(2*(3*sin(f*x + e)^3
 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
+ e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a*c^4*(2*sin(f*x + e)/(sin(f*x +
 e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*log
(sec(f*x + e) + tan(f*x + e)) - 720*a*c^4*tan(f*x + e))/f
```

mupad [B] time = 6.64, size = 176, normalized size = 1.68

$$\frac{7\,a\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{7\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{79\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{6}-\frac{224\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{49\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6}-\frac{7\,a\,c}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (7*a*c^4*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((49*a*c^4*tan(e/2 + (f*x)/2)^3
)/6 - (7*a*c^4*tan(e/2 + (f*x)/2))/4 - (224*a*c^4*tan(e/2 + (f*x)/2)^5)/15
+ (79*a*c^4*tan(e/2 + (f*x)/2)^7)/6 + (7*a*c^4*tan(e/2 + (f*x)/2)^9)/4)/(f*
(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6
 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^4\left(\int\sec\left(e+fx\right)dx+\int\left(-3\sec^2\left(e+fx\right)\right)dx+\int 2\sec^3\left(e+fx\right)dx+\int 2\sec^4\left(e+fx\right)dx+\int\left(-3\sec^5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*c**4*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + Integ
ral(2*sec(e + f*x)**3, x) + Integral(2*sec(e + f*x)**4, x) + Integral(-3*se
c(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

3.2 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=86

$$\frac{2ac^3\tan^3(e+fx)}{3f}+\frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^3\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3ac^3\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] 5/8*a*c^3*arctanh(sin(f*x+e))/f-3/8*a*c^3*sec(f*x+e)*tan(f*x+e)/f-1/4*a*c^3
*sec(f*x+e)^3*tan(f*x+e)/f+2/3*a*c^3*tan(f*x+e)^3/f
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$\frac{2ac^3\tan^3(e+fx)}{3f}+\frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^3\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3ac^3\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a*c^3*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (2*a*c^3*Tan[e + f*x]^
3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx = -\left((ac)\int \left(c^2\sec(e+fx)\tan^2(e+fx) - 2c^2\sec^2(e+\right.\right.$$

$$= -\left(\left(ac^3\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right) - \left(ac^3\right)\int \text{se}$$

$$= -\frac{ac^3\sec(e+fx)\tan(e+fx)}{2f} - \frac{ac^3\sec^3(e+fx)\tan(}{4f}$$

$$= \frac{ac^3\tanh^{-1}(\sin(e+fx))}{2f} - \frac{3ac^3\sec(e+fx)\tan(e+f}{8f}$$

$$= \frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3ac^3\sec(e+fx)\tan(e+}{8f}$$

Mathematica [B] time = 6.48, size = 887, normalized size = 10.31

$$a\left(\frac{5\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c-c\sec(e+fx))^3\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{64f} - \frac{5\cos^3(e+fx)\log\left(\cos\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a*((5*Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*Log[Cos[e/2 + (f*x)/2] - Sin[e/2
+ (f*x)/2]]*(c - c*Sec[e + f*x])^3)/(64*f) - (5*Cos[e + f*x]^3*Csc[e/2 + (f
*x)/2]^6*Log[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^
3)/(64*f) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3)/(1
28*f*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^4) - (Cos[e + f*x]^3*Csc[e/2
 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(24*f*(Cos[e/2] - Sin[e/
2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Csc[e/2
+ (f*x)/2]^6*(c - c*Sec[e + f*x])^3*(Cos[e/2] - 17*Sin[e/2]))/(384*f*(Cos[e
/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (Cos[e + f*x
]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(12*f*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) - (Cos[e + f*x]^3
*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3)/(128*f*(Cos[e/2 + (f*x)/2] +
Sin[e/2 + (f*x)/2])^4) - (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e
+ f*x])^3*Sin[(f*x)/2])/(24*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + S
in[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e +
 f*x])^3*(-Cos[e/2] - 17*Sin[e/2]))/(384*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
 (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(
c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(12*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
(f*x)/2] + Sin[e/2 + (f*x)/2])))
```

fricas [A] time = 0.47, size = 117, normalized size = 1.36

$$\frac{15\,ac^{3}\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-15\,ac^{3}\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(16\,ac^{3}\cos\left(fx+e\right)\right.}{48\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a*c^3*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) - 2*(16*a*c^3*cos(f*x + e)^3 + 9*a*c^3*cos(f*x +
e)^2 - 16*a*c^3*cos(f*x + e) + 6*a*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(5*a*c^3/16*ln(abs(tan((f*x+exp(1))/2)-1))-5*a*c^3/16*ln(abs(t
an((f*x+exp(1))/2)+1))+(15*tan((f*x+exp(1))/2)^7*a*c^3+73*tan((f*x+exp(1))/
2)^5*a*c^3-55*tan((f*x+exp(1))/2)^3*a*c^3+15*tan((f*x+exp(1))/2)*a*c^3)*1/2
4/(tan((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.31, size = 107, normalized size = 1.24

$$-\frac{2a\,c^3\tan\left(fx+e\right)}{3f}+\frac{2a\,c^3\tan\left(fx+e\right)\left(\sec^2\left(fx+e\right)\right)}{3f}+\frac{5a\,c^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}-\frac{a\,c^3\left(\sec^3\left(fx+\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -2/3/f*a*c^3*tan(f*x+e)+2/3/f*a*c^3*tan(f*x+e)*sec(f*x+e)^2+5/8/f*a*c^3*ln(
sec(f*x+e)+tan(f*x+e))-1/4*a*c^3*sec(f*x+e)^3*tan(f*x+e)/f-3/8*a*c^3*sec(f*
x+e)*tan(f*x+e)/f
```

maxima [A] time = 0.42, size = 133, normalized size = 1.55

$$\frac{32\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac^3+3\,ac^3\left(\frac{2\left(3\,\sin\left(fx+e\right)^3-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\right.\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^3 + 3*a*c^3*(2*(3*sin(f*x +
e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(
f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 48*a*c^3*log(sec(f*x + e) + tan(
f*x + e)) - 96*a*c^3*tan(f*x + e))/f
```

mupad [B] time = 5.20, size = 146, normalized size = 1.70

$$\frac{5\,a\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{73\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}-\frac{55\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (5*a*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a*c^3*tan(e/2 + (f*x)/2))/4
 - (55*a*c^3*tan(e/2 + (f*x)/2)^3)/12 + (73*a*c^3*tan(e/2 + (f*x)/2)^5)/12
+ (5*a*c^3*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2
+ (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac^3\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int 2\sec^2\left(e+fx\right)dx+\int\left(-2\sec^4\left(e+fx\right)\right)dx+\int\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*c**3*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Inte
gral(-2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

3.3 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=61

$$\frac{ac^2\tan^3(e+fx)}{3f}+\frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*a*c^2*arctanh(sin(f*x+e))/f-1/2*a*c^2*sec(f*x+e)*tan(f*x+e)/f+1/3*a*c^2
*tan(f*x+e)^3/f
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3958, 2611, 3770, 2607, 30}

$$\frac{ac^2\tan^3(e+fx)}{3f}+\frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (a*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = -\left((ac)\int \left(c\sec(e+fx)\tan^2(e+fx)-c\sec^2(e+fx)\,\text{t}\right.\right.$$

$$= -\left(\left(ac^2\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)+\left(ac^2\right)\int \text{sec}$$

$$= -\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(ac^2\right)\int \sec(e+fx)$$

$$= \frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}$$

Mathematica [B] time = 0.72, size = 313, normalized size = 5.13

$$-\frac{ac^2\sec(e)\sec^3(e+fx)\left(-12\sin(2e+fx)+6\sin(e+2fx)+6\sin(3e+2fx)+4\sin(2e+3fx)+3\cos(2e+3f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/48*(a*c^2*Sec[e]*Sec[e + f*x]^3*(3*Cos[2*e + 3*f*x]*Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] + 3*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] + 9*Cos[f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Cos[2*e + f*x]*(Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 3*Cos[2*e +
 3*f*x]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 3*Cos[4*e + 3*f*x]*Log[C
os[(e + f*x)/2] + Sin[(e + f*x)/2]] - 12*Sin[2*e + f*x] + 6*Sin[e + 2*f*x]
+ 6*Sin[3*e + 2*f*x] + 4*Sin[2*e + 3*f*x]))/f
```

fricas [A] time = 0.45, size = 103, normalized size = 1.69

$$\frac{3\,ac^2\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,ac^2\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,ac^2\cos\left(fx+e\right)^2\right.}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a*c^2*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a*c^2*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) - 2*(2*a*c^2*cos(f*x + e)^2 + 3*a*c^2*cos(f*x + e)
- 2*a*c^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-a*c^2/4*ln(abs(tan((f*x+exp(1))/2)-1))+a*c^2/4*ln(abs(tan((f*
x+exp(1))/2)+1))+(-3*tan((f*x+exp(1))/2)^5*a*c^2-8*tan((f*x+exp(1))/2)^3*a*
c^2+3*tan((f*x+exp(1))/2)*a*c^2)*1/6/(tan((f*x+exp(1))/2)^2-1)^3)
```

maple [A] time = 1.17, size = 84, normalized size = 1.38

$$-\frac{a\,c^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{a\,c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}-\frac{a\,c^2\tan\left(fx+e\right)}{3f}+\frac{a\,c^2\tan\left(fx+e\right)\left(\sec^2\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/2*a*c^2*sec(f*x+e)*tan(f*x+e)/f+1/2/f*a*c^2*ln(sec(f*x+e)+tan(f*x+e))-1/
3*a*c^2*tan(f*x+e)/f+1/3/f*a*c^2*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.59, size = 108, normalized size = 1.77

$$\frac{4\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac^2+3\,ac^2\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+1}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2 + 3*a*c^2*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 12*a
*c^2*log(sec(f*x + e) + tan(f*x + e)) - 12*a*c^2*tan(f*x + e))/f
```

mupad [B] time = 3.81, size = 114, normalized size = 1.87

$$\frac{a\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\,c^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\frac{8\,a\,c^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3}-a\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a*c^2*atanh(tan(e/2 + (f*x)/2)))/f - ((8*a*c^2*tan(e/2 + (f*x)/2)^3)/3 - a
*c^2*tan(e/2 + (f*x)/2) + a*c^2*tan(e/2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)
/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^2\left(\int\sec\left(e+fx\right)dx+\int\left(-\sec^2\left(e+fx\right)\right)dx+\int\left(-\sec^3\left(e+fx\right)\right)dx+\int\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*c**2*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integra
l(-sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

3.4 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=38

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*a*c*arctanh(sin(f*x+e))/f-1/2*a*c*sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3958, 2611, 3770}

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)$$
$$= -\frac{ac\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}(ac)\int \sec(e+fx)\,dx$$
$$= \frac{ac\tanh^{-1}(\sin(e+fx))}{2f} - \frac{ac\sec(e+fx)\tan(e+fx)}{2f}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$-ac\left(\frac{\tan(e+fx)\sec(e+fx)}{2f} - \frac{\tanh^{-1}(\sin(e+fx))}{2f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))
```

fricas [A] time = 0.45, size = 67, normalized size = 1.76

$$\frac{ac\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right) - ac\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right) - 2\,ac\sin\left(fx+e\right)}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*(a*c*cos(f*x + e)^2*log(sin(f*x + e) + 1) - a*c*cos(f*x + e)^2*log(-sin
(f*x + e) + 1) - 2*a*c*sin(f*x + e))/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
```

```
*pi/x/2)-2/f*(a*c/8*ln(abs(sin(f*x+exp(1))-1))-a*c/8*ln(abs(sin(f*x+exp(1))
+1))-sin(f*x+exp(1))*a*c*1/4/(sin(f*x+exp(1))^2-1))
```

maple [A] time = 0.66, size = 42, normalized size = 1.11

$$\frac{ca\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}-\frac{ac\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/2/f*c*a*ln(sec(f*x+e)+tan(f*x+e))-1/2*a*c*sec(f*x+e)*tan(f*x+e)/f
```

maxima [A] time = 0.46, size = 68, normalized size = 1.79

$$\frac{ac\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] 1/4*(a*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) + 4*a*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

mupad [B] time = 2.24, size = 77, normalized size = 2.03

$$\frac{a\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}+a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a*c*atanh(tan(e/2 + (f*x)/2)))/f - (a*c*tan(e/2 + (f*x)/2)^3 + a*c*tan(e/2
 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int\sec^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] -a*c*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**3, x))
```

3.5 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=42

$$-\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -a*arctanh(sin(f*x+e))/c/f-2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3957, 3770}

$$-\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[e + f
*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{a\int\sec(e+fx)\,dx}{c}$$
$$= -\frac{a\tanh^{-1}(\sin(e+fx))}{cf} - \frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

Mathematica [**A**] time = 0.07, size = 77, normalized size = 1.83

$$-\frac{a\left(-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)}{f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*((-2*Cot[(e + f*x)/2])/f - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f
+ Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f))/c)
```

fricas [**A**] time = 0.44, size = 66, normalized size = 1.57

$$-\frac{a\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right) - a\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right) - 4\,a\cos\left(fx+e\right) - 4\,a}{2\,cf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*(a*log(sin(f*x + e) + 1)*sin(f*x + e) - a*log(-sin(f*x + e) + 1)*sin(f
*x + e) - 4*a*cos(f*x + e) - 4*a)/(c*f*sin(f*x + e))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)-2/f*(-a*1/2/c*ln(abs(tan((f*x+exp(1))/2)-1))+a*1/2/c*ln(abs(tan((f
*x+exp(1))/2)+1))-a/c/tan((f*x+exp(1))/2))
```

maple [A] time = 0.87, size = 63, normalized size = 1.50

$$\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fc}-\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{fc}+\frac{2a}{fc\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/f*a/c*ln(tan(1/2*e+1/2*f*x)-1)-1/f*a/c*ln(tan(1/2*e+1/2*f*x)+1)+2/f*a/c/t
an(1/2*e+1/2*f*x)
```

maxima [B] time = 1.17, size = 101, normalized size = 2.40

$$-\frac{a\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-\frac{a(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -(a*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a*(cos(f*x + e)
+ 1)/(c*sin(f*x + e)))/f
```

mupad [B] time = 1.85, size = 31, normalized size = 0.74

$$-\frac{2\,a\,\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] -(2*a*(atanh(tan(e/2 + (f*x)/2)) - cot(e/2 + (f*x)/2)))/(c*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\left(\int\frac{\sec(e+fx)}{\sec(e+fx)-1}\,dx+\int\frac{\sec^2(e+fx)}{\sec(e+fx)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**2
/(sec(e + f*x) - 1), x))/c
```

3.6 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -1/3*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

Mathematica [A] time = 0.25, size = 50, normalized size = 1.39

$$\frac{a\csc\left(\frac{e}{2}\right)\left(\sin\left(e+\frac{3fx}{2}\right)-3\sin\left(e+\frac{fx}{2}\right)\right)\csc^3\left(\frac{1}{2}(e+fx)\right)}{12c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*Csc[e/2]*Csc[(e + f*x)/2]^3*(-3*Sin[e + (f*x)/2] + Sin[e + (3*f*x)/2]))/
(12*c^2*f)
```

fricas [A] time = 0.43, size = 51, normalized size = 1.42

$$\frac{a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)+a}{3\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)/((c^2*f*cos(f*x + e) - c^2*f)
*sin(f*x + e))
```

giac [A] time = 1.58, size = 21, normalized size = 0.58

$$-\frac{a}{3\,c^{2}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*a/(c^2*f*tan(1/2*f*x + 1/2*e)^3)
```

maple [A] time = 0.79, size = 21, normalized size = 0.58

$$-\frac{a}{3f\,c^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/3/f*a/c^2/tan(1/2*e+1/2*f*x)^3
```

maxima [B] time = 0.60, size = 97, normalized size = 2.69

$$-\frac{\frac{a\left(\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)\left(\cos\left(fx+e\right)+1\right)^{3}}{c^{2}\sin\left(fx+e\right)^{3}}-\frac{a\left(\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-1\right)\left(\cos\left(fx+e\right)+1\right)^{3}}{c^{2}\sin\left(fx+e\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c
^2*sin(f*x + e)^3) - a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x
 + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

mupad [B] time = 2.06, size = 20, normalized size = 0.56

$$-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] -(a*cot(e/2 + (f*x)/2)^3)/(3*c^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

3.7 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

```
[Out] -1/5*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^3-1/15*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx}{5c}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}-\frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}$$

Mathematica [A] time = 0.41, size = 87, normalized size = 1.14

$$-\frac{a\csc\left(\frac{e}{2}\right)\left(15\sin\left(e+\frac{fx}{2}\right)-5\sin\left(e+\frac{3fx}{2}\right)-15\sin\left(2e+\frac{3fx}{2}\right)+4\sin\left(2e+\frac{5fx}{2}\right)+25\sin\left(\frac{fx}{2}\right)\right)\csc^5\left(\frac{1}{2}(e+fx\right.}{240c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/240*(a*Csc[e/2]*Csc[(e + f*x)/2]^5*(25*Sin[(f*x)/2] + 15*Sin[e + (f*x)/2
] - 5*Sin[e + (3*f*x)/2] - 15*Sin[2*e + (3*f*x)/2] + 4*Sin[2*e + (5*f*x)/2]
))/(c^3*f)
```

fricas [A] time = 0.44, size = 78, normalized size = 1.03

$$\frac{4\,a\cos\left(fx+e\right)^3+7\,a\cos\left(fx+e\right)^2+2\,a\cos\left(fx+e\right)-a}{15\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*a*cos(f*x + e)^3 + 7*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/((c^3
*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

giac [A] time = 0.51, size = 39, normalized size = 0.51

$$-\frac{5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,a}{30\,c^3f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -1/30*(5*a*tan(1/2*f*x + 1/2*e)^2 - 3*a)/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

maple [A] time = 0.72, size = 37, normalized size = 0.49

$$\frac{a\left(-\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}\right)}{2f\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f*a/c^3*(-1/3/tan(1/2*e+1/2*f*x)^3+1/5/tan(1/2*e+1/2*f*x)^5)
```

maxima [A] time = 0.34, size = 117, normalized size = 1.54

$$-\frac{\frac{a\left(\frac{10\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{15\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-3\right)\left(\cos\left(fx+e\right)+1\right)^{5}}{c^{3}\sin\left(fx+e\right)^{5}}+\frac{3a\left(\frac{5\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-1\right)\left(\cos\left(fx+e\right)+1\right)^{5}}{c^{3}\sin\left(fx+e\right)^{5}}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(a*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 3*a*(5*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)
^5))/f
```

mupad [B] time = 1.71, size = 35, normalized size = 0.46

$$\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}\,\left(3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-5\right)}{30\,c^{3}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (a*cot(e/2 + (f*x)/2)^3*(3*cot(e/2 + (f*x)/2)^2 - 5))/(30*c^3*f)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx\right)}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e +
f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.8 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=116

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -1/7*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-2/35*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c^2
-c^2*sec(f*x+e))^2
```

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4) - (2*(a +
 a*Sec[e + f*x])*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(105*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx}{7c}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3} + \frac{2}{}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3} - \frac{2}{}$$

Mathematica [A] time = 0.46, size = 113, normalized size = 0.97

$$-\frac{a\csc\left(\frac{e}{2}\right)\left(455\sin\left(e+\frac{fx}{2}\right)-273\sin\left(e+\frac{3fx}{2}\right)-210\sin\left(2e+\frac{3fx}{2}\right)+56\sin\left(2e+\frac{5fx}{2}\right)+105\sin\left(3e+\frac{5fx}{2}\right)-\right.}{6720c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/6720*(a*Csc[e/2]*Csc[(e + f*x)/2]^7*(350*Sin[(f*x)/2] + 455*Sin[e + (f*x
)/2] - 273*Sin[e + (3*f*x)/2] - 210*Sin[2*e + (3*f*x)/2] + 56*Sin[2*e + (5*
f*x)/2] + 105*Sin[3*e + (5*f*x)/2] - 23*Sin[3*e + (7*f*x)/2]))/(c^4*f)
```

fricas [A] time = 0.44, size = 104, normalized size = 0.90

$$\frac{23\,a\cos\left(fx+e\right)^4+36\,a\cos\left(fx+e\right)^3+5\,a\cos\left(fx+e\right)^2-6\,a\cos\left(fx+e\right)+2\,a}{105\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*(23*a*cos(f*x + e)^4 + 36*a*cos(f*x + e)^3 + 5*a*cos(f*x + e)^2 - 6*a
*cos(f*x + e) + 2*a)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^
4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

giac [A] time = 0.35, size = 54, normalized size = 0.47

$$-\frac{35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,a}{420\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/420*(35*a*tan(1/2*f*x + 1/2*e)^4 - 42*a*tan(1/2*f*x + 1/2*e)^2 + 15*a)/(
c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

maple [A] time = 0.76, size = 50, normalized size = 0.43

$$\frac{a\left(-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}-\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{2}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}\right)}{4f\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/4/f*a/c^4*(-1/7/tan(1/2*e+1/2*f*x)^7-1/3/tan(1/2*e+1/2*f*x)^3+2/5/tan(1/2
*e+1/2*f*x)^5)
```

maxima [A] time = 0.46, size = 177, normalized size = 1.53

$$\frac{\frac{a\left(\frac{21\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{35\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{105\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-15\right)(\cos(fx+e)+1)^{7}}{c^{4}\sin(fx+e)^{7}}+\frac{3a\left(\frac{21\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{35\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{35\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-5\right)(\cos(fx+e)+1)^{7}}{c^{4}\sin(fx+e)^{7}}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

mupad [B] time = 1.75, size = 61, normalized size = 0.53

$$\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{10\,c^{4}\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{12\,c^{4}\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{28\,c^{4}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (a*cot(e/2 + (f*x)/2)^5)/(10*c^4*f) - (a*cot(e/2 + (f*x)/2)^3)/(12*c^4*f) -
 (a*cot(e/2 + (f*x)/2)^7)/(28*c^4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**
4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

3.9 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=158

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105c^2f(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{21cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx}{9f(c-}$$

```
[Out] -1/9*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/21*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/c^2/f/
(c-c*sec(f*x+e))^3-2/315*(a+a*sec(f*x+e))*tan(f*x+e)/c/f/(c^2-c^2*sec(f*x+e
))^2
```

Rubi [A] time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105c^2f(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{21cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx}{9f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(21*c*f*(c - c*Sec[e + f*x])^4) - (2*(a + a*Se
c[e + f*x])*Tan[e + f*x])/(105*c^2*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*Se
c[e + f*x])*Tan[e + f*x])/(315*c*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

```
&&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx}{3c}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} + \frac{2}{}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(}{}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(}{}$$

Mathematica [A] time = 0.36, size = 139, normalized size = 0.88

$$-\frac{a\csc\left(\frac{e}{2}\right)\left(3465\sin\left(e+\frac{fx}{2}\right)-2247\sin\left(e+\frac{3fx}{2}\right)-2625\sin\left(2e+\frac{3fx}{2}\right)+1143\sin\left(2e+\frac{5fx}{2}\right)+945\sin\left(3e+\right.\right.}{80640c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/80640*(a*Csc[e/2]*Csc[(e + f*x)/2]^9*(3843*Sin[(f*x)/2] + 3465*Sin[e + (
f*x)/2] - 2247*Sin[e + (3*f*x)/2] - 2625*Sin[2*e + (3*f*x)/2] + 1143*Sin[2*
e + (5*f*x)/2] + 945*Sin[3*e + (5*f*x)/2] - 207*Sin[3*e + (7*f*x)/2] - 315*
Sin[4*e + (7*f*x)/2] + 58*Sin[4*e + (9*f*x)/2]))/(c^5*f)
```

fricas [A] time = 0.42, size = 128, normalized size = 0.81

$$\frac{58\,a\cos\left(fx+e\right)^5+83\,a\cos\left(fx+e\right)^4+4\,a\cos\left(fx+e\right)^3-11\,a\cos\left(fx+e\right)^2+8\,a\cos\left(fx+e\right)-2\,a}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="fric
as")
```

```
[Out] 1/315*(58*a*cos(f*x + e)^5 + 83*a*cos(f*x + e)^4 + 4*a*cos(f*x + e)^3 - 11*
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - 2*a)/((c^5*f*cos(f*x + e)^4 - 4*c^5*f
```

```
*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*si
n(f*x + e))
```

giac [A] time = 0.47, size = 69, normalized size = 0.44

$$-\frac{105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-189\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+135\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-35\,a}{2520\,c^{5}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="giac
")
```

```
[Out] -1/2520*(105*a*tan(1/2*f*x + 1/2*e)^6 - 189*a*tan(1/2*f*x + 1/2*e)^4 + 135*
a*tan(1/2*f*x + 1/2*e)^2 - 35*a)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

maple [A] time = 0.77, size = 63, normalized size = 0.40

$$\frac{a\left(-\frac{3}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}-\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{3}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}\right)}{8f\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/8/f*a/c^5*(-3/7/tan(1/2*e+1/2*f*x)^7+1/9/tan(1/2*e+1/2*f*x)^9-1/3/tan(1/2
*e+1/2*f*x)^3+3/5/tan(1/2*e+1/2*f*x)^5)
```

maxima [A] time = 0.70, size = 197, normalized size = 1.25

$$-\frac{\frac{a\left(\frac{180\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{378\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{420\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{315\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-35\right)\left(\cos\left(fx+e\right)+1\right)^{9}}{c^{5}\sin\left(fx+e\right)^{9}}+\frac{5\,a\left(\frac{18\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{42\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{63\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-7\right)(\cos}{c^{5}\sin\left(fx+e\right)^{9}}}{5040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="maxi
ma")
```

```
[Out] -1/5040*(a*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x
```

```
+ e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)
 + 5*a*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) +
1)^9/(c^5*sin(f*x + e)^9))/f
```

mupad [B] time = 1.84, size = 106, normalized size = 0.67

$$\frac{a\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(35\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-135\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+189\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-105\,\mathrm{s}\right.}{2520\,c^5\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a*cos(e/2 + (f*x)/2)^3*(35*cos(e/2 + (f*x)/2)^6 - 105*sin(e/2 + (f*x)/2)^6
 + 189*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 135*cos(e/2 + (f*x)/2)^4
*sin(e/2 + (f*x)/2)^2))/(2520*c^5*f*sin(e/2 + (f*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\right.}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e +
 f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e +
f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(
e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

3.10 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=171

$$-\frac{a^2c^5\tan^7(e+fx)}{7f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}+\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{2f}-\frac{3a^2c^5\tan(e$$

```
[Out] 9/16*a^2*c^5*arctanh(sin(f*x+e))/f-3/16*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f-3/8
*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f+1/
2*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-4/5*a^2*c^5*tan(f*x+e)^5/f-1/7*a^2*c^
5*tan(f*x+e)^7/f
```

Rubi [A] time = 0.27, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3958, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^2c^5\tan^7(e+fx)}{7f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}+\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{2f}-\frac{3a^2c^5\tan(e$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (9*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (3*a^2*c^5*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (3*a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Sec
[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(
2*f) - (4*a^2*c^5*Tan[e + f*x]^5)/(5*f) - (a^2*c^5*Tan[e + f*x]^7)/(7*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \left(a^2c^2\right)\int \left(c^3\sec(e+fx)\tan^4(e+fx) - 3c^3\sec^2(e+f\right.$$
$$= \left(a^2c^5\right)\int \sec(e+fx)\tan^4(e+fx)\,dx - \left(a^2c^5\right)\int \sec^4$$
$$= \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^5\sec^3(e+fx)\tan}{2f}$$
$$= -\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{8f} - \frac{3a^2c^5\sec^3(e+fx)\,t}{8f}$$
$$= \frac{3a^2c^5\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^2c^5\sec(e+fx)\tan(e}{16f}$$
$$= \frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f} - \frac{3a^2c^5\sec(e+fx)\tan(e}{16f}$$

Mathematica [A] time = 1.85, size = 102, normalized size = 0.60

$$\frac{a^2c^5\left(10080\tanh^{-1}(\sin(e+fx)) - (2520\sin(e+fx) - 455\sin(2(e+fx)) - 616\sin(3(e+fx)) + 2380\sin(4(e+\right.}{17920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(10080*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(2520*Sin[e + f*x] -
 455*Sin[2*(e + f*x)] - 616*Sin[3*(e + f*x)] + 2380*Sin[4*(e + f*x)] - 392*
Sin[5*(e + f*x)] + 245*Sin[6*(e + f*x)] + 184*Sin[7*(e + f*x)])))/(17920*f)
```

fricas [A] time = 0.46, size = 177, normalized size = 1.04

$$\frac{315\,a^2c^5\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right) - 315\,a^2c^5\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right) - 2\left(368\,a^2c^5\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1120*(315*a^2*c^5*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 315*a^2*c^5*cos(
f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(368*a^2*c^5*cos(f*x + e)^6 + 245*a^2
*c^5*cos(f*x + e)^5 - 656*a^2*c^5*cos(f*x + e)^4 + 350*a^2*c^5*cos(f*x + e)
```

```
^3 + 208*a^2*c^5*cos(f*x + e)^2 - 280*a^2*c^5*cos(f*x + e) + 80*a^2*c^5)*si
n(f*x + e))/(f*cos(f*x + e)^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(9*a^2*c^5/32*ln(abs(tan((f*x+exp(1))/2)-1))-9*a^2*c^5/32*ln(a
bs(tan((f*x+exp(1))/2)+1))+(315*tan((f*x+exp(1))/2)^13*a^2*c^5-2100*tan((f*
x+exp(1))/2)^11*a^2*c^5-8393*tan((f*x+exp(1))/2)^9*a^2*c^5+9216*tan((f*x+ex
p(1))/2)^7*a^2*c^5-5943*tan((f*x+exp(1))/2)^5*a^2*c^5+2100*tan((f*x+exp(1))
/2)^3*a^2*c^5-315*tan((f*x+exp(1))/2)*a^2*c^5)*1/560/(tan((f*x+exp(1))/2)^2
-1)^7)
```

maple [A] time = 1.94, size = 192, normalized size = 1.12

$$-\frac{23a^2c^5\tan\left(fx+e\right)}{35f}-\frac{13a^2c^5\tan\left(fx+e\right)\left(\sec^4\left(fx+e\right)\right)}{35f}+\frac{41a^2c^5\tan\left(fx+e\right)\left(\sec^2\left(fx+e\right)\right)}{35f}-\frac{5a^2c^5\left(\sec^3\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x)
```

```
[Out] -23/35*a^2*c^5*tan(f*x+e)/f-13/35/f*a^2*c^5*tan(f*x+e)*sec(f*x+e)^4+41/35/f
*a^2*c^5*tan(f*x+e)*sec(f*x+e)^2-5/8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f-7/16
*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f+9/16/f*a^2*c^5*ln(sec(f*x+e)+tan(f*x+e))+1
/2/f*a^2*c^5*tan(f*x+e)*sec(f*x+e)^5-1/7/f*a^2*c^5*tan(f*x+e)*sec(f*x+e)^6
```

maxima [B] time = 0.34, size = 368, normalized size = 2.15

$$-\frac{96\left(5\tan\left(fx+e\right)^7+21\tan\left(fx+e\right)^5+35\tan\left(fx+e\right)^3+35\tan\left(fx+e\right)\right)a^2c^5+224\left(3\tan\left(fx+e\right)^5+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^2*c^5 + 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^2*c^5 - 5600*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 105*a^
2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x
 + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) +
1) + 15*log(sin(f*x + e) - 1)) - 1050*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(
f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1)
 + 3*log(sin(f*x + e) - 1)) + 840*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 -
 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3360*a^2*c^5*log(sec
(f*x + e) + tan(f*x + e)) + 10080*a^2*c^5*tan(f*x + e))/f
```

mupad [B] time = 5.76, size = 251, normalized size = 1.47

$$\frac{-\frac{9\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}+\frac{15\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{2}+\frac{1199\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{40}-\frac{1152\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{849\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{40}-\frac{15\,a^2\,c^5\,\tan}{2}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\,\tan\left(\frac{e}{2}+\frac{f}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

```
[Out] ((849*a^2*c^5*tan(e/2 + (f*x)/2)^5)/40 - (15*a^2*c^5*tan(e/2 + (f*x)/2)^3)/
2 - (1152*a^2*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1199*a^2*c^5*tan(e/2 + (f*x)/
2)^9)/40 + (15*a^2*c^5*tan(e/2 + (f*x)/2)^11)/2 - (9*a^2*c^5*tan(e/2 + (f*x
)/2)^13)/8 + (9*a^2*c^5*tan(e/2 + (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 -
 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^
8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)
^14 - 1)) + (9*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^5\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int 3\sec^2\left(e+fx\right)dx+\int\left(-\sec^3\left(e+fx\right)\right)\,dx+\int\left(-5\sec^4\left(e+fx\right)\right)\,dx+\int 5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**2, x) + I
ntegral(-sec(e + f*x)**3, x) + Integral(-5*sec(e + f*x)**4, x) + Integral(5
*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x) + Integral(-3*sec(e + f
*x)**7, x) + Integral(sec(e + f*x)**8, x))
```

3.11 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=150

$$-\frac{2a^2c^4\tan^5(e+fx)}{5f}+\frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^4\tan^3(e+fx)\sec^3(e+fx)}{6f}-\frac{a^2c^4\tan(e+fx)\sec^3(e+fx)}{8f}$$

```
[Out] 7/16*a^2*c^4*arctanh(sin(f*x+e))/f-5/16*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f-1/8
*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^4*sec(f*x+e)*tan(f*x+e)^3/f+1/
6*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)^3/f-2/5*a^2*c^4*tan(f*x+e)^5/f
```

Rubi [A] time = 0.24, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2c^4\tan^5(e+fx)}{5f}+\frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^4\tan^3(e+fx)\sec^3(e+fx)}{6f}-\frac{a^2c^4\tan(e+fx)\sec^3(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^2*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^4*Sec[e
 + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*
f) - (2*a^2*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = \left(a^2c^2\right)\int \left(c^2\sec(e+fx)\tan^4(e+fx) - 2c^2\sec^2(e+f\right.$$

$$= \left(a^2c^4\right)\int \sec(e+fx)\tan^4(e+fx)\,dx + \left(a^2c^4\right)\int \sec^3$$

$$= \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^4\sec^3(e+fx)\tan}{6f}$$

$$= -\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{8f} - \frac{a^2c^4\sec^3(e+fx)\tan}{8f}$$

$$= \frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{8f} - \frac{5a^2c^4\sec(e+fx)\tan(e}{16f}$$

$$= \frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f} - \frac{5a^2c^4\sec(e+fx)\tan(e}{16f}$$

Mathematica [A] time = 1.40, size = 91, normalized size = 0.61

$$\frac{a^2c^4\left(1680\tanh^{-1}(\sin(e+fx))+(330\sin(e+fx)-240\sin(2(e+fx))-445\sin(3(e+fx))+192\sin(4(e+fx$$
$$3840f$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*(1680*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^6*(330*Sin[e + f*x] - 2
40*Sin[2*(e + f*x)] - 445*Sin[3*(e + f*x)] + 192*Sin[4*(e + f*x)] - 135*Sin
[5*(e + f*x)] - 48*Sin[6*(e + f*x)])))/(3840*f)
```

fricas [A] time = 0.49, size = 161, normalized size = 1.07

$$105\,a^2c^4\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-105\,a^2c^4\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(96\,a^2c^4\cos\left(\right.\right.$$
$$48$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*a^2*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 105*a^2*c^4*cos(f
*x + e)^6*log(-sin(f*x + e) + 1) - 2*(96*a^2*c^4*cos(f*x + e)^5 + 135*a^2*c
^4*cos(f*x + e)^4 - 192*a^2*c^4*cos(f*x + e)^3 + 10*a^2*c^4*cos(f*x + e)^2
+ 96*a^2*c^4*cos(f*x + e) - 40*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-7*a^2*c^4/32*ln(abs(tan((f*x+exp(1))/2)-1))+7*a^2*c^4/32*ln(a
bs(tan((f*x+exp(1))/2)+1))-(105*tan((f*x+exp(1))/2)^11*a^2*c^4-595*tan((f*x
+exp(1))/2)^9*a^2*c^4-1686*tan((f*x+exp(1))/2)^7*a^2*c^4+1386*tan((f*x+exp(
1))/2)^5*a^2*c^4-595*tan((f*x+exp(1))/2)^3*a^2*c^4+105*tan((f*x+exp(1))/2)*
a^2*c^4)*1/240/(tan((f*x+exp(1))/2)^2-1)^6)
```

maple [A] time = 1.66, size = 167, normalized size = 1.11

$$-\frac{a^2c^4\left(\sec^3\left(fx+e\right)\right)\tan\left(fx+e\right)}{24f}-\frac{9a^2c^4\sec\left(fx+e\right)\tan\left(fx+e\right)}{16f}+\frac{7a^2c^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16f}-\frac{2a^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/24*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)/f-9/16*a^2*c^4*sec(f*x+e)*tan(f*x+e)/
f+7/16/f*a^2*c^4*ln(sec(f*x+e)+tan(f*x+e))-2/5*a^2*c^4*tan(f*x+e)/f+4/5/f*a
^2*c^4*tan(f*x+e)*sec(f*x+e)^2-2/5/f*a^2*c^4*tan(f*x+e)*sec(f*x+e)^4+1/6/f*
a^2*c^4*tan(f*x+e)*sec(f*x+e)^5
```

maxima [B] time = 0.56, size = 321, normalized size = 2.14

$$-\frac{64\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^4-640\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^4+5\,a\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/480*(64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4
 - 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 5*a^2*c^4*(2*(15*sin(f*x
 + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x
+ e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x
+ e) - 1)) - 30*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e
)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e)
- 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e)
 + 1) + log(sin(f*x + e) - 1)) - 480*a^2*c^4*log(sec(f*x + e) + tan(f*x + e
)) + 960*a^2*c^4*tan(f*x + e))/f
```

mupad [B] time = 5.61, size = 219, normalized size = 1.46

$$\frac{-\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{119\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{24}+\frac{281\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{20}-\frac{231\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20}+\frac{119\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24}-\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}\cdots\right.}{8}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] ((119*a^2*c^4*tan(e/2 + (f*x)/2)^3)/24 - (231*a^2*c^4*tan(e/2 + (f*x)/2)^5)
/20 + (281*a^2*c^4*tan(e/2 + (f*x)/2)^7)/20 + (119*a^2*c^4*tan(e/2 + (f*x)/
2)^9)/24 - (7*a^2*c^4*tan(e/2 + (f*x)/2)^11)/8 - (7*a^2*c^4*tan(e/2 + (f*x)
/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 +
 (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
 (f*x)/2)^12 + 1)) + (7*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^4\left(\int \sec\left(e+fx\right)dx+\int\left(-2\sec^2\left(e+fx\right)\right)dx+\int\left(-\sec^3\left(e+fx\right)\right)dx+\int 4\sec^4\left(e+fx\right)dx+\int\left(-\mathrm{se}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + In
tegral(-sec(e + f*x)**3, x) + Integral(4*sec(e + f*x)**4, x) + Integral(-se
c(e + f*x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(sec(e + f*x)
**7, x))
```

3.12 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=94

$$-\frac{a^2c^3\tan^5(e+fx)}{5f}+\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^3\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^3\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] 3/8*a^2*c^3*arctanh(sin(f*x+e))/f-3/8*a^2*c^3*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^2*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/5*a^2*c^3*tan(f*x+e)^5/f
```

Rubi [A] time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2611, 3770, 2607, 30}

$$-\frac{a^2c^3\tan^5(e+fx)}{5f}+\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^3\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^3\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^3*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^3*Tan[e +
 f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \left(a^2c^2\right)\int \left(c\sec(e+fx)\tan^4(e+fx)-c\sec^2(e+fx\right.$$
$$= \left(a^2c^3\right)\int \sec(e+fx)\tan^4(e+fx)\,dx-\left(a^2c^3\right)\int \sec$$
$$= \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{1}{4}\left(3a^2c^3\right)\int \sec(e+$$
$$= -\frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^3\sec(e+fx)\,\text{ta}}{4f}$$
$$= \frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^2c^3\sec(e+fx)\tan(e}{8f}$$

Mathematica [A] time = 0.75, size = 82, normalized size = 0.87

$$\frac{a^2c^3\left(120\tanh^{-1}(\sin(e+fx))-(40\sin(e+fx)+10\sin(2(e+fx))-20\sin(3(e+fx))+25\sin(4(e+fx))+4\right.}{320f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(120*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^5*(40*Sin[e + f*x] + 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] + 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

fricas [A] time = 0.48, size = 145, normalized size = 1.54

$$\frac{15\,a^2c^3\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^2c^3\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\left(8\,a^2c^3\cos\left(fx+\right.\right.}{80\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^2*c^3*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^2*c^3*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) - 2*(8*a^2*c^3*cos(f*x + e)^4 + 25*a^2*c^3*co
s(f*x + e)^3 - 16*a^2*c^3*cos(f*x + e)^2 - 10*a^2*c^3*cos(f*x + e) + 8*a^2*
c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(3*a^2*c^3/16*ln(abs(tan((f*x+exp(1))/2)-1))-3*a^2*c^3/16*ln(a
bs(tan((f*x+exp(1))/2)+1))-(-15*tan((f*x+exp(1))/2)^9*a^2*c^3+70*tan((f*x+e
xp(1))/2)^7*a^2*c^3+128*tan((f*x+exp(1))/2)^5*a^2*c^3-70*tan((f*x+exp(1))/2
)^3*a^2*c^3+15*tan((f*x+exp(1))/2)*a^2*c^3)*1/40/(tan((f*x+exp(1))/2)^2-1)^
5)
```

maple [A] time = 1.51, size = 142, normalized size = 1.51

$$-\frac{a^2c^3\tan\left(fx+e\right)}{5f}+\frac{2c^3a^2\tan\left(fx+e\right)\left(\sec^2\left(fx+e\right)\right)}{5f}-\frac{5c^3a^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{3c^3a^2\ln\left(\sec\left(fx+e\right)+\right.}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/5*a^2*c^3*tan(f*x+e)/f+2/5/f*a^2*c^3*tan(f*x+e)*sec(f*x+e)^2-5/8/f*c^3*a
^2*sec(f*x+e)*tan(f*x+e)+3/8/f*c^3*a^2*ln(sec(f*x+e)+tan(f*x+e))+1/4/f*a^2*
c^3*tan(f*x+e)*sec(f*x+e)^3-1/5/f*a^2*c^3*tan(f*x+e)*sec(f*x+e)^4
```

maxima [B] time = 0.40, size = 227, normalized size = 2.41

$$-\frac{16\left(3\,\tan\left(fx+e\right)^{5}+10\,\tan\left(fx+e\right)^{3}+15\,\tan\left(fx+e\right)\right)a^{2}c^{3}-160\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{2}c^{3}+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^3
 - 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 15*a^2*c^3*(2*(3*sin(f*x
 + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(s
in(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^3*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 240*
a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 240*a^2*c^3*tan(f*x + e))/f
```

mupad [B] time = 6.50, size = 187, normalized size = 1.99

$$\frac{-\frac{3\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{7\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{7\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}+\frac{3\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}+\frac{3\,a^2\,c^3\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] ((32*a^2*c^3*tan(e/2 + (f*x)/2)^5)/5 - (7*a^2*c^3*tan(e/2 + (f*x)/2)^3)/2 +
 (7*a^2*c^3*tan(e/2 + (f*x)/2)^7)/2 - (3*a^2*c^3*tan(e/2 + (f*x)/2)^9)/4 +
(3*a^2*c^3*tan(e/2 + (f*x)/2))/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 +
 (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 +
(f*x)/2)^10 - 1)) + (3*a^2*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^3\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int\sec^2\left(e+fx\right)dx+\int 2\sec^3\left(e+fx\right)dx+\int\left(-2\sec^4\left(e+fx\right)\right)dx+\int\left(-\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*c**3*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(-s
ec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

3.13 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=73

$$\frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^2\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] 3/8*a^2*c^2*arctanh(sin(f*x+e))/f-3/8*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^2*c^2*sec(f*x+e)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2611, 3770}

$$\frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^2\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx$$
$$= \frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{1}{4}\left(3a^2c^2\right)\int \sec(e+$$
$$= -\frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^2\sec(e+fx)\,\mathrm{ta}}{4f}$$
$$= \frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^2c^2\sec(e+fx)\tan(e}{8f}$$

Mathematica [A] time = 0.15, size = 51, normalized size = 0.70

$$\frac{a^2c^2\left(6\tanh^{-1}(\sin(e+fx)) - (5\cos(2(e+fx))+1)\tan(e+fx)\sec^3(e+fx)\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*c^2*(6*ArcTanh[Sin[e + f*x]] - (1 + 5*Cos[2*(e + f*x)])*Sec[e + f*x]^3
*Tan[e + f*x]))/(16*f)
```

fricas [A] time = 0.45, size = 99, normalized size = 1.36

$$\frac{3\,a^2c^2\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right) - 3\,a^2c^2\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right) - 2\left(5\,a^2c^2\cos\left(fx+e\right.\right.}{16\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/16*(3*a^2*c^2*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c^2*cos(f*x +
e)^4*log(-sin(f*x + e) + 1) - 2*(5*a^2*c^2*cos(f*x + e)^2 - 2*a^2*c^2)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2/f*(-3*a^2*c^2/32*ln(abs(sin(f*x+exp(1))-1))+3*a^2*c^2/32*ln(abs(s
in(f*x+exp(1))+1))-(-5*sin(f*x+exp(1))^3*a^2*c^2+3*sin(f*x+exp(1))*a^2*c^2)
*1/16/(sin(f*x+exp(1))^2-1)^2)
```

maple [A] time = 0.87, size = 75, normalized size = 1.03

$$-\frac{5a^2c^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{3a^2c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}+\frac{a^2c^2\tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -5/8*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+3/8/f*a^2*c^2*ln(sec(f*x+e)+tan(f*x+e)
)+1/4/f*a^2*c^2*tan(f*x+e)*sec(f*x+e)^3
```

maxima [B] time = 0.54, size = 150, normalized size = 2.05

$$-\frac{a^2c^2\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-8a^2c^2\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\right.\right.}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/16*(a^2*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 8
*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) - 16*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)))/f
```

mupad [B] time = 5.21, size = 155, normalized size = 2.12

$$\frac{-\frac{3\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{11\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{4}+\frac{11\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{4}-\frac{3\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{3\,a^2\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] ((11*a^2*c^2*tan(e/2 + (f*x)/2)^3)/4 + (11*a^2*c^2*tan(e/2 + (f*x)/2)^5)/4
- (3*a^2*c^2*tan(e/2 + (f*x)/2)^7)/4 - (3*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f
*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6
+ tan(e/2 + (f*x)/2)^8 + 1)) + (3*a^2*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^2\left(\int \sec\left(e+fx\right)dx+\int\left(-2\sec^3\left(e+fx\right)\right)dx+\int\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*c**2*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**5, x))
```

3.14 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=61

$$-\frac{a^2 c \tan^3(e+fx)}{3f}+\frac{a^2 c \tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2 c \tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/3*a^2*c
*tan(f*x+e)^3/f
```

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3958, 2611, 3770, 2607, 30}

$$-\frac{a^2 c \tan^3(e+fx)}{3f}+\frac{a^2 c \tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2 c \tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(2*
f) - (a^2*c*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a\sec(e+fx)\tan^2(e+fx)+a\sec^2(e+fx\right.\right.$$
$$= -\left(\left(a^2c\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int \text{se}$$
$$= -\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^2c\right)\int \sec(e+fx$$
$$= \frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2c\sec(e+fx)\tan(e+fx}{2f}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 0.74

$$\frac{a^2c\left(-2\tan^3(e+fx)+3\tanh^{-1}(\sin(e+fx))-3\tan(e+fx)\sec(e+fx)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] - 3*Sec[e + f*x]*Tan[e + f*x] - 2*Tan[e + f
*x]^3))/(6*f)
```

fricas [A] time = 0.46, size = 103, normalized size = 1.69

$$\frac{3\,a^2c\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,a^2c\cos\left(fx+e\right)^2\right.}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a^2*c*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) + 2*(2*a^2*c*cos(f*x + e)^2 - 3*a^2*c*cos(f*x + e)
- 2*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(a^2*c/4*ln(abs(tan((f*x+exp(1))/2)-1))-a^2*c/4*ln(abs(tan((f*
x+exp(1))/2)+1))+(3*tan((f*x+exp(1))/2)^5*a^2*c-8*tan((f*x+exp(1))/2)^3*a^2
*c-3*tan((f*x+exp(1))/2)*a^2*c)*1/6/(tan((f*x+exp(1))/2)^2-1)^3)
```

maple [A] time = 1.10, size = 84, normalized size = 1.38

$$\frac{a^2 c \tan\left(fx+e\right)}{3f}+\frac{a^2 c \ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}-\frac{a^2 c \sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}-\frac{a^2 c \tan\left(fx+e\right)\left(\sec^2\left(fx\right.\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/3*a^2*c*tan(f*x+e)/f+1/2/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))-1/2*a^2*c*sec(
f*x+e)*tan(f*x+e)/f-1/3/f*a^2*c*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.38, size = 108, normalized size = 1.77

$$-\frac{4\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c-3\,a^2c\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-1\!\!\!\!\!\quad}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*
a^2*c*log(sec(f*x + e) + tan(f*x + e)) - 12*a^2*c*tan(f*x + e))/f
```

mupad [B] time = 3.79, size = 113, normalized size = 1.85

$$\frac{-c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}+\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a^2*c*tan(e/2 + (f*x)/2) + (8*a^2*c*tan(e/2 + (f*x)/2)^3)/3 - a^2*c*tan(e/
2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e
/2 + (f*x)/2)^6 - 1)) + (a^2*c*atanh(tan(e/2 + (f*x)/2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2c\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int\left(-\sec^2\left(e+fx\right)\right)dx+\int\sec^3\left(e+fx\right)dx+\int\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integ
ral(sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

3.15 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{3a^2\tan(e+fx)}{cf}-\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

```
[Out] -3*a^2*arctanh(sin(f*x+e))/c/f-3*a^2*tan(f*x+e)/c/f-2*(a^2+a^2*sec(f*x+e))*
tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$-\frac{3a^2\tan(e+fx)}{cf}-\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-3*a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (3*a^2*Tan[e + f*x])/(c*f) - (2*(a^2
 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(3a)\int \sec(e+fx)(a+a\sec(e+$$

$$= -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{\left(3a^2\right)\int \sec(e+fx)\,dx}{c} - \frac{\left(3a^2\right)}{}$$

$$= -\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf} - \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))} + \frac{\left(3a^2\right.}{}$$

$$= -\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf} - \frac{3a^2\tan(e+fx)}{cf} - \frac{2\left(a^2+a^2\sec(e+fx)\right)}{f(c-c\sec(e+}$$

Mathematica [B] time = 1.87, size = 220, normalized size = 2.97

$$\frac{2a^2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(4\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^2*Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]*(4*Csc[e/2]*Sec[(e +
f*x)/2]*Sin[(f*x)/2] + (-3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 3*Log
[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2] - Sin[e/2])*(Co
s[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])))*Tan[(e + f*x)/2]))/(f*(c - c*Sec[e + f*x]))
```

fricas [A] time = 0.45, size = 108, normalized size = 1.46

$$-\frac{3\,a^2\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-3\,a^2\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-}{2\,cf\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*a^2*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*a^2*cos(f*x
 + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) - 10*a^2*cos(f*x + e)^2 - 8*a^2*c
os(f*x + e) + 2*a^2)/(c*f*cos(f*x + e)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(-3*a^2*1/2/c*ln(abs(tan((f*x+exp(1))/2)-1))+3*a^2*1/2/c*ln(ab
s(tan((f*x+exp(1))/2)+1))-(3*tan((f*x+exp(1))/2)^2*a^2-2*a^2)/c/(tan((f*x+e
xp(1))/2)^3-tan((f*x+exp(1))/2)))
```

maple [A] time = 0.64, size = 116, normalized size = 1.57

$$\frac{4a^2}{fc\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{a^2}{fc\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}+\frac{3a^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fc}+\frac{a^2}{fc\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}-\frac{3a^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

```
[Out] 4/f*a^2/c/tan(1/2*e+1/2*f*x)+1/f*a^2/c/(tan(1/2*e+1/2*f*x)-1)+3/f*a^2/c*ln(
tan(1/2*e+1/2*f*x)-1)+1/f*a^2/c/(tan(1/2*e+1/2*f*x)+1)-3/f*a^2/c*ln(tan(1/2
*e+1/2*f*x)+1)
```

maxima [B] time = 0.37, size = 225, normalized size = 3.04

$$-\frac{a^2\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+2a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx}{c\sin(}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(a^2*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x
 + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) + 2*a^2
*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a^2*(cos(f*x + e) +
 1)/(c*sin(f*x + e)))/f
```

mupad [B] time = 1.91, size = 77, normalized size = 1.04

$$\frac{6\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-4\,a^2}{c\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}-\frac{6\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] (6*a^2*tan(e/2 + (f*x)/2)^2 - 4*a^2)/(c*f*tan(e/2 + (f*x)/2)*(tan(e/2 + (f*
x)/2)^2 - 1)) - (6*a^2*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(2*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1),
 x))/c
```

3.16 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2} dx$

Optimal. Leaf size=89

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] a^2*arctanh(sin(f*x+e))/c^2/f-2/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*se
c(f*x+e))^2+2*a^2*tan(f*x+e)/f/(c^2-c^2*sec(f*x+e))
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(c^2*f) - (2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(3*f*(c - c*Sec[e + f*x])^2) + (2*a^2*Tan[e + f*x])/(f*(c^2 - c^2*Sec[
e + f*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^2} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx}{c}$$
$$= -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)} + \frac{a^2\int s$$
$$= \frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f} - \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{2a^2}{f\left(c^2-\right.}$$

Mathematica [A] time = 0.09, size = 109, normalized size = 1.22

$$\frac{a^2\left(-\frac{4\cot\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*((-4*Cot[(e + f*x)/2])/(3*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)
/(3*f) - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f))/c^2
```

fricas [A] time = 0.47, size = 128, normalized size = 1.44

$$-\frac{8\,a^2\cos\left(fx+e\right)^2 - 8\,a^2\cos\left(fx+e\right) - 3\left(a^2\cos\left(fx+e\right)-a^2\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right) + 3\left(a^2\cos\right.}{6\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(8*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) - 3*(a^2*cos(f*x + e) - a^2
)*log(sin(f*x + e) + 1)*sin(f*x + e) + 3*(a^2*cos(f*x + e) - a^2)*log(-sin(
f*x + e) + 1)*sin(f*x + e) - 16*a^2)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x
+ e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-a^2*1/2/c^2*ln(abs(tan((f*x+exp(1))/2)-1))+a^2*1/2/c^2*ln(abs
(tan((f*x+exp(1))/2)+1))+(-3*tan((f*x+exp(1))/2)^2*a^2-a^2)*1/3/c^2/tan((f*
x+exp(1))/2)^3)
```

maple [A] time = 0.79, size = 91, normalized size = 1.02

$$-\frac{2a^2}{3f\,c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}-\frac{2a^2}{f\,c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{a^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,c^2}+\frac{a^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{f\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -2/3/f*a^2/c^2/tan(1/2*e+1/2*f*x)^3-2/f*a^2/c^2/tan(1/2*e+1/2*f*x)-1/f*a^2/
c^2*ln(tan(1/2*e+1/2*f*x)-1)+1/f*a^2/c^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.35, size = 201, normalized size = 2.26

$$\frac{a^2\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}-\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right)-\frac{2a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}+\frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(a^2*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 2*a^2*(3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + a^2*(3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x +
e)^3))/f
```

mupad [B] time = 1.78, size = 63, normalized size = 0.71

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f}-\frac{2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{2\,a^2}{3}}{c^2\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (2*a^2*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) - (2*a^2*tan(e/2 + (f*x)/2)^2 + (
2*a^2)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(2*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

3.17 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=38

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

```
[Out] -1/5*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^3
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

Mathematica [A] time = 0.12, size = 25, normalized size = 0.66

$$\frac{a^2\cot^5\left(\frac{1}{2}(e+fx)\right)}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Cot[(e + f*x)/2]^5)/(5*c^3*f)
```

fricas [B] time = 0.44, size = 83, normalized size = 2.18

$$\frac{a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2}{5\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)/
((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

giac [A] time = 2.38, size = 23, normalized size = 0.61

$$\frac{a^2}{5\,c^3f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*a^2/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

maple [A] time = 0.81, size = 23, normalized size = 0.61

$$\frac{a^2}{5f\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/5/f*a^2/c^3/tan(1/2*e+1/2*f*x)^5
```

maxima [B] time = 0.35, size = 189, normalized size = 4.97

$$\frac{\frac{a^2\left(\frac{10\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+3\right)\left(\cos\left(fx+e\right)+1\right)^5}{c^3\sin\left(fx+e\right)^5}-\frac{a^2\left(\frac{10\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-3\right)\left(\cos\left(fx+e\right)+1\right)^5}{c^3\sin\left(fx+e\right)^5}-\frac{6\,a^2\left(\frac{5\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-1\right)\left(\cos\left(fx+\cdots\right)\right)}{c^3\sin\left(fx+e\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(a^2*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - a^2*(10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - 6*a^2*(5*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

mupad [**B**] time = 1.64, size = 22, normalized size = 0.58

$$\frac{a^2 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (a^2*cot(e/2 + (f*x)/2)^5)/(5*c^3*f)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
*3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.18 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=80

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -1/7*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-1/35*(a+a*sec(f*x+e
))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3
```

Rubi [A] time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4) - ((a +
 a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx}{7c}$$
$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}$$

Mathematica [A] time = 0.44, size = 115, normalized size = 1.44

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\left(140\sin\left(e+\frac{fx}{2}\right)-91\sin\left(e+\frac{3fx}{2}\right)-35\sin\left(2e+\frac{3fx}{2}\right)+7\sin\left(2e+\frac{5fx}{2}\right)+35\sin\left(3e+\frac{5fx}{2}\right)-6\sin}{2240c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/2240*(a^2*Csc[e/2]*Csc[(e + f*x)/2]^7*(70*Sin[(f*x)/2] + 140*Sin[e + (f*
x)/2] - 91*Sin[e + (3*f*x)/2] - 35*Sin[2*e + (3*f*x)/2] + 7*Sin[2*e + (5*f*
x)/2] + 35*Sin[3*e + (5*f*x)/2] - 6*Sin[3*e + (7*f*x)/2]))/(c^4*f)
```

fricas [A] time = 0.47, size = 114, normalized size = 1.42

$$\frac{6a^2\cos\left(fx+e\right)^4+17a^2\cos\left(fx+e\right)^3+15a^2\cos\left(fx+e\right)^2+3a^2\cos\left(fx+e\right)-a^2}{35\left(c^4f\cos\left(fx+e\right)^3-3c^4f\cos\left(fx+e\right)^2+3c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(6*a^2*cos(f*x + e)^4 + 17*a^2*cos(f*x + e)^3 + 15*a^2*cos(f*x + e)^2
+ 3*a^2*cos(f*x + e) - a^2)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

giac [A] time = 0.39, size = 43, normalized size = 0.54

$$\frac{7a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5a^2}{70c^4f\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/70*(7*a^2*tan(1/2*f*x + 1/2*e)^2 - 5*a^2)/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

maple [A] time = 0.80, size = 39, normalized size = 0.49

$$\frac{a^{2}\left(-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}\right)}{2f\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/2/f*a^2/c^4*(-1/7/tan(1/2*e+1/2*f*x)^7+1/5/tan(1/2*e+1/2*f*x)^5)
```

maxima [B] time = 0.35, size = 270, normalized size = 3.38

$$\frac{\frac{2\,a^{2}\left(\frac{21\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{35\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{105\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-15\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}+\frac{3\,a^{2}\left(\frac{21\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{35\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{35\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(2*a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x
 + e) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a^2*(21*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^2*(21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^4*sin
(f*x + e)^7))/f
```

mupad [B] time = 1.62, size = 37, normalized size = 0.46

$$-\frac{a^{2}\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}\left(5\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-7\right)}{70\,c^{4}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(a^2*cot(e/2 + (f*x)/2)^5*(5*cot(e/2 + (f*x)/2)^2 - 7))/(70*c^4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx + \int \frac{2\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx + \int \frac{}{\mathrm{se}}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

3.19 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx$

Optimal. Leaf size=121

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{315c^2f(c-c\sec(e+fx))^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

```
[Out] -1/9*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-2/63*(a+a*sec(f*x+e
))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/315*(a+a*sec(f*x+e))^2*tan(f*x+e)/
c^2/f/(c-c*sec(f*x+e))^3
```

Rubi [A] time = 0.23, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{315c^2f(c-c\sec(e+fx))^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4) - (2*(a
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(315*c^2*f*(c - c*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx}{9c}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4} +$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4} -$$

Mathematica [A] time = 0.44, size = 141, normalized size = 1.17

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\left(2520\sin\left(e+\frac{fx}{2}\right)-1638\sin\left(e+\frac{3fx}{2}\right)-2310\sin\left(2e+\frac{3fx}{2}\right)+1062\sin\left(2e+\frac{5fx}{2}\right)+630\sin\left(3e+\right.\right.}{80640c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/80640*(a^2*Csc[e/2]*Csc[(e + f*x)/2]^9*(3402*Sin[(f*x)/2] + 2520*Sin[e +
 (f*x)/2] - 1638*Sin[e + (3*f*x)/2] - 2310*Sin[2*e + (3*f*x)/2] + 1062*Sin[
2*e + (5*f*x)/2] + 630*Sin[3*e + (5*f*x)/2] - 108*Sin[3*e + (7*f*x)/2] - 31
5*Sin[4*e + (7*f*x)/2] + 47*Sin[4*e + (9*f*x)/2]))/(c^5*f)
```

fricas [A] time = 0.43, size = 140, normalized size = 1.16

$$\frac{47\,a^2\cos\left(fx+e\right)^5+127\,a^2\cos\left(fx+e\right)^4+101\,a^2\cos\left(fx+e\right)^3+11\,a^2\cos\left(fx+e\right)^2-8\,a^2\cos\left(fx+e\right)+2\,a^2}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/315*(47*a^2*cos(f*x + e)^5 + 127*a^2*cos(f*x + e)^4 + 101*a^2*cos(f*x + e
)^3 + 11*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) + 2*a^2)/((c^5*f*cos(f*x +
 e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x +
 e) + c^5*f)*sin(f*x + e))
```

giac [A] time = 0.46, size = 60, normalized size = 0.50

$$\frac{63\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-90\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+35\,a^{2}}{1260\,c^{5}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/1260*(63*a^2*tan(1/2*f*x + 1/2*e)^4 - 90*a^2*tan(1/2*f*x + 1/2*e)^2 + 35*
a^2)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

maple [A] time = 0.83, size = 52, normalized size = 0.43

$$\frac{a^{2}\left(-\frac{2}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}\right)}{4f\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/4/f*a^2/c^5*(-2/7/tan(1/2*e+1/2*f*x)^7+1/9/tan(1/2*e+1/2*f*x)^9+1/5/tan(1
/2*e+1/2*f*x)^5)
```

maxima [B] time = 0.36, size = 269, normalized size = 2.22

$$-\frac{\frac{a^{2}\left(\frac{180\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{378\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{420\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{315\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-35\right)\left(\cos\left(fx+e\right)+1\right)^{9}}{c^{5}\sin\left(fx+e\right)^{9}}+\frac{10\,a^{2}\left(\frac{18\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{42\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{63\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-7}{c^{5}\sin\left(fx+e\right)^{9}}}{5040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^2*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 10*a^2*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
```

```
e) + 1)^9/(c^5*sin(f*x + e)^9) + 7*a^2*(18*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5
*sin(f*x + e)^9))/f
```

mupad [B] time = 1.63, size = 67, normalized size = 0.55

$$\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20\,c^5\,f}-\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14\,c^5\,f}+\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{36\,c^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a^2*cot(e/2 + (f*x)/2)^5)/(20*c^5*f) - (a^2*cot(e/2 + (f*x)/2)^7)/(14*c^5*
f) + (a^2*cot(e/2 + (f*x)/2)^9)/(36*c^5*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(\right.}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(2*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(
e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 +
 5*sec(e + f*x) - 1), x))/c**5
```

3.20 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=163

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{231c^2f(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{33cf(c-c\sec(e+fx))^5}-\frac{\tan(e+}{11}$$

```
[Out] -1/11*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-1/33*(a+a*sec(f*x+
e))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/231*(a+a*sec(f*x+e))^2*tan(f*x+e)
/c^2/f/(c-c*sec(f*x+e))^4-2/1155*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c^2-c^2*s
ec(f*x+e))^3
```

Rubi [A] time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{231c^2f(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{33cf(c-c\sec(e+fx))^5}-\frac{\tan(e+}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(11*f*(c - c*Sec[e + f*x])^6) - ((a
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(33*c*f*(c - c*Sec[e + f*x])^5) - (2*(a +
 a*Sec[e + f*x])^2*Tan[e + f*x])/(231*c^2*f*(c - c*Sec[e + f*x])^4) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*(c^2 - c^2*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

```
&& !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx}{11c}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} + \cdots$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \cdots$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \cdots$$

Mathematica [A] time = 0.67, size = 167, normalized size = 1.02

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\left(37422\sin\left(e+\frac{fx}{2}\right)-27060\sin\left(e+\frac{3fx}{2}\right)-23100\sin\left(2e+\frac{3fx}{2}\right)+11220\sin\left(2e+\frac{5fx}{2}\right)+13860\sin\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/1182720*(a^2*Csc[e/2]*Csc[(e + f*x)/2]^11*(32802*Sin[(f*x)/2] + 37422*Si
n[e + (f*x)/2] - 27060*Sin[e + (3*f*x)/2] - 23100*Sin[2*e + (3*f*x)/2] + 11
220*Sin[2*e + (5*f*x)/2] + 13860*Sin[3*e + (5*f*x)/2] - 4895*Sin[3*e + (7*f
*x)/2] - 3465*Sin[4*e + (7*f*x)/2] + 517*Sin[4*e + (9*f*x)/2] + 1155*Sin[5*
e + (9*f*x)/2] - 152*Sin[5*e + (11*f*x)/2]))/(c^6*f)
```

fricas [A] time = 0.45, size = 168, normalized size = 1.03

$$\frac{152\,a^2\cos\left(fx+e\right)^6+395\,a^2\cos\left(fx+e\right)^5+289\,a^2\cos\left(fx+e\right)^4+15\,a^2\cos\left(fx+e\right)^3-19\,a^2\cos\left(fx+e\right)^2+1\cdots}{1155\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/1155*(152*a^2*cos(f*x + e)^6 + 395*a^2*cos(f*x + e)^5 + 289*a^2*cos(f*x +
 e)^4 + 15*a^2*cos(f*x + e)^3 - 19*a^2*cos(f*x + e)^2 + 10*a^2*cos(f*x + e)
 - 2*a^2)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*
x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x
+ e))
```

giac [A] time = 0.49, size = 77, normalized size = 0.47

$$\frac{231\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-495\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+385\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-105\,a^{2}}{9240\,c^{6}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/9240*(231*a^2*tan(1/2*f*x + 1/2*e)^6 - 495*a^2*tan(1/2*f*x + 1/2*e)^4 + 3
85*a^2*tan(1/2*f*x + 1/2*e)^2 - 105*a^2)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

maple [A] time = 0.90, size = 65, normalized size = 0.40

$$\frac{a^{2}\left(-\frac{1}{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}-\frac{3}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}\right)}{8f\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x)
```

```
[Out] 1/8/f*a^2/c^6*(-1/11/tan(1/2*e+1/2*f*x)^11-3/7/tan(1/2*e+1/2*f*x)^7+1/3/tan
(1/2*e+1/2*f*x)^9+1/5/tan(1/2*e+1/2*f*x)^5)
```

maxima [B] time = 0.36, size = 389, normalized size = 2.39

$$\frac{\frac{a^{2}\left(\frac{385\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{990\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{1386\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{1155\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}+\frac{3465\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}-315\right)\left(\cos\left(fx+e\right)+1\right)^{11}}{c^{6}\sin\left(fx+e\right)^{11}}+\frac{6\,a^{2}\left(\frac{385\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{330\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 6*a^2*(385*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) + 1)
^11/(c^6*sin(f*x + e)^11) + 5*a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

mupad [B] time = 1.69, size = 108, normalized size = 0.66

$$-\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(105\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-385\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+495\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-23\right.}{9240\,c^6\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -(a^2*cos(e/2 + (f*x)/2)^5*(105*cos(e/2 + (f*x)/2)^6 - 231*sin(e/2 + (f*x)/
2)^6 + 495*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 385*cos(e/2 + (f*x)/
2)^4*sin(e/2 + (f*x)/2)^2))/(9240*c^6*f*sin(e/2 + (f*x)/2)^11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e+fx\right)-20\sec^3\left(e+fx\right)+15\sec^2\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int\frac{}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 +
15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*
x) + 1), x))/c**6
```

3.21 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$

Optimal. Leaf size=227

$$\frac{a^3c^6\tan^9(e+fx)}{9f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{3a^3c^6\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^6\tan}{}$$

```
[Out] 55/128*a^3*c^6*arctanh(sin(f*x+e))/f-25/128*a^3*c^6*sec(f*x+e)*tan(f*x+e)/f
-15/64*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^6*sec(f*x+e)*tan(f*x+e)
^3/f+5/16*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^6*sec(f*x+e)*tan(f*
x+e)^5/f-3/8*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^5/f+4/7*a^3*c^6*tan(f*x+e)^7/f
+1/9*a^3*c^6*tan(f*x+e)^9/f
```

Rubi [A] time = 0.34, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2611, 3770, 2607, 30, 3768, 14}

$$\frac{a^3c^6\tan^9(e+fx)}{9f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{3a^3c^6\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^6\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (55*a^3*c^6*ArcTanh[Sin[e + f*x]])/(128*f) - (25*a^3*c^6*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (15*a^3*c^6*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3
*c^6*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^6*Sec[e + f*x]^3*Tan[e
+ f*x]^3)/(16*f) - (a^3*c^6*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (3*a^3*c^6
*Sec[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (4*a^3*c^6*Tan[e + f*x]^7)/(7*f) +
(a^3*c^6*Tan[e + f*x]^9)/(9*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx = -\left(\left(a^3c^3\right)\int \left(c^3\sec(e+fx)\tan^6(e+fx)-3c^3\sec^2(\right.\right.$$
$$= -\left(\left(a^3c^6\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+\left(a^3c^6\right)\int$$
$$= -\frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{3a^3c^6\sec^3(e+fx)}{8f}$$
$$= \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f}+\frac{5a^3c^6\sec^3(e+fx)}{16f}$$
$$= -\frac{5a^3c^6\sec(e+fx)\tan(e+fx)}{16f}-\frac{15a^3c^6\sec^3(e+fx}{64f}$$
$$= \frac{5a^3c^6\tanh^{-1}(\sin(e+fx))}{16f}-\frac{25a^3c^6\sec(e+fx)\tan}{128f}$$
$$= \frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{25a^3c^6\sec(e+fx)\tan}{128f}$$

Mathematica [A] time = 3.39, size = 122, normalized size = 0.54

$$\frac{a^3c^6\left(443520\tanh^{-1}(\sin(e+fx))-(-88704\sin(e+fx)+88074\sin(2(e+fx))+37632\sin(3(e+fx))-2142\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(443520*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^9*(-88704*Sin[e + f*x
] + 88074*Sin[2*(e + f*x)] + 37632*Sin[3*(e + f*x)] - 2142*Sin[4*(e + f*x)]
 + 2304*Sin[5*(e + f*x)] + 39858*Sin[6*(e + f*x)] - 7488*Sin[7*(e + f*x)] +
 4599*Sin[8*(e + f*x)] + 1856*Sin[9*(e + f*x)])))/(1032192*f)
```

fricas [A] time = 0.52, size = 209, normalized size = 0.92

$$\frac{3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(\sin\left(fx+e\right)+1\right)-3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(-\sin\left(fx+e\right)+1\right)-2\left(3712\,a^3c^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/16128*(3465*a^3*c^6*cos(f*x + e)^9*log(sin(f*x + e) + 1) - 3465*a^3*c^6*c
os(f*x + e)^9*log(-sin(f*x + e) + 1) - 2*(3712*a^3*c^6*cos(f*x + e)^8 + 459
9*a^3*c^6*cos(f*x + e)^7 - 10240*a^3*c^6*cos(f*x + e)^6 + 3066*a^3*c^6*cos(
f*x + e)^5 + 8448*a^3*c^6*cos(f*x + e)^4 - 7224*a^3*c^6*cos(f*x + e)^3 - 10
24*a^3*c^6*cos(f*x + e)^2 + 3024*a^3*c^6*cos(f*x + e) - 896*a^3*c^6)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-55*a^3*c^6/256*ln(abs(tan((f*x+exp(1))/2)-1))+55*a^3*c^6/256*
ln(abs(tan((f*x+exp(1))/2)+1))-(3465*tan((f*x+exp(1))/2)^17*a^3*c^6-30030*t
an((f*x+exp(1))/2)^15*a^3*c^6+115038*tan((f*x+exp(1))/2)^13*a^3*c^6+334602*
tan((f*x+exp(1))/2)^11*a^3*c^6-360448*tan((f*x+exp(1))/2)^9*a^3*c^6+255222*
tan((f*x+exp(1))/2)^7*a^3*c^6-115038*tan((f*x+exp(1))/2)^5*a^3*c^6+30030*ta
n((f*x+exp(1))/2)^3*a^3*c^6-3465*tan((f*x+exp(1))/2)*a^3*c^6)*1/8064/(tan((
f*x+exp(1))/2)^2-1)^9)
```

maple [A] time = 1.93, size = 242, normalized size = 1.07

$$-\frac{29a^3c^6\tan\left(fx+e\right)}{63f}+\frac{a^3c^6\tan\left(fx+e\right)\left(\sec^8\left(fx+e\right)\right)}{9f}+\frac{8a^3c^6\tan\left(fx+e\right)\left(\sec^6\left(fx+e\right)\right)}{63f}-\frac{22a^3c^6\tan\left(fx+e\right.}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x)
```

```
[Out] -29/63/f*a^3*c^6*tan(f*x+e)+1/9/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^8+8/63/f*a^
3*c^6*tan(f*x+e)*sec(f*x+e)^6-22/21/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^4+80/63
/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^2+55/128/f*a^3*c^6*ln(sec(f*x+e)+tan(f*x+e
))-73/192*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)/f-73/128*a^3*c^6*sec(f*x+e)*tan(f
*x+e)/f+43/48/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^5-3/8/f*a^3*c^6*tan(f*x+e)*se
c(f*x+e)^7
```

maxima [B] time = 0.35, size = 443, normalized size = 1.95

$$256\left(35\tan\left(fx+e\right)^9+180\tan\left(fx+e\right)^7+378\tan\left(fx+e\right)^5+420\tan\left(fx+e\right)^3+315\tan\left(fx+e\right)\right)a^3c^6-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/80640*(256*(35*tan(f*x + e)^9 + 180*tan(f*x + e)^7 + 378*tan(f*x + e)^5 +
 420*tan(f*x + e)^3 + 315*tan(f*x + e))*a^3*c^6 - 32256*(3*tan(f*x + e)^5 +
 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^6 + 215040*(tan(f*x + e)^3 + 3*
tan(f*x + e))*a^3*c^6 + 315*a^3*c^6*(2*(105*sin(f*x + e)^7 - 385*sin(f*x +
e)^5 + 511*sin(f*x + e)^3 - 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x +
 e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1
) + 105*log(sin(f*x + e) - 1)) - 6720*a^3*c^6*(2*(15*sin(f*x + e)^5 - 40*si
n(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(
f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 30
240*a^3*c^6*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 8064
0*a^3*c^6*log(sec(f*x + e) + tan(f*x + e)) - 241920*a^3*c^6*tan(f*x + e))/f
```

mupad [B] time = 5.58, size = 316, normalized size = 1.39

$$\frac{55\,a^3\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f}-\frac{\frac{55\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{17}}{64}-\frac{715\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{96}+\frac{913\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{32}+\frac{18589\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{224}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{18}-9\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}+36\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-84\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6)/cos(e + f*x),x)
```

```
[Out] (55*a^3*c^6*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((715*a^3*c^6*tan(e/2 + (f*
x)/2)^3)/96 - (913*a^3*c^6*tan(e/2 + (f*x)/2)^5)/32 + (14179*a^3*c^6*tan(e/
2 + (f*x)/2)^7)/224 - (5632*a^3*c^6*tan(e/2 + (f*x)/2)^9)/63 + (18589*a^3*c
^6*tan(e/2 + (f*x)/2)^11)/224 + (913*a^3*c^6*tan(e/2 + (f*x)/2)^13)/32 - (7
15*a^3*c^6*tan(e/2 + (f*x)/2)^15)/96 + (55*a^3*c^6*tan(e/2 + (f*x)/2)^17)/6
4 - (55*a^3*c^6*tan(e/2 + (f*x)/2))/64)/(f*(9*tan(e/2 + (f*x)/2)^2 - 36*tan
(e/2 + (f*x)/2)^4 + 84*tan(e/2 + (f*x)/2)^6 - 126*tan(e/2 + (f*x)/2)^8 + 12
6*tan(e/2 + (f*x)/2)^10 - 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^
14 - 9*tan(e/2 + (f*x)/2)^16 + tan(e/2 + (f*x)/2)^18 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3c^6\left(\int\sec\left(e+fx\right)dx+\int\left(-3\sec^2\left(e+fx\right)\right)dx+\int 8\sec^4\left(e+fx\right)dx+\int\left(-6\sec^5\left(e+fx\right)\right)dx+\int\left(-6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*c**6*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + In
tegral(8*sec(e + f*x)**4, x) + Integral(-6*sec(e + f*x)**5, x) + Integral(-
6*sec(e + f*x)**6, x) + Integral(8*sec(e + f*x)**7, x) + Integral(-3*sec(e
+ f*x)**9, x) + Integral(sec(e + f*x)**10, x))
```

3.22 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=206

$$\frac{2a^3c^5\tan^7(e+fx)}{7f}+\frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{a^3c^5\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^5\tan^3(e+fx)\sec^3(e+f}{48f}$$

```
[Out] 45/128*a^3*c^5*arctanh(sin(f*x+e))/f-35/128*a^3*c^5*sec(f*x+e)*tan(f*x+e)/f
-5/64*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^5*sec(f*x+e)*tan(f*x+e)^
3/f+5/48*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^5*sec(f*x+e)*tan(f*x
+e)^5/f-1/8*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^5/f+2/7*a^3*c^5*tan(f*x+e)^7/f
```

Rubi [A] time = 0.30, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$\frac{2a^3c^5\tan^7(e+fx)}{7f}+\frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{a^3c^5\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^5\tan^3(e+fx)\sec^3(e+f}{48f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (45*a^3*c^5*ArcTanh[Sin[e + f*x]])/(128*f) - (35*a^3*c^5*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (5*a^3*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3*
c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^5*Sec[e + f*x]^3*Tan[e +
 f*x]^3)/(48*f) - (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (a^3*c^5*Se
c[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (2*a^3*c^5*Tan[e + f*x]^7)/(7*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -\left(\left(a^3c^3\right)\int\left(c^2\sec(e+fx)\tan^6(e+fx)-2c^2\sec^2(\right.\right.$$

$$= -\left(\left(a^3c^5\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)-\left(a^3c^5\right)\int$$

$$= -\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{a^3c^5\sec^3(e+fx)\,\mathrm{t}}{8f}$$

$$= \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f}+\frac{5a^3c^5\sec^3(e+fx)}{48f}$$

$$= -\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{16f}-\frac{5a^3c^5\sec^3(e+fx)}{64f}$$

$$= \frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{35a^3c^5\sec(e+fx)\tan}{128f}$$

$$= \frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{35a^3c^5\sec(e+fx)\tan}{128f}$$

Mathematica [A] time = 2.54, size = 111, normalized size = 0.54

$$-\frac{a^3c^5\left((5705\sin(e+fx)-1792\sin(2(e+fx))+21\sin(3(e+fx))+1792\sin(4(e+fx))+2065\sin(5(e+fx))\right.}{5734}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/57344*(a^3*c^5*(-20160*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^8*(5705*Sin[
e + f*x] - 1792*Sin[2*(e + f*x)] + 21*Sin[3*(e + f*x)] + 1792*Sin[4*(e + f*
x)] + 2065*Sin[5*(e + f*x)] - 768*Sin[6*(e + f*x)] + 581*Sin[7*(e + f*x)] +
 128*Sin[8*(e + f*x)])))/f
```

fricas [A] time = 0.50, size = 193, normalized size = 0.94

$$\frac{315\,a^3c^5\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-315\,a^3c^5\cos\left(fx+e\right)^8\log\left(-\sin\left(fx+e\right)+1\right)-2\left(256\,a^3c^5\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1792*(315*a^3*c^5*cos(f*x + e)^8*log(sin(f*x + e) + 1) - 315*a^3*c^5*cos(
f*x + e)^8*log(-sin(f*x + e) + 1) - 2*(256*a^3*c^5*cos(f*x + e)^7 + 581*a^3
*c^5*cos(f*x + e)^6 - 768*a^3*c^5*cos(f*x + e)^5 - 210*a^3*c^5*cos(f*x + e)
^4 + 768*a^3*c^5*cos(f*x + e)^3 - 168*a^3*c^5*cos(f*x + e)^2 - 256*a^3*c^5*
cos(f*x + e) + 112*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(45*a^3*c^5/256*ln(abs(tan((f*x+exp(1))/2)-1))-45*a^3*c^5/256*
ln(abs(tan((f*x+exp(1))/2)+1))+(315*tan((f*x+exp(1))/2)^15*a^3*c^5-2415*tan
((f*x+exp(1))/2)^13*a^3*c^5+8043*tan((f*x+exp(1))/2)^11*a^3*c^5+17609*tan((
f*x+exp(1))/2)^9*a^3*c^5-15159*tan((f*x+exp(1))/2)^7*a^3*c^5+8043*tan((f*x+
exp(1))/2)^5*a^3*c^5-2415*tan((f*x+exp(1))/2)^3*a^3*c^5+315*tan((f*x+exp(1)
)/2)*a^3*c^5)*1/896/(tan((f*x+exp(1))/2)^2-1)^8)
```

maple [A] time = 1.91, size = 217, normalized size = 1.05

$$-\frac{2a^3c^5\tan\left(fx+e\right)}{7f}-\frac{6a^3c^5\tan\left(fx+e\right)\left(\sec^4\left(fx+e\right)\right)}{7f}+\frac{6a^3c^5\tan\left(fx+e\right)\left(\sec^2\left(fx+e\right)\right)}{7f}-\frac{83a^3c^5\sec\left(fx+e\right)}{128f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x)
```

```
[Out] -2/7*a^3*c^5*tan(f*x+e)/f-6/7/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^4+6/7/f*a^3*c
^5*tan(f*x+e)*sec(f*x+e)^2-83/128*a^3*c^5*sec(f*x+e)*tan(f*x+e)/f+45/128/f*
a^3*c^5*ln(sec(f*x+e)+tan(f*x+e))+3/16/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^5+15
/64*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)/f+2/7/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^6
-1/8/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^7
```

maxima [B] time = 0.35, size = 408, normalized size = 1.98

$$\frac{1536\left(5\tan\left(fx+e\right)^7+21\tan\left(fx+e\right)^5+35\tan\left(fx+e\right)^3+35\tan\left(fx+e\right)\right)a^3c^5-10752\left(3\tan\left(fx+e\right)^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/26880*(1536*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 3
5*tan(f*x + e))*a^3*c^5 - 10752*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*
tan(f*x + e))*a^3*c^5 + 53760*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^5 + 3
5*a^3*c^5*(2*(105*sin(f*x + e)^7 - 385*sin(f*x + e)^5 + 511*sin(f*x + e)^3
- 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 -
 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1) + 105*log(sin(f*x + e) -
 1)) - 560*a^3*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x +
 e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(si
n(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 13440*a^3*c^5*(2*sin(f*x + e)
/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 26
880*a^3*c^5*log(sec(f*x + e) + tan(f*x + e)) - 53760*a^3*c^5*tan(f*x + e))/
f
```

mupad [B] time = 5.52, size = 284, normalized size = 1.38

$$\frac{45\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f}-\frac{\frac{45\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{64}-\frac{345\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{64}+\frac{1149\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{64}+\frac{17609\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{448}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}-8\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}+28\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-56\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

```
[Out] (45*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((1149*a^3*c^5*tan(e/2 + (f
*x)/2)^5)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^3)/64 - (15159*a^3*c^5*tan(e
/2 + (f*x)/2)^7)/448 + (17609*a^3*c^5*tan(e/2 + (f*x)/2)^9)/448 + (1149*a^3
*c^5*tan(e/2 + (f*x)/2)^11)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^13)/64 + (
45*a^3*c^5*tan(e/2 + (f*x)/2)^15)/64 + (45*a^3*c^5*tan(e/2 + (f*x)/2))/64)/
(f*(28*tan(e/2 + (f*x)/2)^4 - 8*tan(e/2 + (f*x)/2)^2 - 56*tan(e/2 + (f*x)/2
)^6 + 70*tan(e/2 + (f*x)/2)^8 - 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*
x)/2)^12 - 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^3c^5\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int 2\sec^2\left(e+fx\right)dx+\int 2\sec^3\left(e+fx\right)dx+\int\left(-6\sec^4\left(e+fx\right)\right)dx+\int 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*c**5*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + I
ntegral(2*sec(e + f*x)**3, x) + Integral(-6*sec(e + f*x)**4, x) + Integral(
```

```
6*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x) + Integral(-2*sec(e
 + f*x)**8, x) + Integral(sec(e + f*x)**9, x))
```

3.23 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=121

$$\frac{a^3c^4\tan^7(e+fx)}{7f}+\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^4\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^4\tan^3(e+fx)\sec(e+fx)}{24f}-$$

```
[Out] 5/16*a^3*c^4*arctanh(sin(f*x+e))/f-5/16*a^3*c^4*sec(f*x+e)*tan(f*x+e)/f+5/2
4*a^3*c^4*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^4*sec(f*x+e)*tan(f*x+e)^5/f+1
/7*a^3*c^4*tan(f*x+e)^7/f
```

Rubi [A] time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2611, 3770, 2607, 30}

$$\frac{a^3c^4\tan^7(e+fx)}{7f}+\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^4\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^4\tan^3(e+fx)\sec(e+fx)}{24f}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^4*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f) + (a^3*c^4*Tan[e + f*x]^7)/(7*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\left(\left(a^3c^3\right)\int \left(c\sec(e+fx)\tan^6(e+fx)-c\sec^2(e+f\right.\right.$$

$$= -\left(\left(a^3c^4\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+\left(a^3c^4\right)\int s$$

$$= -\frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{1}{6}\left(5a^3c^4\right)\int \sec(e+$$

$$= \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^4\sec(e+fx)\tan}{6f}$$

$$= -\frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^4\sec(e+fx)\text{ta}}{24f}$$

$$= \frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^3c^4\sec(e+fx)\tan(e-}{16f}$$

Mathematica [A] time = 1.64, size = 102, normalized size = 0.84

$$\frac{a^3c^4\left(3360\tanh^{-1}(\sin(e+fx))-(-840\sin(e+fx)+595\sin(2(e+fx))+504\sin(3(e+fx))+196\sin(4(e+f\right.}{10752f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(3360*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(-840*Sin[e + f*x] +
595*Sin[2*(e + f*x)] + 504*Sin[3*(e + f*x)] + 196*Sin[4*(e + f*x)] - 168*Si
n[5*(e + f*x)] + 231*Sin[6*(e + f*x)] + 24*Sin[7*(e + f*x)])))/(10752*f)
```

fricas [A] time = 0.51, size = 177, normalized size = 1.46

$$\frac{105\,a^{3}c^{4}\cos\left(fx+e\right)^{7}\log\left(\sin\left(fx+e\right)+1\right)-105\,a^{3}c^{4}\cos\left(fx+e\right)^{7}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(48\,a^{3}c^{4}\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/672*(105*a^3*c^4*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 105*a^3*c^4*cos(f
*x + e)^7*log(-sin(f*x + e) + 1) - 2*(48*a^3*c^4*cos(f*x + e)^6 + 231*a^3*c
^4*cos(f*x + e)^5 - 144*a^3*c^4*cos(f*x + e)^4 - 182*a^3*c^4*cos(f*x + e)^3
 + 144*a^3*c^4*cos(f*x + e)^2 + 56*a^3*c^4*cos(f*x + e) - 48*a^3*c^4)*sin(f
*x + e))/(f*cos(f*x + e)^7)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-5*a^3*c^4/32*ln(abs(tan((f*x+exp(1))/2)-1))+5*a^3*c^4/32*ln(a
bs(tan((f*x+exp(1))/2)+1))+(-105*tan((f*x+exp(1))/2)^13*a^3*c^4+700*tan((f*
x+exp(1))/2)^11*a^3*c^4-1981*tan((f*x+exp(1))/2)^9*a^3*c^4-3072*tan((f*x+ex
p(1))/2)^7*a^3*c^4+1981*tan((f*x+exp(1))/2)^5*a^3*c^4-700*tan((f*x+exp(1))/
2)^3*a^3*c^4+105*tan((f*x+exp(1))/2)*a^3*c^4)*1/336/(tan((f*x+exp(1))/2)^2-
1)^7)
```

maple [A] time = 1.95, size = 192, normalized size = 1.59

$$\frac{13a^{3}c^{4}\tan\left(fx+e\right)\left(\sec^{3}\left(fx+e\right)\right)}{24f}-\frac{11a^{3}c^{4}\sec\left(fx+e\right)\tan\left(fx+e\right)}{16f}+\frac{5a^{3}c^{4}\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x)
```

```
[Out] 13/24/f*a^3*c^4*tan(f*x+e)*sec(f*x+e)^3-11/16*a^3*c^4*sec(f*x+e)*tan(f*x+e)
/f+5/16/f*a^3*c^4*ln(sec(f*x+e)+tan(f*x+e))-1/7*a^3*c^4*tan(f*x+e)/f+3/7/f*
a^3*c^4*tan(f*x+e)*sec(f*x+e)^2-3/7/f*a^3*c^4*tan(f*x+e)*sec(f*x+e)^4-1/6/f
*a^3*c^4*tan(f*x+e)*sec(f*x+e)^5+1/7/f*a^3*c^4*tan(f*x+e)*sec(f*x+e)^6
```

maxima [B] time = 0.36, size = 368, normalized size = 3.04

$$\frac{96\left(5\tan\left(fx+e\right)^{7}+21\tan\left(fx+e\right)^{5}+35\tan\left(fx+e\right)^{3}+35\tan\left(fx+e\right)\right)a^{3}c^{4}-672\left(3\tan\left(fx+e\right)^{5}+10\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*t
an(f*x + e))*a^3*c^4 - 672*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f
*x + e))*a^3*c^4 + 3360*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 + 35*a^3*
c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x +
 e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1)
 + 15*log(sin(f*x + e) - 1)) - 630*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x
 + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) +
3*log(sin(f*x + e) - 1)) + 2520*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1
) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^4*log(sec(f
*x + e) + tan(f*x + e)) - 3360*a^3*c^4*tan(f*x + e))/f
```

mupad [B] time = 5.71, size = 252, normalized size = 2.08

$$\frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}-\frac{\frac{5\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}-\frac{25\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{6}+\frac{283\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{128\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{7}-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((25*a^3*c^4*tan(e/2 + (f*x)/
2)^3)/6 - (283*a^3*c^4*tan(e/2 + (f*x)/2)^5)/24 + (128*a^3*c^4*tan(e/2 + (f
*x)/2)^7)/7 + (283*a^3*c^4*tan(e/2 + (f*x)/2)^9)/24 - (25*a^3*c^4*tan(e/2 +
 (f*x)/2)^11)/6 + (5*a^3*c^4*tan(e/2 + (f*x)/2)^13)/8 - (5*a^3*c^4*tan(e/2
+ (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*ta
```

```
n(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7
*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3c^4\left(\int \sec\left(e+fx\right)dx + \int\left(-\sec^2\left(e+fx\right)\right)dx + \int\left(-3\sec^3\left(e+fx\right)\right)dx + \int 3\sec^4\left(e+fx\right)dx + \int 3\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Inte
gral(-3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4, x) + Integral(3*s
ec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + Integral(-sec(e + f*
x)**7, x) + Integral(sec(e + f*x)**8, x))
```

3.24 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=100

$$\frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^3\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^3\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^3\tan(e+fx)\sec}{16f}$$

```
[Out] 5/16*a^3*c^3*arctanh(sin(f*x+e))/f-5/16*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+5/2
4*a^3*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^3*sec(f*x+e)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2611, 3770}

$$\frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^3\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^3\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^3\tan(e+fx)\sec}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^3*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^3*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{1}{6}\left(5a^3c^3\right)\int \sec(e$$
$$= \frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^3\sec(e+fx)\,\mathrm{ta}}{6f}$$
$$= -\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f} + \frac{5a^3c^3\sec(e+fx)}{24f}$$
$$= \frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f} - \frac{5a^3c^3\sec(e+fx)\tan(e}{16f}$$

Mathematica [A] time = 0.25, size = 60, normalized size = 0.60

$$-\frac{a^3c^3\left((28\cos(2(e+fx))+33\cos(4(e+fx))+59)\tan(e+fx)\sec^5(e+fx)-120\tanh^{-1}(\sin(e+fx))\right)}{384f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/384*(a^3*c^3*(-120*ArcTanh[Sin[e + f*x]] + (59 + 28*Cos[2*(e + f*x)] + 3
3*Cos[4*(e + f*x)])*Sec[e + f*x]^5*Tan[e + f*x]))/f
```

fricas [A] time = 0.48, size = 115, normalized size = 1.15

$$\frac{15\,a^3c^3\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^3\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(33\,a^3c^3\cos\left(fx\right.\right.}{96\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/96*(15*a^3*c^3*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*a^3*c^3*cos(f*x
+ e)^6*log(-sin(f*x + e) + 1) - 2*(33*a^3*c^3*cos(f*x + e)^4 - 26*a^3*c^3*c
os(f*x + e)^2 + 8*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)-2/f*(5*a^3*c^3/64*ln(abs(sin(f*x+exp(1))-1))-5*a^3*c^3/64*ln(abs(s
in(f*x+exp(1))+1))+(-33*sin(f*x+exp(1))^5*a^3*c^3+40*sin(f*x+exp(1))^3*a^3*
c^3-15*sin(f*x+exp(1))*a^3*c^3)*1/96/(sin(f*x+exp(1))^2-1)^3)
```

maple [A] time = 1.14, size = 100, normalized size = 1.00

$$-\frac{11a^3c^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{16f}+\frac{5a^3c^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16f}+\frac{13a^3c^3\tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{24f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -11/16*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+5/16/f*a^3*c^3*ln(sec(f*x+e)+tan(f*x
+e))+13/24/f*a^3*c^3*tan(f*x+e)*sec(f*x+e)^3-1/6/f*a^3*c^3*tan(f*x+e)*sec(f
*x+e)^5
```

maxima [B] time = 0.35, size = 244, normalized size = 2.44

$$a^3c^3\left(\frac{2\left(15\sin\left(fx+e\right)^5-40\sin\left(fx+e\right)^3+33\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^6-3\sin\left(fx+e\right)^4+3\sin\left(fx+e\right)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-18a^3c^3\left(\frac{2(}{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/96*(a^3*c^3*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/
(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
 + e) + 1) + 15*log(sin(f*x + e) - 1)) - 18*a^3*c^3*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e
) + 1) + 3*log(sin(f*x + e) - 1)) + 72*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e
)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 96*a^3*c^3*log(
sec(f*x + e) + tan(f*x + e)))/f
```

mupad [B] time = 5.56, size = 220, normalized size = 2.20

$$\frac{5\,a^3\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}-\frac{\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{4}+\frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{4}-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (5*a^3*c^3*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((33*a^3*c^3*tan(e/2 + (f*x)/
2)^5)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^3)/24 + (33*a^3*c^3*tan(e/2 + (f*x
)/2)^7)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (5*a^3*c^3*tan(e/2 + (f*
x)/2)^11)/8 + (5*a^3*c^3*tan(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4
 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)
^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^3c^3\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int 3\sec^3\left(e+fx\right)dx+\int\left(-3\sec^5\left(e+fx\right)\right)dx+\int\sec^7\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**3, x) + I
ntegral(-3*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**7, x))
```

3.25 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=94

$$\frac{a^3c^2\tan^5(e+fx)}{5f}+\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^3c^2\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] 3/8*a^3*c^2*arctanh(sin(f*x+e))/f-3/8*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^3*c^2*sec(f*x+e)*tan(f*x+e)^3/f+1/5*a^3*c^2*tan(f*x+e)^5/f
```

Rubi [A] time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2611, 3770, 2607, 30}

$$\frac{a^3c^2\tan^5(e+fx)}{5f}+\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^3c^2\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^3*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^3*c^2*Tan[e +
 f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx &= \left(a^2c^2\right)\int \left(a\sec(e+fx)\tan^4(e+fx)+a\sec^2(e+fx\right.\\ &= \left(a^3c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx+\left(a^3c^2\right)\int \sec\\ &= \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{1}{4}\left(3a^3c^2\right)\int \sec(e+\\ &= -\frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^2\sec(e+fx)\,\mathrm{ta}}{4f}\\ &= \frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^3c^2\sec(e+fx)\tan(e}{8f}\end{aligned}$$

Mathematica [A] time = 0.81, size = 81, normalized size = 0.86

$$\frac{a^3c^2\left(120\tanh^{-1}(\sin(e+fx))+(40\sin(e+fx)-10\sin(2(e+fx))-20\sin(3(e+fx))-25\sin(4(e+fx))+4\right.}{320f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(120*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^5*(40*Sin[e + f*x] - 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] - 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

fricas [A] time = 0.49, size = 145, normalized size = 1.54

$$\frac{15\,a^3c^2\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^2\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(8\,a^3c^2\cos\left(fx+\right.\right.}{80\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^3*c^2*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c^2*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^3*c^2*cos(f*x + e)^4 - 25*a^3*c^2*co
s(f*x + e)^3 - 16*a^3*c^2*cos(f*x + e)^2 + 10*a^3*c^2*cos(f*x + e) + 8*a^3*
c^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-3*a^3*c^2/16*ln(abs(tan((f*x+exp(1))/2)-1))+3*a^3*c^2/16*ln(a
bs(tan((f*x+exp(1))/2)+1))-(15*tan((f*x+exp(1))/2)^9*a^3*c^2-70*tan((f*x+ex
p(1))/2)^7*a^3*c^2+128*tan((f*x+exp(1))/2)^5*a^3*c^2+70*tan((f*x+exp(1))/2)
^3*a^3*c^2-15*tan((f*x+exp(1))/2)*a^3*c^2)*1/40/(tan((f*x+exp(1))/2)^2-1)^5
)
```

maple [A] time = 1.56, size = 142, normalized size = 1.51

$$-\frac{5a^3c^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{3a^3c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}+\frac{a^3c^2\tan\left(fx+e\right)}{5f}-\frac{2a^3c^2\tan\left(fx+e\right)\left(\right.}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -5/8*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+3/8/f*a^3*c^2*ln(sec(f*x+e)+tan(f*x+e)
)+1/5*a^3*c^2*tan(f*x+e)/f-2/5/f*a^3*c^2*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a^3*
c^2*tan(f*x+e)*sec(f*x+e)^3+1/5/f*a^3*c^2*tan(f*x+e)*sec(f*x+e)^4
```

maxima [B] time = 0.35, size = 227, normalized size = 2.41

$$16\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)a^3c^2-160\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3c^2-15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^2
- 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 - 15*a^3*c^2*(2*(3*sin(f*x
+ e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(si
n(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 120*a^3*c^2*(2*sin(f*x + e)/(s
in(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a
^3*c^2*log(sec(f*x + e) + tan(f*x + e)) + 240*a^3*c^2*tan(f*x + e))/f
```

mupad [B] time = 6.26, size = 188, normalized size = 2.00

$$\frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{3\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}-\frac{7\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}+\frac{7\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}-\frac{3\,a}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\,\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (3*a^3*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((7*a^3*c^2*tan(e/2 + (f*x)/2
)^3)/2 + (32*a^3*c^2*tan(e/2 + (f*x)/2)^5)/5 - (7*a^3*c^2*tan(e/2 + (f*x)/2
)^7)/2 + (3*a^3*c^2*tan(e/2 + (f*x)/2)^9)/4 - (3*a^3*c^2*tan(e/2 + (f*x)/2)
)/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f
*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3c^2\left(\int\sec\left(e+fx\right)dx+\int\sec^2\left(e+fx\right)dx+\int\left(-2\sec^3\left(e+fx\right)\right)dx+\int\left(-2\sec^4\left(e+fx\right)\right)dx+\int\sec^5\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*c**2*(Integral(sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Integ
ral(-2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(sec
(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

3.26 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=86

$$-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3a^3c\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] 5/8*a^3*c*arctanh(sin(f*x+e))/f-3/8*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^3*c
*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3a^3c\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*c*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a^2\sec(e+fx)\tan^2(e+fx)+2a^2\sec^2(e+\right.\right.$$

$$= -\left(\left(a^3c\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^3c\right)\int \mathrm{se}$$

$$= -\frac{a^3c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^3c\sec^3(e+fx)\tan(}{4f}$$

$$= \frac{a^3c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{3a^3c\sec(e+fx)\tan(e+f}{8f}$$

$$= \frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^3c\sec(e+fx)\tan(e+}{8f}$$

Mathematica [A] time = 0.60, size = 70, normalized size = 0.81

$$\frac{a^3c\left(60\tanh^{-1}(\sin(e+fx))-(33\sin(e+fx)+16\sin(2(e+fx))+9\sin(3(e+fx))-8\sin(4(e+fx)))\sec^4(e+\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*(60*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^4*(33*Sin[e + f*x] + 16*Sin
[2*(e + f*x)] + 9*Sin[3*(e + f*x)] - 8*Sin[4*(e + f*x)])))/(96*f)
```

fricas [A] time = 0.49, size = 117, normalized size = 1.36

$$\frac{15\,a^3c\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left(16\,a^3c\cos\left(fx+e\right)\right.}{48\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a^3*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) + 2*(16*a^3*c*cos(f*x + e)^3 - 9*a^3*c*cos(f*x +
e)^2 - 16*a^3*c*cos(f*x + e) - 6*a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(5*a^3*c/16*ln(abs(tan((f*x+exp(1))/2)-1))-5*a^3*c/16*ln(abs(t
an((f*x+exp(1))/2)+1))+(15*tan((f*x+exp(1))/2)^7*a^3*c-55*tan((f*x+exp(1))/
2)^5*a^3*c+73*tan((f*x+exp(1))/2)^3*a^3*c+15*tan((f*x+exp(1))/2)*a^3*c)*1/2
4/(tan((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.12, size = 107, normalized size = 1.24

$$\frac{2a^3c\tan\left(fx+e\right)}{3f}+\frac{5a^3c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}-\frac{2a^3c\tan\left(fx+e\right)\left(\sec^2\left(fx+e\right)\right)}{3f}-\frac{a^3c\left(\sec^3\left(fx+e\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

```
[Out] 2/3*a^3*c*tan(f*x+e)/f+5/8/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))-2/3/f*a^3*c*ta
n(f*x+e)*sec(f*x+e)^2-1/4*a^3*c*sec(f*x+e)^3*tan(f*x+e)/f-3/8*a^3*c*sec(f*x
+e)*tan(f*x+e)/f
```

maxima [A] time = 0.34, size = 133, normalized size = 1.55

$$-\frac{32\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c-3\,a^{3}c\left(\frac{2\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)-48\,a^{3}c\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-96\,a^{3}c\tan\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 3*a^3*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 48*a^3*c*log(sec(f*x + e) + tan
(f*x + e)) - 96*a^3*c*tan(f*x + e))/f
```

mupad [B] time = 4.79, size = 146, normalized size = 1.70

$$\frac{5\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{55\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{73\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (5*a^3*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a^3*c*tan(e/2 + (f*x)/2))/4
 + (73*a^3*c*tan(e/2 + (f*x)/2)^3)/12 - (55*a^3*c*tan(e/2 + (f*x)/2)^5)/12
+ (5*a^3*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2
+ (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^{3}c\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int\left(-2\sec^{2}\left(e+fx\right)\right)\,dx+\int 2\sec^{4}\left(e+fx\right)dx+\int\sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

3.27 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=100

$$-\frac{10a^3\tan(e+fx)}{cf}-\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{5a^3\tan(e+fx)\sec(e+fx)}{2cf}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

```
[Out] -15/2*a^3*arctanh(sin(f*x+e))/c/f-10*a^3*tan(f*x+e)/c/f-5/2*a^3*sec(f*x+e)*
tan(f*x+e)/c/f-2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$-\frac{10a^3\tan(e+fx)}{cf}-\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{5a^3\tan(e+fx)\sec(e+fx)}{2cf}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-15*a^3*ArcTanh[Sin[e + f*x]])/(2*c*f) - (10*a^3*Tan[e + f*x])/(c*f) - (5*
a^3*Sec[e + f*x]*Tan[e + f*x])/(2*c*f) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(5a)\int \sec(e+fx)(a+a\sec(e+$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(5a)\int \sec(e+fx)\left(a^2+a^2\sec^2\right.}{c}$$

$$= -\frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \Big($$

$$= -\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf} - \frac{10a^3\tan(e+fx)}{cf} - \frac{5a^3\sec(e+fx)\tan(e}{2cf}$$

Mathematica [B] time = 2.64, size = 287, normalized size = 2.87

$$a^3\cos^2(e+fx)\tan\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(32\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)+\tan\left(\frac{1}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^3*Tan[(e + f*x)/2
]*(32*Csc[e/2]*Sec[(e + f*x)/2]*Sin[(f*x)/2] + (-30*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 30*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(
-2) + (16*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*Tan[(e
+ f*x)/2]))/(16*f*(c - c*Sec[e + f*x]))
```

fricas [A] time = 0.49, size = 125, normalized size = 1.25

$$-\frac{15\,a^3\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-15\,a^3\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{4\,cf\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*a^3*cos(f*x + e)^2*log(sin(f*x + e) + 1)*sin(f*x + e) - 15*a^3*cos
(f*x + e)^2*log(-sin(f*x + e) + 1)*sin(f*x + e) - 48*a^3*cos(f*x + e)^3 - 3
4*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 2*a^3)/(c*f*cos(f*x + e)^2*sin
(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(-4*a^3/c/tan((f*x+exp(1))/2)-(7*tan((f*x+exp(1))/2)^3*a^3-9*t
an((f*x+exp(1))/2)*a^3)*1/2/c/(tan((f*x+exp(1))/2)^2-1)^2-15*a^3*1/4/c*ln(a
bs(tan((f*x+exp(1))/2)-1))+15*a^3*1/4/c*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.70, size = 166, normalized size = 1.66

$$\frac{8a^3}{fc\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{a^3}{2fc\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}+\frac{7a^3}{2fc\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}+\frac{15a^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{2fc}+\frac{a^3}{2fc\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

```
[Out] 8/f*a^3/c/tan(1/2*e+1/2*f*x)-1/2/f*a^3/c/(tan(1/2*e+1/2*f*x)-1)^2+7/2/f*a^3
/c/(tan(1/2*e+1/2*f*x)-1)+15/2/f*a^3/c*ln(tan(1/2*e+1/2*f*x)-1)+1/2/f*a^3/c
/(tan(1/2*e+1/2*f*x)+1)^2+7/2/f*a^3/c/(tan(1/2*e+1/2*f*x)+1)-15/2/f*a^3/c*l
n(tan(1/2*e+1/2*f*x)+1)
```

maxima [**B**] time = 0.34, size = 387, normalized size = 3.87

$$-\frac{a^3\left(\frac{2\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+6a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\cdots}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(a^3*(2*(5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*c*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/c - 3*log(sin(f*x + e)/(cos(f*x + e) + 1
) - 1)/c) + 6*a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x +
 e)/(cos(f*x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/c) + 6*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 2*a^3*(
cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

mupad [**B**] time = 3.17, size = 105, normalized size = 1.05

$$\frac{15\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-25\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] (15*a^3*tan(e/2 + (f*x)/2)^4 - 25*a^3*tan(e/2 + (f*x)/2)^2 + 8*a^3)/(f*(c*t
an(e/2 + (f*x)/2) - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^5)) - (
15*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x) - 1
), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) - 1), x))/c
```

3.28 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{5a^3\tan(e+fx)}{c^2f}+\frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{10\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2a\tan(e+fx)(a\sec(e+fx)+}{3f(c-c\sec(e+fx))^2}$$

```
[Out] 5*a^3*arctanh(sin(f*x+e))/c^2/f+5*a^3*tan(f*x+e)/c^2/f-2/3*a*(a+a*sec(f*x+e
))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^2+10/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f
/(c^2-c^2*sec(f*x+e))
```

Rubi [A] time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$\frac{5a^3\tan(e+fx)}{c^2f}+\frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{10\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2a\tan(e+fx)(a\sec(e+fx)+}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) + (5*a^3*Tan[e + f*x])/(c^2*f) - (2*a
*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) + (10*(a
^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c^2 - c^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx}{3c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10\left(a^3+a^3\sec(e+fx)\right)\tan(e+f}{3f\left(c^2-c^2\sec(e+fx)\right)}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10\left(a^3+a^3\sec(e+fx)\right)\tan(e+f}{3f\left(c^2-c^2\sec(e+fx)\right)}$$

$$= \frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10\left(a^3+\right.}{3}$$

$$= \frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan}{3f(c-c\sec(e+fx)}$$

Mathematica [B] time = 3.45, size = 402, normalized size = 3.38

$$\frac{a^3(\cos(e+fx)+1)^3\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-48\sin(e)\csc^3\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^7\left(\frac{1}{2}(e+fx)\right)\csc^4(e+fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(((-74 + 42*C
os[e] - 76*Cos[f*x] + 120*Cos[e + f*x] - 46*Cos[2*(e + f*x)] - 76*Cos[2*e +
 f*x] + 23*Cos[e + 2*f*x] + 23*Cos[3*e + 2*f*x])*Csc[e/2]^3*Sec[(e + f*x)/2
]^5*Sin[(f*x)/2])/16 - 48*Csc[e/2]^3*Csc[e + f*x]^4*Sin[e]*Sin[(f*x)/2]*Sin
[(e + f*x)/2]^7 + Cos[e]*Cos[e + f*x]*Csc[e/2]^2*Sec[(e + f*x)/2]^4*(4*Cot[
e/2] + 15*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]])*Sin[(e + f*x)/2]^2)*Tan[(e + f*x)/2] - 4*(-1 + 5*Cos[e
 + f*x])*Cot[e/2]^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sin[(f*x)/2]*Tan[(e + f*x)/
2]^2))/(6*c^2*f*(-1 + Cos[e + f*x])^2*(-1 + Cot[e/2])*(1 + Cot[e/2])*(-1 +
Tan[(e + f*x)/2])*(1 + Tan[(e + f*x)/2]))
```

fricas [A] time = 0.49, size = 165, normalized size = 1.39

$$-\frac{46\,a^3\cos\left(fx+e\right)^3-22\,a^3\cos\left(fx+e\right)^2-62\,a^3\cos\left(fx+e\right)+6\,a^3-15\left(a^3\cos\left(fx+e\right)^2-a^3\cos\left(fx+e\right)\right)\mathrm{l}}{6\left(c^2f\cos\left(fx+e\right)^2-c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(46*a^3*cos(f*x + e)^3 - 22*a^3*cos(f*x + e)^2 - 62*a^3*cos(f*x + e) +
 6*a^3 - 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(sin(f*x + e) + 1)*s
in(f*x + e) + 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(-sin(f*x + e)
+ 1)*sin(f*x + e))/((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e
))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-tan((f*x+exp(1))/2)*a^3/c^2/(tan((f*x+exp(1))/2)^2-1)-(12*tan
((f*x+exp(1))/2)^2*a^3+2*a^3)*1/3/c^2/tan((f*x+exp(1))/2)^3-5*a^3*1/2/c^2*l
n(abs(tan((f*x+exp(1))/2)-1))+5*a^3*1/2/c^2*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.69, size = 140, normalized size = 1.18

$$-\frac{4a^3}{3f\,c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}-\frac{8a^3}{f\,c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{a^3}{f\,c^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}-\frac{5a^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,c^2}-\frac{a^3}{f\,c^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -4/3/f*a^3/c^2/tan(1/2*e+1/2*f*x)^3-8/f*a^3/c^2/tan(1/2*e+1/2*f*x)-1/f*a^3/
c^2/(tan(1/2*e+1/2*f*x)-1)-5/f*a^3/c^2*ln(tan(1/2*e+1/2*f*x)-1)-1/f*a^3/c^2
/(tan(1/2*e+1/2*f*x)+1)+5/f*a^3/c^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.36, size = 349, normalized size = 2.93

$$-\frac{a^3\Bigg(\frac{\frac{14\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{27\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1}{\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{12\log\Big(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\Big)}{c^2}+\frac{12\log\Big(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\Big)}{c^2}\Bigg)-3a^3\Bigg(\frac{6\log\Big(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\Big)}{c^2}-\frac{6\log\Big(\frac{\sin(fx+e)}{\cos(fx+e)+}}{c^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(a^3*((14*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 27*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 1)/(c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - c^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/c^2 + 12*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^2) - 3*a^3*(6*log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1
)^3/(c^2*sin(f*x + e)^3)) + 3*a^3*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) - a^3*(3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

mupad [B] time = 2.04, size = 93, normalized size = 0.78

$$\frac{10\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f}+\frac{-10\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{20\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{4\,a^3}{3}}{c^2\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (10*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) + ((20*a^3*tan(e/2 + (f*x)/2)^2)
/3 - 10*a^3*tan(e/2 + (f*x)/2)^4 + (4*a^3)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3*(
tan(e/2 + (f*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e\right.}$$
$$c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(3*sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**4/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

3.29 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3 f}-\frac{2a^3\tan(e+fx)}{f\left(c^3-c^3\sec(e+fx)\right)}+\frac{2\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf(c-c\sec(e+fx))^2}-\frac{2a\tan(e+fx)(a\sec(e+}{5f(c-c\sec(e+fx}$$

```
[Out] -a^3*arctanh(sin(f*x+e))/c^3/f-2/5*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*s
ec(f*x+e))^3+2/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2-2*a
^3*tan(f*x+e)/f/(c^3-c^3*sec(f*x+e))
```

Rubi [A] time = 0.21, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$-\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3 f}-\frac{2a^3\tan(e+fx)}{f\left(c^3-c^3\sec(e+fx)\right)}+\frac{2\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf(c-c\sec(e+fx))^2}-\frac{2a\tan(e+fx)(a\sec(e+}{5f(c-c\sec(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Sin[e + f*x]])/(c^3*f)) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(5*f*(c - c*Sec[e + f*x])^3) + (2*(a^3 + a^3*Sec[e + f*x])*Tan[e +
f*x])/(3*c*f*(c - c*Sec[e + f*x])^2) - (2*a^3*Tan[e + f*x])/(f*(c^3 - c^3*S
ec[e + f*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx}{c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2\left(a^3+a^3\sec(e+fx)\right)\tan(e+}{3cf(c-c\sec(e+fx))^2}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2\left(a^3+a^3\sec(e+fx)\right)\tan(e+}{3cf(c-c\sec(e+fx))^2}$$

$$= -\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2\left(a^3\right.}{}$$

Mathematica [A] time = 0.11, size = 139, normalized size = 1.05

$$-\frac{a^3\left(-\frac{26\cot\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)}{5f} + \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}\right.\right.}{}\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*((-26*Cot[(e + f*x)/2])/(15*f) + (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2
]^2)/(15*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4)/(5*f) - Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f)
)/c^3)
```

fricas [A] time = 0.49, size = 176, normalized size = 1.33

$$\frac{52\,a^3\cos\left(fx+e\right)^3 - 44\,a^3\cos\left(fx+e\right)^2 - 4\,a^3\cos\left(fx+e\right) + 92\,a^3 - 15\left(a^3\cos\left(fx+e\right)^2 - 2\,a^3\cos\left(fx+e\right) +\right.}{30\left(c^3f\cos\left(fx+e\right)^2 - 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(52*a^3*cos(f*x + e)^3 - 44*a^3*cos(f*x + e)^2 - 4*a^3*cos(f*x + e) +
92*a^3 - 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*log(sin(f*x + e
) + 1)*sin(f*x + e) + 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*lo
```

```
g(-sin(f*x + e) + 1)*sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
 + e) + c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(-a^3*1/2/c^3*ln(abs(tan((f*x+exp(1))/2)-1))+a^3*1/2/c^3*ln(ab
s(tan((f*x+exp(1))/2)+1))+(-15*tan((f*x+exp(1))/2)^4*a^3-5*tan((f*x+exp(1))
/2)^2*a^3-3*a^3)*1/15/c^3/tan((f*x+exp(1))/2)^5)
```

maple [A] time = 0.92, size = 113, normalized size = 0.86

$$\frac{2a^3}{5f\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}+\frac{2a^3}{3f\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}+\frac{2a^3}{f\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}+\frac{a^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,c^3}-\frac{a^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{f\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 2/5/f*a^3/c^3/tan(1/2*e+1/2*f*x)^5+2/3/f*a^3/c^3/tan(1/2*e+1/2*f*x)^3+2/f*a
^3/c^3/tan(1/2*e+1/2*f*x)+1/f*a^3/c^3*ln(tan(1/2*e+1/2*f*x)-1)-1/f*a^3/c^3*
ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.36, size = 309, normalized size = 2.34

$$-\frac{a^3\left(\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)-\frac{3a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+}{(\cos(fx+e)+}\right.}{c^3\sin(fx+}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(a^3*(60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/c^3 - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^
```

```
2 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*
sin(f*x + e)^5)) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^
5) + a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 9*a^3*(5*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x +
e)^5))/f
```

mupad [B] time = 1.78, size = 78, normalized size = 0.59

$$\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{2\,a^3}{5}}{c^3\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] ((2*a^3*tan(e/2 + (f*x)/2)^2)/3 + 2*a^3*tan(e/2 + (f*x)/2)^4 + (2*a^3)/5)/(
c^3*f*tan(e/2 + (f*x)/2)^5) - (2*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+\cdots}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)*
*4/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

3.30 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} dx$

Optimal. Leaf size=38

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -1/7*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^4
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

Mathematica [A] time = 0.16, size = 25, normalized size = 0.66

$$-\frac{a^3\cot^7\left(\frac{1}{2}(e+fx)\right)}{7c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*(a^3*Cot[(e + f*x)/2]^7)/(c^4*f)
```

fricas [B] time = 0.45, size = 111, normalized size = 2.92

$$\frac{a^3 \cos\left(fx+e\right)^4 + 4\,a^3 \cos\left(fx+e\right)^3 + 6\,a^3 \cos\left(fx+e\right)^2 + 4\,a^3 \cos\left(fx+e\right) + a^3}{7\left(c^4 f \cos\left(fx+e\right)^3 - 3\,c^4 f \cos\left(fx+e\right)^2 + 3\,c^4 f \cos\left(fx+e\right) - c^4 f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*
c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

giac [A] time = 0.42, size = 23, normalized size = 0.61

$$-\frac{a^3}{7\,c^4 f \tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/7*a^3/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

maple [A] time = 0.92, size = 23, normalized size = 0.61

$$-\frac{a^3}{7 f\,c^4 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/7/f*a^3/c^4/tan(1/2*e+1/2*f*x)^7
```

maxima [B] time = 0.37, size = 356, normalized size = 9.37

$$-\frac{\frac{a^3\left(\frac{21\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{35\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{35\,\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+5\right)\left(\cos\left(fx+e\right)+1\right)^7}{c^4\sin\left(fx+e\right)^7}-\frac{a^3\left(\frac{21\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{35\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{105\,\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-15\right)\left(\cos\left(fx+e\right)+1\right)^7}{c^4\sin\left(fx+e\right)^7}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/280*(a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5)*(cos(f*x +
e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
+ e)^7) + a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*
x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

mupad [B] time = 1.81, size = 22, normalized size = 0.58

$$-\frac{a^3 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{7\,c^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(a^3*cot(e/2 + (f*x)/2)^7)/(7*c^4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\mathrm{se}}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)
**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

3.31 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=80

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

```
[Out] -1/9*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/63*(a+a*sec(f*x+e
))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4
```

Rubi [A] time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - ((a +
 a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx}{9c}$$
$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))^3\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}$$

Mathematica [A] time = 0.40, size = 141, normalized size = 1.76

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\left(315\sin\left(e+\frac{fx}{2}\right)-189\sin\left(e+\frac{3fx}{2}\right)-483\sin\left(2e+\frac{3fx}{2}\right)+225\sin\left(2e+\frac{5fx}{2}\right)+63\sin\left(3e+\frac{5fx}{2}\right)-\right.}{16128c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/16128*(a^3*Csc[e/2]*Csc[(e + f*x)/2]^9*(693*Sin[(f*x)/2] + 315*Sin[e + (
f*x)/2] - 189*Sin[e + (3*f*x)/2] - 483*Sin[2*e + (3*f*x)/2] + 225*Sin[2*e +
 (5*f*x)/2] + 63*Sin[3*e + (5*f*x)/2] - 9*Sin[3*e + (7*f*x)/2] - 63*Sin[4*e
 + (7*f*x)/2] + 8*Sin[4*e + (9*f*x)/2]))/(c^5*f)
```

fricas [A] time = 0.45, size = 140, normalized size = 1.75

$$\frac{8a^3\cos\left(fx+e\right)^5+31a^3\cos\left(fx+e\right)^4+44a^3\cos\left(fx+e\right)^3+26a^3\cos\left(fx+e\right)^2+4a^3\cos\left(fx+e\right)-a^3}{63\left(c^5f\cos\left(fx+e\right)^4-4c^5f\cos\left(fx+e\right)^3+6c^5f\cos\left(fx+e\right)^2-4c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(8*a^3*cos(f*x + e)^5 + 31*a^3*cos(f*x + e)^4 + 44*a^3*cos(f*x + e)^3
+ 26*a^3*cos(f*x + e)^2 + 4*a^3*cos(f*x + e) - a^3)/((c^5*f*cos(f*x + e)^4
- 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) +
c^5*f)*sin(f*x + e))
```

giac [A] time = 0.89, size = 43, normalized size = 0.54

$$-\frac{9a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-7a^3}{126c^5f\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] -1/126*(9*a^3*tan(1/2*f*x + 1/2*e)^2 - 7*a^3)/(c^5*f*tan(1/2*f*x + 1/2*e)^9
)
```

maple [A] time = 0.95, size = 39, normalized size = 0.49

$$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}+\frac{1}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9}\right)}{2fc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/2/f*a^3/c^5*(-1/7/tan(1/2*e+1/2*f*x)^7+1/9/tan(1/2*e+1/2*f*x)^9)
```

maxima [B] time = 0.37, size = 357, normalized size = 4.46

$$-\frac{\frac{a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{15a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\cdots\right.}{c^5\sin(fx+e)^9}}{5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^3*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 15*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 + 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) + 21*a^3*(18*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 -
5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

mupad [B] time = 1.62, size = 37, normalized size = 0.46

$$\frac{a^3 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7 \left(7\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-9\right)}{126\,c^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a^3*cot(e/2 + (f*x)/2)^7*(7*cot(e/2 + (f*x)/2)^2 - 9))/(126*c^5*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx + \int \frac{3\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2 (}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(se
c(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2
 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**5 - 5*
sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x)
- 1), x))/c**5
```

3.32 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=121

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{693c^2f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{99cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

```
[Out] -1/11*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-2/99*(a+a*sec(f*x+
e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/693*(a+a*sec(f*x+e))^3*tan(f*x+e)
/c^2/f/(c-c*sec(f*x+e))^4
```

Rubi [A] time = 0.23, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{693c^2f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{99cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(11*f*(c - c*Sec[e + f*x])^6) - (2*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(99*c*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*c^2*f*(c - c*Sec[e + f*x])^4)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx}{11c}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5} +$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5} -$$

Mathematica [A] time = 0.70, size = 167, normalized size = 1.38

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\left(21252\sin\left(e+\frac{fx}{2}\right)-15444\sin\left(e+\frac{3fx}{2}\right)-10626\sin\left(2e+\frac{3fx}{2}\right)+4950\sin\left(2e+\frac{5fx}{2}\right)+8085\sin\left(3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/709632*(a^3*Csc[e/2]*Csc[(e + f*x)/2]^11*(15246*Sin[(f*x)/2] + 21252*Sin
[e + (f*x)/2] - 15444*Sin[e + (3*f*x)/2] - 10626*Sin[2*e + (3*f*x)/2] + 495
0*Sin[2*e + (5*f*x)/2] + 8085*Sin[3*e + (5*f*x)/2] - 2959*Sin[3*e + (7*f*x)
/2] - 1386*Sin[4*e + (7*f*x)/2] + 176*Sin[4*e + (9*f*x)/2] + 693*Sin[5*e +
(9*f*x)/2] - 79*Sin[5*e + (11*f*x)/2]))/(c^6*f)
```

fricas [A] time = 0.50, size = 168, normalized size = 1.39

$$\frac{79\,a^3\cos\left(fx+e\right)^6+298\,a^3\cos\left(fx+e\right)^5+404\,a^3\cos\left(fx+e\right)^4+216\,a^3\cos\left(fx+e\right)^3+19\,a^3\cos\left(fx+e\right)^2-1}{693\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/693*(79*a^3*cos(f*x + e)^6 + 298*a^3*cos(f*x + e)^5 + 404*a^3*cos(f*x + e
)^4 + 216*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 10*a^3*cos(f*x + e)
+ 2*a^3)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x
 + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x +
 e))
```

giac [A] time = 1.50, size = 60, normalized size = 0.50

$$-\frac{99\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-154\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+63\,a^3}{2772\,c^6f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] -1/2772*(99*a^3*tan(1/2*f*x + 1/2*e)^4 - 154*a^3*tan(1/2*f*x + 1/2*e)^2 + 6
3*a^3)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

maple [A] time = 0.96, size = 52, normalized size = 0.43

$$\frac{a^3\left(-\frac{1}{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{2}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}\right)}{4f\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

```
[Out] 1/4/f*a^3/c^6*(-1/11/tan(1/2*e+1/2*f*x)^11-1/7/tan(1/2*e+1/2*f*x)^7+2/9/tan
(1/2*e+1/2*f*x)^9)
```

maxima [B] time = 0.38, size = 518, normalized size = 4.28

$$\frac{\frac{3\,a^3\left(\frac{385\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{990\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{1386\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{1155\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}+\frac{3465\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}-315\right)\left(\cos\left(fx+e\right)+1\right)^{11}}{c^6\sin\left(fx+e\right)^{11}}+\frac{9\,a^3\left(\frac{385\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{330\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\cdots}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(3*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1
)^10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 9*a^3*(385*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4
```

```
62*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) +
1)^11/(c^6*sin(f*x + e)^11) + 5*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^1
0/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) -
 a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 990*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 315)
*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

mupad [B] time = 1.86, size = 67, normalized size = 0.55

$$\frac{a^3 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{18\,c^6\,f}-\frac{a^3 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{28\,c^6\,f}-\frac{a^3 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{44\,c^6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

```
[Out] (a^3*cot(e/2 + (f*x)/2)^9)/(18*c^6*f) - (a^3*cot(e/2 + (f*x)/2)^7)/(28*c^6*
f) - (a^3*cot(e/2 + (f*x)/2)^11)/(44*c^6*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e+fx\right)-20\sec^3\left(e+fx\right)+15\sec^2\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5
+ 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e +
f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**6 - 6*sec(e + f*x)*
*5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e
 + f*x) + 1), x))/c**6
```

3.33 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7} dx$

Optimal. Leaf size=162

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{3003c^3f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{429c^2f(c-c\sec(e+fx))^5}-\frac{3\tan(e+fx)(a\sec(e+fx)+a)^3}{143cf(c-c\sec(e+fx))^6}-\frac{\tan(e}{1}$$

```
[Out] -1/13*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^7-3/143*(a+a*sec(f*x
+e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^6-2/429*(a+a*sec(f*x+e))^3*tan(f*x+e
)/c^2/f/(c-c*sec(f*x+e))^5-2/3003*(a+a*sec(f*x+e))^3*tan(f*x+e)/c^3/f/(c-c*
sec(f*x+e))^4
```

Rubi [A] time = 0.32, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{3003c^3f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{429c^2f(c-c\sec(e+fx))^5}-\frac{3\tan(e+fx)(a\sec(e+fx)+a)^3}{143cf(c-c\sec(e+fx))^6}-\frac{\tan(e}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(13*f*(c - c*Sec[e + f*x])^7) - (3*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(143*c*f*(c - c*Sec[e + f*x])^6) - (2*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(429*c^2*f*(c - c*Sec[e + f*x])^5) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*c^3*f*(c - c*Sec[e + f*x])^4)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

```
&& !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx}{13c}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} +$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} -$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} -$$

Mathematica [A] time = 0.59, size = 193, normalized size = 1.19

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\left(246246\sin\left(e+\frac{fx}{2}\right)-182754\sin\left(e+\frac{3fx}{2}\right)-216216\sin\left(2e+\frac{3fx}{2}\right)+122551\sin\left(2e+\frac{5fx}{2}\right)+9909\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -1/12300288*(a^3*Csc[e/2]*Csc[(e + f*x)/2]^13*(285714*Sin[(f*x)/2] + 246246
*Sin[e + (f*x)/2] - 182754*Sin[e + (3*f*x)/2] - 216216*Sin[2*e + (3*f*x)/2]
 + 122551*Sin[2*e + (5*f*x)/2] + 99099*Sin[3*e + (5*f*x)/2] - 37609*Sin[3*e
 + (7*f*x)/2] - 51051*Sin[4*e + (7*f*x)/2] + 15171*Sin[4*e + (9*f*x)/2] + 9
009*Sin[5*e + (9*f*x)/2] - 1027*Sin[5*e + (11*f*x)/2] - 3003*Sin[6*e + (11*
f*x)/2] + 310*Sin[6*e + (13*f*x)/2]))/(c^7*f)
```

fricas [A] time = 0.46, size = 194, normalized size = 1.20

$$\frac{310\,a^3\cos\left(fx+e\right)^7+1143\,a^3\cos\left(fx+e\right)^6+1492\,a^3\cos\left(fx+e\right)^5+736\,a^3\cos\left(fx+e\right)^4+34\,a^3\cos\left(fx+e\right)^3}{3003\left(c^7f\cos\left(fx+e\right)^6-6\,c^7f\cos\left(fx+e\right)^5+15\,c^7f\cos\left(fx+e\right)^4-20\,c^7f\cos\left(fx+e\right)^3+15\,c^7f\cos\left(fx-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="fr
icas")
```

[Out] 1/3003*(310*a^3*cos(f*x + e)^7 + 1143*a^3*cos(f*x + e)^6 + 1492*a^3*cos(f*x + e)^5 + 736*a^3*cos(f*x + e)^4 + 34*a^3*cos(f*x + e)^3 - 29*a^3*cos(f*x + e)^2 + 12*a^3*cos(f*x + e) - 2*a^3)/((c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f*x + e)^5 + 15*c^7*f*cos(f*x + e)^4 - 20*c^7*f*cos(f*x + e)^3 + 15*c^7*f*cos(f*x + e)^2 - 6*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e))

giac [A] time = 0.71, size = 77, normalized size = 0.48

$$-\frac{429\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1001\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+819\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-231\,a^3}{24024\,c^7 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="giac")

[Out] -1/24024*(429*a^3*tan(1/2*f*x + 1/2*e)^6 - 1001*a^3*tan(1/2*f*x + 1/2*e)^4 + 819*a^3*tan(1/2*f*x + 1/2*e)^2 - 231*a^3)/(c^7*f*tan(1/2*f*x + 1/2*e)^13)

maple [A] time = 0.96, size = 65, normalized size = 0.40

$$\frac{a^3\left(-\frac{3}{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}+\frac{1}{13\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{13}}-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}\right)}{8f\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x)

[Out] 1/8/f*a^3/c^7*(-3/11/tan(1/2*e+1/2*f*x)^11+1/13/tan(1/2*e+1/2*f*x)^13-1/7/tan(1/2*e+1/2*f*x)^7+1/3/tan(1/2*e+1/2*f*x)^9)

maxima [B] time = 0.39, size = 517, normalized size = 3.19

$$-\frac{\frac{a^3\left(\frac{8190\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{5005\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{25740\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{9009\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}+\frac{30030\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}-\frac{45045\sin\left(fx+e\right)^{12}}{\left(\cos\left(fx+e\right)+1\right)^{12}}-3465\right)\left(\cos\left(fx+e\right)+1\right)^{13}}{c^7\sin\left(fx+e\right)^{13}}+\frac{5\,a^3\left(\frac{1638\sin\cdots}{\left(\cos\left(f\cdots\right.\right.}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="maxima")

```
[Out] -1/960960*(a^3*(8190*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5005*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 25740*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9009
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30030*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 - 45045*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 3465)*(cos(f*x + e)
 + 1)^13/(c^7*sin(f*x + e)^13) + 5*a^3*(1638*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 5005*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8580*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 9009*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 6006*sin(f*x
+ e)^10/(cos(f*x + e) + 1)^10 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- 231)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13) + 35*a^3*(468*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 715*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1287*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 1716*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 1287*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 99)*(cos(f*x + e) + 1)
^13/(c^7*sin(f*x + e)^13) + 77*a^3*(65*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 117*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^12/(cos(f*x +
e) + 1)^12 - 15)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13))/f
```

mupad [B] time = 1.72, size = 108, normalized size = 0.67

$$\frac{a^3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(231\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6-819\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1001\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-42\right.}{24024\,c^7\,f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

```
[Out] (a^3*cos(e/2 + (f*x)/2)^7*(231*cos(e/2 + (f*x)/2)^6 - 429*sin(e/2 + (f*x)/2
)^6 + 1001*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 819*cos(e/2 + (f*x)/
2)^4*sin(e/2 + (f*x)/2)^2))/(24024*c^7*f*sin(e/2 + (f*x)/2)^13)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int\frac{\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx+\int\frac{}{\sec^7(e+fx)-7\sec^6(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(
e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2
+ 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**7 - 7
*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x
)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
```

```
x)**3/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e
+ f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1),
x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec
(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2
 + 7*sec(e + f*x) - 1), x))/c**7
```

3.34 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{7c^4\tan^3(e+fx)}{3af}+\frac{28c^4\tan(e+fx)}{af}-\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af}-\frac{21c^4\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-}{f(a\sec(e+}$$

```
[Out] -35/2*c^4*arctanh(sin(f*x+e))/a/f+28*c^4*tan(f*x+e)/a/f-21/2*c^4*sec(f*x+e)
*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))+7/3*c^
4*tan(f*x+e)^3/a/f
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3791, 3770, 3767, 8, 3768}

$$\frac{7c^4\tan^3(e+fx)}{3af}+\frac{28c^4\tan(e+fx)}{af}-\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af}-\frac{21c^4\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-}{f(a\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-35*c^4*ArcTanh[Sin[e + f*x]])/(2*a*f) + (28*c^4*Tan[e + f*x])/(a*f) - (21
*c^4*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^3*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x])) + (7*c^4*Tan[e + f*x]^3)/(3*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int \sec(e+fx)(c-c\sec(e+fx}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int \left(c^3\sec(e+fx)-3c^3\sec^2(e\right.}{}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(7c^4\right)\int \sec(e+fx)\,dx}{a} + \frac{\left(7c^4\right)\int s}{}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx)}{2af} + \frac{2c(c-c\sec}{f(a}$$

$$= -\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af} + \frac{28c^4\tan(e+fx)}{af} - \frac{21c^4\sec(e+fx)\tan(e}{2af}$$

Mathematica [B] time = 6.42, size = 1036, normalized size = 8.56

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*Log[Cos[e/2 +
(f*x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(16*f*(a + a*Sec[e +
 f*x])) - (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*Log[
Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(16*f*(a +
 a*Sec[e + f*x])) + (2*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]
^7*Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(f*(a + a*Sec[e + f*x])) +
 (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f
*x])^4*Sin[(f*x)/2])/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] - Sin[e/2])*(Cos[
e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2
]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*(-7*Cos[e/2] + 8*Sin[e/2]))
/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin
[e/2 + (f*x)/2])^2) + (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*
x)/2]^6*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(24*f*(a + a*Sec[e + f*x])*(Co
s[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) + (Cos[e + f*
x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*Sin[(
f*x)/2])/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/
2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2
+ (f*x)/2]^6*(c - c*Sec[e + f*x])^4*(7*Cos[e/2] + 8*Sin[e/2]))/(48*f*(a + a
*Sec[e + f*x])*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/
2])^2) + (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c -
c*Sec[e + f*x])^4*Sin[(f*x)/2])/(24*f*(a + a*Sec[e + f*x])*(Cos[e/2] + Sin[
e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

fricas [A] time = 0.52, size = 153, normalized size = 1.26

$$-\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\log\left(\right.}{12\left(af\cos\left(fx+e\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/12*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(sin(f*x + e) + 1)
- 105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*
(166*c^4*cos(f*x + e)^3 + 55*c^4*cos(f*x + e)^2 - 13*c^4*cos(f*x + e) + 2*c
^4)*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(8*tan((f*x+exp(1))/2)*c^4/a+(-87*tan((f*x+exp(1))/2)^5*c^4+136
*tan((f*x+exp(1))/2)^3*c^4-57*tan((f*x+exp(1))/2)*c^4)*1/6/a/(tan((f*x+exp(
1))/2)^2-1)^3+35*c^4*1/4/a*ln(abs(tan((f*x+exp(1))/2)-1))-35*c^4*1/4/a*ln(a
bs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.75, size = 212, normalized size = 1.75

$$\frac{16c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{fa}-\frac{c^4}{3fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3}-\frac{3c^4}{fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}-\frac{29c^4}{2fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}+\frac{35c^4\ln\left(\tan\left(\frac{e}{2}\right.\right.}{2fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x)
```

```
[Out] 16/f*c^4/a*tan(1/2*e+1/2*f*x)-1/3/f*c^4/a/(tan(1/2*e+1/2*f*x)-1)^3-3/f*c^4/
a/(tan(1/2*e+1/2*f*x)-1)^2-29/2/f*c^4/a/(tan(1/2*e+1/2*f*x)-1)+35/2/f*c^4/a
*ln(tan(1/2*e+1/2*f*x)-1)-1/3/f*c^4/a/(tan(1/2*e+1/2*f*x)+1)^3+3/f*c^4/a/(t
an(1/2*e+1/2*f*x)+1)^2-29/2/f*c^4/a/(tan(1/2*e+1/2*f*x)+1)-35/2/f*c^4/a*ln(
tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.35, size = 591, normalized size = 4.88

$$c^4\left(\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{16\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a-\frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{6\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+12c^4\left(\frac{2\left(\frac{\sin}{\cos(\cdots}\right.}{a-\frac{2a\sin\cdots}{(\cos(\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(c^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) + 12*c^4*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
```

```
 a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)
/(a*(cos(f*x + e) + 1))) - 36*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(c
os(f*x + e) + 1))) - 24*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) +
 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.85, size = 112, normalized size = 0.93

$$\frac{16\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}-\frac{29\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{136\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+19\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}-\frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (16*c^4*tan(e/2 + (f*x)/2))/(a*f) - (29*c^4*tan(e/2 + (f*x)/2)^5 - (136*c^4
*tan(e/2 + (f*x)/2)^3)/3 + 19*c^4*tan(e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)
/2)^2 - 1)^3) - (35*c^4*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^5\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x) + 1
), x) + Integral(-4*sec(e + f*x)**4/(sec(e + f*x) + 1), x) + Integral(sec(e
 + f*x)**5/(sec(e + f*x) + 1), x))/a
```

3.35 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{10c^3\tan(e+fx)}{af}-\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}-\frac{5c^3\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

```
[Out] -15/2*c^3*arctanh(sin(f*x+e))/a/f+10*c^3*tan(f*x+e)/a/f-5/2*c^3*sec(f*x+e)*
tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$\frac{10c^3\tan(e+fx)}{af}-\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}-\frac{5c^3\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-15*c^3*ArcTanh[Sin[e + f*x]])/(2*a*f) + (10*c^3*Tan[e + f*x])/(a*f) - (5*
c^3*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(5c)\int \sec(e+fx)(c-c\sec(e+fx))}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(5c)\int \sec(e+fx)\left(c^2+c^2\sec^2(e+\right.}{a}$$

$$= -\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(15c}{}$$

$$= -\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af} + \frac{10c^3\tan(e+fx)}{af} - \frac{5c^3\sec(e+fx)\tan(e+}{2af}$$

Mathematica [B] time = 2.65, size = 287, normalized size = 2.87

$$\frac{\cos^2(e+fx)\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^3\left(\cot\left(\frac{1}{2}(e+fx)\right)\left(-\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\mathrm{co}\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*(c - c*Sec[e + f*x])^3*
(-32*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2] + Cot[(e + f*x)/2]*(-30*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]] + 30*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]] + (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^(-2) - (16*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[
e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]))))))/(16*a*f*(1 + Sec[e + f*x]))
```

fricas [A] time = 0.46, size = 140, normalized size = 1.40

$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\log\left(-\right.}{4\left(af\cos\left(fx+e\right)^3+af\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(sin(f*x + e) + 1) -
15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(24
*c^3*cos(f*x + e)^2 + 7*c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a*f*cos(f*x
+ e)^3 + a*f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(-4*tan((f*x+exp(1))/2)*c^3/a-(-9*tan((f*x+exp(1))/2)^3*c^3+7*
tan((f*x+exp(1))/2)*c^3)*1/2/a/(tan((f*x+exp(1))/2)^2-1)^2-15*c^3*1/4/a*ln(
abs(tan((f*x+exp(1))/2)-1))+15*c^3*1/4/a*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.70, size = 164, normalized size = 1.64

$$\frac{8c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{fa}-\frac{c^3}{2fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}-\frac{9c^3}{2fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}+\frac{15c^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{2fa}+\frac{c^3}{2fa\left(\tan\left(\frac{e}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x)
```

```
[Out] 8/f*c^3/a*tan(1/2*e+1/2*f*x)-1/2/f*c^3/a/(tan(1/2*e+1/2*f*x)-1)^2-9/2/f*c^3
/a/(tan(1/2*e+1/2*f*x)-1)+15/2/f*c^3/a*ln(tan(1/2*e+1/2*f*x)-1)+1/2/f*c^3/a
/(tan(1/2*e+1/2*f*x)+1)^2-9/2/f*c^3/a/(tan(1/2*e+1/2*f*x)+1)-15/2/f*c^3/a*l
n(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.35, size = 386, normalized size = 3.86

$$\frac{c^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-6c^3\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(c^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
+ 1))) - 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)))
- 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^3*sin(f*
x + e)/(a*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.66, size = 96, normalized size = 0.96

$$\frac{8\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}-\frac{9\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-7\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^2}-\frac{15\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (8*c^3*tan(e/2 + (f*x)/2))/(a*f) - (9*c^3*tan(e/2 + (f*x)/2)^3 - 7*c^3*tan(
e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)/2)^2 - 1)^2) - (15*c^3*atanh(tan(e/2
+ (f*x)/2)))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(3*sec(e + f
*x)**2/(sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f*x) +
 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) + 1), x))/a
```

3.36 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{3c^2\tan(e+fx)}{af}-\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}$$

```
[Out] -3*c^2*arctanh(sin(f*x+e))/a/f+3*c^2*tan(f*x+e)/a/f+2*(c^2-c^2*sec(f*x+e))*
tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$\frac{3c^2\tan(e+fx)}{af}-\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-3*c^2*ArcTanh[Sin[e + f*x]])/(a*f) + (3*c^2*Tan[e + f*x])/(a*f) + (2*(c^2
 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(3c)\int \sec(e+fx)(c-c\sec(e+f$$

$$= \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(3c^2\right)\int \sec(e+fx)\,dx}{a} + \frac{\left(3c^2\right)\int s$$

$$= -\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af} + \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(3c^2\right)$$

$$= -\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af} + \frac{3c^2\tan(e+fx)}{af} + \frac{2\left(c^2-c^2\sec(e+fx)\right)ta}{f(a+a\sec(e+f}$$

Mathematica [B] time = 1.66, size = 220, normalized size = 2.97

$$\frac{2c^2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(4\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)+\cot\left(\frac{1}{2}(e+fx)\right)\left(\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin(\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c^2*Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]*(4*Csc[(e + f*x)/2]*S
ec[e/2]*Sin[(f*x)/2] + Cot[(e + f*x)/2]*(3*Log[Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]] - 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2]
 - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(1 + Sec[e + f*x]))
```

fricas [A] time = 0.49, size = 119, normalized size = 1.61

$$-\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(-\sin\right.}{2\left(af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*(
c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(5*c^2*co
s(f*x + e) + c^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(2*tan((f*x+exp(1))/2)*c^2/a-tan((f*x+exp(1))/2)*c^2/a/(tan((f*
x+exp(1))/2)^2-1)+3*c^2*1/2/a*ln(abs(tan((f*x+exp(1))/2)-1))-3*c^2*1/2/a*ln
(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.70, size = 116, normalized size = 1.57

$$\frac{4c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{fa}-\frac{c^2}{fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}+\frac{3c^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fa}-\frac{c^2}{fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}-\frac{3c^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x)
```

```
[Out] 4/f*c^2/a*tan(1/2*e+1/2*f*x)-1/f*c^2/a/(tan(1/2*e+1/2*f*x)-1)+3/f*c^2/a*ln(
tan(1/2*e+1/2*f*x)-1)-1/f*c^2/a/(tan(1/2*e+1/2*f*x)+1)-3/f*c^2/a*ln(tan(1/2
*e+1/2*f*x)+1)
```

maxima [B] time = 0.35, size = 224, normalized size = 3.03

$$-\frac{c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+2c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(c^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^2*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.64, size = 77, normalized size = 1.04

$$\frac{4\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}+\frac{2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\right)}-\frac{6\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (4*c^2*tan(e/2 + (f*x)/2))/(a*f) + (2*c^2*tan(e/2 + (f*x)/2))/(f*(a - a*tan
(e/2 + (f*x)/2)^2)) - (6*c^2*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-2*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) + 1),
 x))/a
```

3.37 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\tanh^{-1}(\sin(e+fx))}{af}$$

```
[Out] -c*arctanh(sin(f*x+e))/a/f+2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3957, 3770}

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\tanh^{-1}(\sin(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f
*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{c\int\sec(e+fx)\,dx}{a}$$
$$= -\frac{c\tanh^{-1}(\sin(e+fx))}{af} + \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.88

$$-\frac{c\left(-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (2*Tan[(e + f*x)/2])/f))/a)
```

fricas [A] time = 0.45, size = 70, normalized size = 1.71

$$-\frac{\left(c\cos\left(fx+e\right)+c\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(c\cos\left(fx+e\right)+c\right)\log\left(-\sin\left(fx+e\right)+1\right)-4\,c\sin\left(fx+e\right)}{2\left(af\cos\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*((c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - (c*cos(f*x + e) + c)*log
(-sin(f*x + e) + 1) - 4*c*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)-2/f*(-c*1/2/a*ln(abs(tan((f*x+exp(1))/2)-1))+c*1/2/a*ln(abs(tan((f
*x+exp(1))/2)+1))-tan((f*x+exp(1))/2)*c/a)
```

maple [A] time = 0.78, size = 61, normalized size = 1.49

$$\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{fa}+\frac{c\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fa}-\frac{c\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] 2/f/a*c*tan(1/2*e+1/2*f*x)+1/f/a*c*ln(tan(1/2*e+1/2*f*x)-1)-1/f/a*c*ln(tan(
1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.33, size = 101, normalized size = 2.46

$$-\frac{c\left(\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a}-\frac{\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}\right)-\frac{c\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -(c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c*sin(f*x + e)/(
a*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.58, size = 31, normalized size = 0.76

$$-\frac{2\,c\,\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(2*c*(atanh(tan(e/2 + (f*x)/2)) - tan(e/2 + (f*x)/2)))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

3.38 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=16

$$\frac{\csc(e+fx)}{acf}$$

[Out] csc(f*x+e)/a/c/f

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2606, 8}

$$\frac{\csc(e+fx)}{acf}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]

[Out] Csc[e + f*x]/(a*c*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = -\frac{\int \cot(e+fx)\csc(e+fx)\,dx}{ac}$$
$$= \frac{\text{Subst}(\int 1\,dx, x, \csc(e+fx))}{acf}$$
$$= \frac{\csc(e+fx)}{acf}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$\frac{\csc(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
[Out] Csc[e + f*x]/(a*c*f)
```

fricas [A] time = 0.44, size = 18, normalized size = 1.12

$$\frac{1}{acf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
[Out] 1/(a*c*f*sin(f*x + e))
```

giac [A] time = 0.42, size = 19, normalized size = 1.19

$$\frac{1}{acf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] 1/(a*c*f*sin(f*x + e))
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)\left(c-c\sec\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

maxima [A] time = 0.32, size = 18, normalized size = 1.12

$$\frac{1}{acf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

mupad [B] time = 1.56, size = 18, normalized size = 1.12

$$\frac{1}{a\,c\,f\,\sin\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))),x)
```

```
[Out] 1/(a*c*f*sin(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-1}\,dx}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**2 - 1), x)/(a*c)
```

3.39 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\cot^3(e+fx)}{3ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}$$

```
[Out] -1/3*cot(f*x+e)^3/a/c^2/f+csc(f*x+e)/a/c^2/f-1/3*csc(f*x+e)^3/a/c^2/f
```

Rubi [A] time = 0.14, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3958, 2606, 2607, 30}

$$-\frac{\cot^3(e+fx)}{3ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -Cot[e + f*x]^3/(3*a*c^2*f) + Csc[e + f*x]/(a*c^2*f) - Csc[e + f*x]^3/(3*a*
c^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
```

```
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx &= \frac{\int \left(a\cot^3(e+fx)\csc(e+fx)+a\cot^2(e+fx)\csc^2(e+fx)\right)\,dx}{a^2c^2}\\ &= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{ac^2}+\frac{\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{ac^2}\\ &= \frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(e+fx)\right)}{ac^2f}-\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(\right.}{ac^2f}\\ &= -\frac{\cot^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}\end{aligned}$$

Mathematica [A] time = 0.48, size = 81, normalized size = 1.37

$$\frac{\csc(e)(-10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)+2\sin(e+2fx)+6\sin(e)+2\sin(fx))\csc^2\left(\frac{1}{2}(e+fx\right.}{24ac^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]*(6*Sin[e] + 2*Sin[f*x] - 10*Sin[e +
 f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] + 2*Sin[e + 2*f*x]))/(24*a*c^
2*f)
```

fricas [A] time = 0.43, size = 50, normalized size = 0.85

$$\frac{\cos\left(fx+e\right)^2+2\cos\left(fx+e\right)-2}{3\left(ac^2f\cos\left(fx+e\right)-ac^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2*cos(f*x + e) - 2)/((a*c^2*f*cos(f*x + e) - a*c^2*f)
*sin(f*x + e))
```

giac [A] time = 0.31, size = 59, normalized size = 1.00

$$\frac{\frac{3\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^2}+\frac{6\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}{ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/12*(3*tan(1/2*f*x + 1/2*e)/(a*c^2) + (6*tan(1/2*f*x + 1/2*e)^2 - 1)/(a*c^
2*tan(1/2*f*x + 1/2*e)^3))/f
```

maple [A] time = 0.78, size = 48, normalized size = 0.81

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}+\frac{2}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{4fac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/4/f/a/c^2*(tan(1/2*e+1/2*f*x)-1/3/tan(1/2*e+1/2*f*x)^3+2/tan(1/2*e+1/2*f*
x))
```

maxima [A] time = 0.34, size = 77, normalized size = 1.31

$$\frac{\frac{\left(\frac{6\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-1\right)\left(\cos\left(fx+e\right)+1\right)^3}{ac^2\sin\left(fx+e\right)^3}+\frac{3\,\sin\left(fx+e\right)}{ac^2\left(\cos\left(fx+e\right)+1\right)}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a*c
^2*sin(f*x + e)^3) + 3*sin(f*x + e)/(a*c^2*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.64, size = 50, normalized size = 0.85

$$\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}{12\,a\,c^2\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (6*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^4 - 1)/(12*a*c^2*f*tan(e/2 +
 (f*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-\sec^2\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) + 1
), x)/(a*c**2)
```

3.40 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=78

$$\frac{2\cot^5(e+fx)}{5ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{\csc(e+fx)}{ac^3f}$$

```
[Out] 2/5*cot(f*x+e)^5/a/c^3/f+csc(f*x+e)/a/c^3/f-csc(f*x+e)^3/a/c^3/f+2/5*csc(f*
x+e)^5/a/c^3/f
```

Rubi [A] time = 0.18, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 14}

$$\frac{2\cot^5(e+fx)}{5ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{\csc(e+fx)}{ac^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*Cot[e + f*x]^5)/(5*a*c^3*f) + Csc[e + f*x]/(a*c^3*f) - Csc[e + f*x]^3/(a
*c^3*f) + (2*Csc[e + f*x]^5)/(5*a*c^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx &= -\frac{\int \left(a^2\cot^5(e+fx)\csc(e+fx)+2a^2\cot^4(e+fx)\csc^2(e+fx)+\right.}{a^3c^3} \\ &= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{ac^3}-\frac{\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{ac^3} \\ &= \frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(e+fx)\right)}{ac^3f}+\frac{\mathrm{Subst}\left(\int\left(-1+x^2\right)^2\right.}{ac^3} \\ &= \frac{2\cot^5(e+fx)}{5ac^3f}+\frac{\mathrm{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{ac^3f}+\frac{\mathrm{Su}}{} \\ &= \frac{2\cot^5(e+fx)}{5ac^3f}+\frac{\csc(e+fx)}{ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}\end{aligned}$$

Mathematica [A] time = 0.85, size = 107, normalized size = 1.37

$$-\frac{\csc(e)(65\sin(e+fx)-52\sin(2(e+fx))+13\sin(3(e+fx))+40\sin(2e+fx)-12\sin(e+2fx)-20\sin(3e+}{320ac^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -1/320*(Csc[e]*Csc[(e + f*x)/2]^4*Csc[e + f*x]*(-40*Sin[e] + 65*Sin[e + f*x
] - 52*Sin[2*(e + f*x)] + 13*Sin[3*(e + f*x)] + 40*Sin[2*e + f*x] - 12*Sin[
e + 2*f*x] - 20*Sin[3*e + 2*f*x] + 8*Sin[2*e + 3*f*x]))/(a*c^3*f)
```

fricas [A] time = 0.44, size = 74, normalized size = 0.95

$$\frac{2\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-4\cos\left(fx+e\right)+2}{5\left(ac^{3}f\cos\left(fx+e\right)^{2}-2\,ac^{3}f\cos\left(fx+e\right)+ac^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/5*(2*cos(f*x + e)^3 + cos(f*x + e)^2 - 4*cos(f*x + e) + 2)/((a*c^3*f*cos(
f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.36, size = 73, normalized size = 0.94

$$\frac{\frac{5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^{3}}+\frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1}{ac^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/40*(5*tan(1/2*f*x + 1/2*e)/(a*c^3) + (15*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1
/2*f*x + 1/2*e)^2 + 1)/(a*c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

maple [A] time = 1.03, size = 61, normalized size = 0.78

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}+\frac{3}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{8fa\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/8/f/a/c^3*(tan(1/2*e+1/2*f*x)-1/tan(1/2*e+1/2*f*x)^3+1/5/tan(1/2*e+1/2*f*
x)^5+3/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.34, size = 97, normalized size = 1.24

$$-\frac{\frac{\left(\frac{5\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-1\right)\left(\cos\left(fx+e\right)+1\right)^5}{ac^3\sin\left(fx+e\right)^5}-\frac{5\sin\left(fx+e\right)}{ac^3\left(\cos\left(fx+e\right)+1\right)}}{40f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/40*((5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a*c^3*sin(f*x + e)^5) - 5*sin(f*x +
e)/(a*c^3*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.72, size = 63, normalized size = 0.81

$$\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1}{40\,a\,c^3\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (15*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^2 + 5*tan(e/2 + (f*x)/2)^6
 + 1)/(40*a*c^3*f*tan(e/2 + (f*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-2\sec^3\left(e+fx\right)+2\sec\left(e+fx\right)-1}\,dx}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x
) - 1), x)/(a*c**3)
```

3.41 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=120

$$-\frac{4\cot^7(e+fx)}{7ac^4f}-\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{\csc(e+fx)}{ac^4f}$$

```
[Out] -1/5*cot(f*x+e)^5/a/c^4/f-4/7*cot(f*x+e)^7/a/c^4/f+csc(f*x+e)/a/c^4/f-2*csc
(f*x+e)^3/a/c^4/f+9/5*csc(f*x+e)^5/a/c^4/f-4/7*csc(f*x+e)^7/a/c^4/f
```

Rubi [A] time = 0.23, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2606, 194, 2607, 30, 270, 14}

$$-\frac{4\cot^7(e+fx)}{7ac^4f}-\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{\csc(e+fx)}{ac^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -Cot[e + f*x]^5/(5*a*c^4*f) - (4*Cot[e + f*x]^7)/(7*a*c^4*f) + Csc[e + f*x]
/(a*c^4*f) - (2*Csc[e + f*x]^3)/(a*c^4*f) + (9*Csc[e + f*x]^5)/(5*a*c^4*f)
- (4*Csc[e + f*x]^7)/(7*a*c^4*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(a^3\cot^7(e+fx)\csc(e+fx)+3a^3\cot^6(e+fx)\csc^2(e+fx)+3\right.}{a^4c}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{ac^4} + \frac{\int \cot^4(e+fx)\csc^4(e+fx)\,dx}{ac^4} +$$

$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \csc(e+fx)\right)}{ac^4f} + \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)\right.}{ac^4}$$

$$= -\frac{3\cot^7(e+fx)}{7ac^4f} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx, x, \csc(e+\right.}{ac^4f}$$

$$= -\frac{\cot^5(e+fx)}{5ac^4f} - \frac{4\cot^7(e+fx)}{7ac^4f} + \frac{\csc(e+fx)}{ac^4f} - \frac{2\csc^3(e+fx)}{ac^4f} +$$

Mathematica [A] time = 0.90, size = 145, normalized size = 1.21

$$\csc(e)(-1946\sin(e+fx)+1946\sin(2(e+fx))-834\sin(3(e+fx))+139\sin(4(e+fx))-1400\sin(2e+fx)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^6*Csc[e + f*x]*(840*Sin[e] - 56*Sin[f*x] - 1946*Si
n[e + f*x] + 1946*Sin[2*(e + f*x)] - 834*Sin[3*(e + f*x)] + 139*Sin[4*(e +
f*x)] - 1400*Sin[2*e + f*x] + 616*Sin[e + 2*f*x] + 840*Sin[3*e + 2*f*x] - 3
44*Sin[2*e + 3*f*x] - 280*Sin[4*e + 3*f*x] + 104*Sin[3*e + 4*f*x]))/(17920*
a*c^4*f)
```

fricas [A] time = 0.44, size = 102, normalized size = 0.85

$$\frac{13\cos\left(fx+e\right)^{4}-4\cos\left(fx+e\right)^{3}-20\cos\left(fx+e\right)^{2}+24\cos\left(fx+e\right)-8}{35\left(ac^{4}f\cos\left(fx+e\right)^{3}-3\,ac^{4}f\cos\left(fx+e\right)^{2}+3\,ac^{4}f\cos\left(fx+e\right)-ac^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/35*(13*cos(f*x + e)^4 - 4*cos(f*x + e)^3 - 20*cos(f*x + e)^2 + 24*cos(f*x
 + e) - 8)/((a*c^4*f*cos(f*x + e)^3 - 3*a*c^4*f*cos(f*x + e)^2 + 3*a*c^4*f*
cos(f*x + e) - a*c^4*f)*sin(f*x + e))
```

giac [A] time = 0.35, size = 87, normalized size = 0.72

$$\frac{\frac{35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^{4}}+\frac{140\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+28\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-5}{ac^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/560*(35*tan(1/2*f*x + 1/2*e)/(a*c^4) + (140*tan(1/2*f*x + 1/2*e)^6 - 70*t
an(1/2*f*x + 1/2*e)^4 + 28*tan(1/2*f*x + 1/2*e)^2 - 5)/(a*c^4*tan(1/2*f*x +
 1/2*e)^7))/f
```

maple [A] time = 0.96, size = 74, normalized size = 0.62

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}-\frac{2}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{4}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}+\frac{4}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{16fac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/16/f/a/c^4*(tan(1/2*e+1/2*f*x)-1/7/tan(1/2*e+1/2*f*x)^7-2/tan(1/2*e+1/2*f
*x)^3+4/5/tan(1/2*e+1/2*f*x)^5+4/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.34, size = 117, normalized size = 0.98

$$\frac{\frac{\left(\frac{28\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{70\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{140\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{ac^{4}\sin\left(fx+e\right)^{7}}+\frac{35\sin\left(fx+e\right)}{ac^{4}\left(\cos\left(fx+e\right)+1\right)}}{560f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/560*((28*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 140*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) +
 1)^7/(a*c^4*sin(f*x + e)^7) + 35*sin(f*x + e)/(a*c^4*(cos(f*x + e) + 1)))/
f
```

mupad [B] time = 2.13, size = 83, normalized size = 0.69

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{16ac^{4}f}+\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}}{4}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}}{8}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}}{20}-\frac{1}{112}}{ac^{4}f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(16*a*c^4*f) + (tan(e/2 + (f*x)/2)^2/20 - tan(e/2 + (f*x
)/2)^4/8 + tan(e/2 + (f*x)/2)^6/4 - 1/112)/(a*c^4*f*tan(e/2 + (f*x)/2)^7)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-3\sec^4\left(e+fx\right)+2\sec^3\left(e+fx\right)+2\sec^2\left(e+fx\right)-3\sec\left(e+fx\right)+1}\,dx}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 - 3*sec(e + f*x)**4 + 2*sec(e + f*x)
**3 + 2*sec(e + f*x)**2 - 3*sec(e + f*x) + 1), x)/(a*c**4)
```

3.42 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=164

$$-\frac{7c^5\tan^3(e+fx)}{a^2f}-\frac{84c^5\tan(e+fx)}{a^2f}+\frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{63c^5\tan(e+fx)\sec(e+fx)}{2a^2f}-\frac{6c^2\tan(e+fx)}{f\left(a^2\sec(\right.}$$

```
[Out] 105/2*c^5*arctanh(sin(f*x+e))/a^2/f-84*c^5*tan(f*x+e)/a^2/f+63/2*c^5*sec(f*
x+e)*tan(f*x+e)/a^2/f-6*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*
x+e))+2/3*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-7*c^5*tan(f*
x+e)^3/a^2/f
```

Rubi [A] time = 0.25, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3957, 3791, 3770, 3767, 8, 3768}

$$-\frac{7c^5\tan^3(e+fx)}{a^2f}-\frac{84c^5\tan(e+fx)}{a^2f}+\frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{63c^5\tan(e+fx)\sec(e+fx)}{2a^2f}-\frac{6c^2\tan(e+fx)}{f\left(a^2\sec(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (105*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (84*c^5*Tan[e + f*x])/(a^2*f) +
 (63*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^2*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^4
*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (7*c^5*Tan[e + f*x]^3)/(a^2*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx}{a}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

$$= \frac{21c^5\tanh^{-1}(\sin(e+fx))}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} - \frac{6c^2(c-c\sec(}{f\left(a^2+\right.}$$

$$= \frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+}{2a^2f}$$

Mathematica [B] time = 1.20, size = 380, normalized size = 2.32

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^6\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(\sec\left(\frac{e}{2}\right)\sec(e)\left(-2901\sin\left(e-\frac{fx}{2}\right)+1197\sin\left(e+\frac{fx}{2}\right)-302\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^6*(c - c*Sec[e + f*x])^5*(20160*Cos[e +
f*x]^3*Cot[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[C
os[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Csc[(e + f*x)/2]^3*Sec[e/2]*Sec[e]*(
-1323*Sin[(f*x)/2] + 3247*Sin[(3*f*x)/2] - 2901*Sin[e - (f*x)/2] + 1197*Sin
[e + (f*x)/2] - 3027*Sin[2*e + (f*x)/2] - 273*Sin[e + (3*f*x)/2] + 1827*Sin
[2*e + (3*f*x)/2] - 1693*Sin[3*e + (3*f*x)/2] + 1995*Sin[e + (5*f*x)/2] - 1
17*Sin[2*e + (5*f*x)/2] + 1143*Sin[3*e + (5*f*x)/2] - 969*Sin[4*e + (5*f*x)
/2] + 1173*Sin[2*e + (7*f*x)/2] + 117*Sin[3*e + (7*f*x)/2] + 747*Sin[4*e +
(7*f*x)/2] - 309*Sin[5*e + (7*f*x)/2] + 494*Sin[3*e + (9*f*x)/2] + 142*Sin[
4*e + (9*f*x)/2] + 352*Sin[5*e + (9*f*x)/2])))/(3072*a^2*f*(1 + Sec[e + f*x
])^2)
```

fricas [A] time = 0.47, size = 210, normalized size = 1.28

$$\frac{315\left(c^5\cos\left(fx+e\right)^5+2\,c^5\cos\left(fx+e\right)^4+c^5\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-315\left(c^5\cos\left(fx+e\right)^5+2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^5*cos(f*x + e)^3)*
log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^
5*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*(494*c^5*cos(f*x + e)^4 + 679*
c^5*cos(f*x + e)^3 + 102*c^5*cos(f*x + e)^2 - 17*c^5*cos(f*x + e) + 2*c^5)*
sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f*cos(f*
x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*((8/3*tan((f*x+exp(1))/2)^3*c^5*a^4+32*tan((f*x+exp(1))/2)*c^5
*a^4)/a^6+(-165*tan((f*x+exp(1))/2)^5*c^5+280*tan((f*x+exp(1))/2)^3*c^5-123
*tan((f*x+exp(1))/2)*c^5)*1/6/a^2/(tan((f*x+exp(1))/2)^2-1)^3+105*c^5*1/4/a
^2*ln(abs(tan((f*x+exp(1))/2)-1))-105*c^5*1/4/a^2*ln(abs(tan((f*x+exp(1))/2
)+1)))
```

maple [A] time = 0.80, size = 234, normalized size = 1.43

$$-\frac{16c^5\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^2}-\frac{64c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^2}+\frac{c^5}{3f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3}+\frac{4c^5}{f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}+\frac{55c}{2f\,a^2\left(\tan\left(\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -16/3/f*c^5/a^2*tan(1/2*e+1/2*f*x)^3-64/f*c^5/a^2*tan(1/2*e+1/2*f*x)+1/3/f*
c^5/a^2/(tan(1/2*e+1/2*f*x)-1)^3+4/f*c^5/a^2/(tan(1/2*e+1/2*f*x)-1)^2+55/2/
f*c^5/a^2/(tan(1/2*e+1/2*f*x)-1)-105/2/f*c^5/a^2*ln(tan(1/2*e+1/2*f*x)-1)+1
/3/f*c^5/a^2/(tan(1/2*e+1/2*f*x)+1)^3-4/f*c^5/a^2/(tan(1/2*e+1/2*f*x)+1)^2+
55/2/f*c^5/a^2/(tan(1/2*e+1/2*f*x)+1)+105/2/f*c^5/a^2*ln(tan(1/2*e+1/2*f*x)
+1)
```

maxima [B] time = 0.35, size = 765, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e)
+ 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(
f*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
+ 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^5*((15*s
in(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 -
 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x + e
) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.73, size = 170, normalized size = 1.04

$$\frac{55\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{280\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+41\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}-\frac{64\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}-\frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}+\frac{1}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (55*c^5*tan(e/2 + (f*x)/2)^5 - (280*c^5*tan(e/2 + (f*x)/2)^3)/3 + 41*c^5*ta
n(e/2 + (f*x)/2))/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)
^4 + a^2*tan(e/2 + (f*x)/2)^6 - a^2)) - (64*c^5*tan(e/2 + (f*x)/2))/(a^2*f)
 - (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (105*c^5*atanh(tan(e/2 + (f*x)
/2)))/(a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^5\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{5\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{10\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{1}{\sec^2\left(e+\right.}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(5*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(10*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integr
al(-5*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.43 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{70c^4\tan(e+fx)}{3a^2f}+\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{35c^4\tan(e+fx)\sec(e+fx)}{6a^2f}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}$$

```
[Out] 35/2*c^4*arctanh(sin(f*x+e))/a^2/f-70/3*c^4*tan(f*x+e)/a^2/f+35/6*c^4*sec(f
*x+e)*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e
))^2-14/3*(c^2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$-\frac{70c^4\tan(e+fx)}{3a^2f}+\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{35c^4\tan(e+fx)\sec(e+fx)}{6a^2f}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (35*c^4*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (70*c^4*Tan[e + f*x])/(3*a^2*f)
+ (35*c^4*Sec[e + f*x]*Tan[e + f*x])/(6*a^2*f) + (2*c*(c - c*Sec[e + f*x])^
3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (14*(c^2 - c^2*Sec[e + f*x])
^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(7c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+f}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+f}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14}{}$$

$$= \frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{70c^4\tan(e+fx)}{3a^2f} + \frac{35c^4\sec(e+fx)\tan(e}{6a^2f}$$

Mathematica [B] time = 1.97, size = 349, normalized size = 2.33

$$\frac{c^4 \sin^3\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(-32\tan\left(\frac{e}{2}\right)\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)-32\sec\left(\frac{e}{2}\right)\sin\left(\frac{f}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^4*Cos[(e + f*x)/2]*Sec[e + f*x]^2*Sin[(e + f*x)/2]^3*(-256*Cot[(e + f*x)
/2]^2*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2] - 32*Csc[(e + f*x)/2]^3*Sec[e/
2]*Sin[(f*x)/2] + 3*Cot[(e + f*x)/2]^3*(-70*Log[Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]] + 70*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(-2) - (2
4*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))) - 32*Cot[(e + f
*x)/2]*Csc[(e + f*x)/2]^2*Tan[e/2]))/(3*a^2*f*(1 + Sec[e + f*x])^2)
```

fricas [A] time = 0.46, size = 197, normalized size = 1.31

$$\frac{105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx+e\right)^3+c^4\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\right.}{12\left(a^2f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*
log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^
4*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(164*c^4*cos(f*x + e)^3 + 229*
c^4*cos(f*x + e)^2 + 30*c^4*cos(f*x + e) - 3*c^4)*sin(f*x + e))/(a^2*f*cos(
f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-4/3*tan((f*x+exp(1))/2)^3*c^4*a^4-12*tan((f*x+exp(1))/2)*c^4
*a^4)/a^6-(-13*tan((f*x+exp(1))/2)^3*c^4+11*tan((f*x+exp(1))/2)*c^4)*1/2/a^
2/(tan((f*x+exp(1))/2)^2-1)^2-35*c^4*1/4/a^2*ln(abs(tan((f*x+exp(1))/2)-1))
+35*c^4*1/4/a^2*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.74, size = 186, normalized size = 1.24

$$-\frac{8c^4\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^2}-\frac{24c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^2}+\frac{c^4}{2f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}+\frac{13c^4}{2f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}-\frac{35c^4\ln\left(\tan\left(\frac{e}{2}\right.\right.}{2f\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -8/3/f*c^4/a^2*tan(1/2*e+1/2*f*x)^3-24/f*c^4/a^2*tan(1/2*e+1/2*f*x)+1/2/f*c
^4/a^2/(tan(1/2*e+1/2*f*x)-1)^2+13/2/f*c^4/a^2/(tan(1/2*e+1/2*f*x)-1)-35/2/
f*c^4/a^2*ln(tan(1/2*e+1/2*f*x)-1)-1/2/f*c^4/a^2/(tan(1/2*e+1/2*f*x)+1)^2+1
3/2/f*c^4/a^2/(tan(1/2*e+1/2*f*x)+1)+35/2/f*c^4/a^2*ln(tan(1/2*e+1/2*f*x)+1
)
```

maxima [B] time = 0.35, size = 531, normalized size = 3.54

$$-\frac{c^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+4c^4\left(\frac{\frac{15\sin(fx}{\cos(fx+e)}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 4*c^4*((15
*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(c
os(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^4*((9*sin(f*x + e)/(cos(f*x +
```

```
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
+ 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.68, size = 136, normalized size = 0.91

$$\frac{13\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-11\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)}-\frac{24\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}-\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}+\frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (13*c^4*tan(e/2 + (f*x)/2)^3 - 11*c^4*tan(e/2 + (f*x)/2))/(f*(a^2*tan(e/2 +
 (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (24*c^4*tan(e/2 + (f*x)/
2))/(a^2*f) - (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (35*c^4*atanh(tan(e/
2 + (f*x)/2)))/(a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+\cdots}\right)\cdots\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(6*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integra
l(-4*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.44 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{5c^3\tan(e+fx)}{a^2f}+\frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{10\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+f}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] 5*c^3*arctanh(sin(f*x+e))/a^2/f-5*c^3*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e
))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-10/3*(c^3-c^3*sec(f*x+e))*tan(f*x+e)/f
/(a^2+a^2*sec(f*x+e))
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$-\frac{5c^3\tan(e+fx)}{a^2f}+\frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{10\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+f}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (5*c^3*Tan[e + f*x])/(a^2*f) + (2*c
*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (10*(c
^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(5c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10\left(c^3-c^3\sec(e+fx)\right)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10\left(c^3-c^3\sec(e+fx)\right)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10\left(c^3-c\right.}{3f\left(\right.}$$

$$= \frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e}{3f(a+a\sec(e+fx))}$$

Mathematica [B] time = 4.32, size = 485, normalized size = 4.08

$$\frac{c^3(\cos(e+fx)-1)^3\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(-\frac{1}{16}\sec^3\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)(-80\cos(e+fx)-40\cos(2(e+fx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^3*(-1 + Cos[e + f*x])^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(-15*Cos[e]*
Cot[(e + f*x)/2]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]])*Sec[e/2]^2 + 2*Cot[(e + f*x)/2]*Csc[(e + f*x)
/2]^2*Sec[e/2]^3*(-Sin[e/2] + Sin[(3*e)/2]) + 20*Cot[(e + f*x)/2]^2*Csc[(e
+ f*x)/2]*Sec[e/2]*Sin[(f*x)/2] - 26*Cot[(e + f*x)/2]^4*Csc[(e + f*x)/2]*Se
c[e/2]*Sin[(f*x)/2] - ((-40 + 40*Cos[e] + 78*Cos[f*x] - 80*Cos[e + f*x] - 4
0*Cos[2*(e + f*x)] + 66*Cos[2*e + f*x] + 23*Cos[e + 2*f*x] + 17*Cos[3*e + 2
*f*x])*Csc[(e + f*x)/2]^5*Sec[e/2]^3*Sin[(f*x)/2])/16 - Cot[(e + f*x)/2]^3*
(15*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]]) - 4*Csc[(e + f*x)/2]^2*Tan[e/2])*(-1 + Tan[e/2]^2)))/(6*a^2*
f*(1 + Cos[e + f*x])^2*(-1 + Cot[(e + f*x)/2])*(1 + Cot[(e + f*x)/2])*(-1 +
 Tan[e/2])*(1 + Tan[e/2]))
```

fricas [A] time = 0.49, size = 178, normalized size = 1.50

$$\frac{15\left(c^3\cos\left(fx+e\right)^3+2\,c^3\cos\left(fx+e\right)^2+c^3\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+2\,c^3\,c}{6\left(a^2f\cos\left(fx+e\right)^3+2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*log(
sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos
(f*x + e))*log(-sin(f*x + e) + 1) - 2*(23*c^3*cos(f*x + e)^2 + 34*c^3*cos(f
*x + e) + 3*c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*((2/3*tan((f*x+exp(1))/2)^3*c^3*a^4+4*tan((f*x+exp(1))/2)*c^3*
a^4)/a^6-tan((f*x+exp(1))/2)*c^3/a^2/(tan((f*x+exp(1))/2)^2-1)+5*c^3*1/2/a^
2*ln(abs(tan((f*x+exp(1))/2)-1))-5*c^3*1/2/a^2*ln(abs(tan((f*x+exp(1))/2)+1
)))
```

maple [A] time = 0.68, size = 136, normalized size = 1.14

$$-\frac{4c^3\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^2}-\frac{8c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^2}+\frac{c^3}{f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}-\frac{5c^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,a^2}+\frac{c^3}{f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -4/3/f*c^3/a^2*tan(1/2*e+1/2*f*x)^3-8/f*c^3/a^2*tan(1/2*e+1/2*f*x)+1/f*c^3/
a^2/(tan(1/2*e+1/2*f*x)-1)-5/f*c^3/a^2*ln(tan(1/2*e+1/2*f*x)-1)+1/f*c^3/a^2
/(tan(1/2*e+1/2*f*x)+1)+5/f*c^3/a^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.34, size = 341, normalized size = 2.87

$$-\frac{c^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+3\,c^3\left(\frac{9\sin(fx+\right.}{\cos(fx+e)}}{6\,j}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 3*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(
cos(f*x + e) + 1)^3)/a^2 - c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.65, size = 104, normalized size = 0.87

$$\frac{10\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}-\frac{4\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}-\frac{8\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}+\frac{2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (10*c^3*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (4*c^3*tan(e/2 + (f*x)/2)^3)/(
3*a^2*f) - (8*c^3*tan(e/2 + (f*x)/2))/(a^2*f) + (2*c^3*tan(e/2 + (f*x)/2))/
(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}dx+\int\left(-\frac{3\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{}{\sec^2\left(e+\right.}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-3*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.45 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] c^2*arctanh(sin(f*x+e))/a^2/f-2*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*(
c^2-c^2*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$\frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*ArcTanh[Sin[e + f*x]])/(a^2*f) - (2*c^2*Tan[e + f*x])/(f*(a^2 + a^2*Se
c[e + f*x])) + (2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e
+ f*x])^2)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}dx}{a}$$

$$= -\frac{2c^2\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{c^2\int \mathrm{se}}{}$$

$$= \frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{2c^2\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2\left(c^2-c^2\sec(e+f\right.}{3f(a+a\sec(}$$

Mathematica [A] time = 0.11, size = 109, normalized size = 1.24

$$\frac{c^2\left(-\frac{4\tan\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (4*Tan[(e + f*x)/2])/(3*f) - (2*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/(3*f)))/a^2
```

fricas [A] time = 0.63, size = 138, normalized size = 1.57

$$\frac{3\left(c^2\cos\left(fx+e\right)^2+2\,c^2\cos\left(fx+e\right)+c^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^2\cos\left(fx+e\right)^2+2\,c^2\cos\left(fx+e\right)+c^2\right.}{6\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(sin(f*x + e) + 1
) - 3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(-sin(f*x + e) + 1
) - 8*(c^2*cos(f*x + e) + 2*c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^
2*f*cos(f*x + e) + a^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-c^2*1/2/a^2*ln(abs(tan((f*x+exp(1))/2)-1))+c^2*1/2/a^2*ln(abs
(tan((f*x+exp(1))/2)+1))+(-1/3*tan((f*x+exp(1))/2)^3*c^2*a^4-tan((f*x+exp(1
))/2)*c^2*a^4)/a^6)
```

maple [A] time = 0.82, size = 89, normalized size = 1.01

$$-\frac{2c^2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^2}-\frac{2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^2}-\frac{c^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,a^2}+\frac{c^2\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/f*c^2/a^2*tan(1/2*e+1/2*f*x)^3-2/f*c^2/a^2*tan(1/2*e+1/2*f*x)-1/f*c^2/
a^2*ln(tan(1/2*e+1/2*f*x)-1)+1/f*c^2/a^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.33, size = 196, normalized size = 2.23

$$-\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\cdots\right.}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.61, size = 46, normalized size = 0.52

$$-\frac{2\,c^2\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] -(2*c^2*(3*tan(e/2 + (f*x)/2) - 3*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*
x)/2)^3))/(3*a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-2*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.46 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] 1/3*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] time = 0.09, size = 23, normalized size = 0.64

$$-\frac{c\tan^3\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(c*Tan[(e + f*x)/2]^3)/(a^2*f)
```

fricas [A] time = 0.45, size = 53, normalized size = 1.47

$$\frac{\left(c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(c*cos(f*x + e) - c)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f
*x + e) + a^2*f)
```

giac [A] time = 0.30, size = 21, normalized size = 0.58

$$-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{3\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*c*tan(1/2*f*x + 1/2*e)^3/(a^2*f)
```

maple [A] time = 0.80, size = 21, normalized size = 0.58

$$-\frac{c\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -1/3/f*c/a^2*tan(1/2*e+1/2*f*x)^3
```

maxima [B] time = 0.33, size = 94, normalized size = 2.61

$$-\frac{\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2}-\frac{c\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.57, size = 20, normalized size = 0.56

$$-\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] -(c*tan(e/2 + (f*x)/2)^3)/(3*a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.47 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=59

$$\frac{\cot^3(e+fx)}{3a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}$$

```
[Out] 1/3*cot(f*x+e)^3/a^2/c/f+csc(f*x+e)/a^2/c/f-1/3*csc(f*x+e)^3/a^2/c/f
```

Rubi [A] time = 0.14, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3958, 2606, 2607, 30}

$$\frac{\cot^3(e+fx)}{3a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] Cot[e + f*x]^3/(3*a^2*c*f) + Csc[e + f*x]/(a^2*c*f) - Csc[e + f*x]^3/(3*a^2
*c*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
```

```
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx &= \frac{\int \left(c\cot^3(e+fx)\csc(e+fx)-c\cot^2(e+fx)\csc^2(e+fx)\right)dx}{a^2c^2}\\ &= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c}-\frac{\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{a^2c}\\ &= -\frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(e+fx)\right)}{a^2cf}-\frac{\text{Subst}\left(\int \left(-1+x^2\right)dx,x,\csc\right.}{a^2cf}\\ &= \frac{\cot^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}\end{aligned}$$

Mathematica [A] time = 0.57, size = 83, normalized size = 1.41

$$-\frac{\csc(e)\sin^2\left(\frac{1}{2}(e+fx)\right)(10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)-2\sin(e+2fx)-6\sin(e)+2\sin(fx}{6a^2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/6*(Csc[e]*Csc[e + f*x]^3*Sin[(e + f*x)/2]^2*(-6*Sin[e] + 2*Sin[f*x] + 10
*Sin[e + f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] - 2*Sin[e + 2*f*x]))/
(a^2*c*f)
```

fricas [A] time = 0.47, size = 49, normalized size = 0.83

$$-\frac{\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)-2}{3\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/3*(cos(f*x + e)^2 - 2*cos(f*x + e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f
)*sin(f*x + e))
```

giac [A] time = 0.51, size = 72, normalized size = 1.22

$$\frac{\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)} - \frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 6a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/12*(3/(a^2*c*tan(1/2*f*x + 1/2*e)) - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*
a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

maple [A] time = 0.64, size = 48, normalized size = 0.81

$$\frac{-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3} + 2\tan\left(\frac{e}{2}+\frac{fx}{2}\right) + \frac{1}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{4fa^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/4/f/a^2/c*(-1/3*tan(1/2*e+1/2*f*x)^3+2*tan(1/2*e+1/2*f*x)+1/tan(1/2*e+1/2
*f*x))
```

maxima [A] time = 0.33, size = 76, normalized size = 1.29

$$\frac{\frac{\frac{6\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c} + \frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/(a^2*c) + 3*(cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

mupad [B] time = 1.62, size = 61, normalized size = 1.03

$$-\frac{4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4 - 8\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2 + 1}{12a^2cf\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^4 - 8*cos(e/2 + (f*x)/2)^2 + 1)/(12*a^2*c*f*cos(e/2
+ (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+\sec^2\left(e+fx\right)-\sec\left(e+fx\right)-1}\,dx}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) -
1), x)/(a**2*c)
```

3.48 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}$$

```
[Out] csc(f*x+e)/a^2/c^2/f-1/3*csc(f*x+e)^3/a^2/c^2/f
```

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3958, 2606}

$$\frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Csc[e + f*x]/(a^2*c^2*f) - Csc[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int\left(-1+x^2\right)\,dx, x, \csc(e+fx)\right)}{a^2c^2f}$$
$$= \frac{\csc(e+fx)}{a^2c^2f} - \frac{\csc^3(e+fx)}{3a^2c^2f}$$

Mathematica [A] time = 0.05, size = 33, normalized size = 0.87

$$\frac{\frac{\csc(e+fx)}{f} - \frac{\csc^3(e+fx)}{3f}}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e + f*x]/f - Csc[e + f*x]^3/(3*f))/(a^2*c^2)
```

fricas [A] time = 0.45, size = 50, normalized size = 1.32

$$\frac{3\cos\left(fx+e\right)^2-2}{3\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*cos(f*x + e)^2 - 2)/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x
+ e))
```

giac [A] time = 0.32, size = 33, normalized size = 0.87

$$\frac{3\sin\left(fx+e\right)^2-1}{3\,a^2c^2f\sin\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)^{2}\left(c-c\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

maxima [A] time = 0.34, size = 31, normalized size = 0.82

$$\frac{3\sin\left(fx+e\right)^{2}-1}{3\,a^{2}c^{2}f\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

mupad [B] time = 1.57, size = 28, normalized size = 0.74

$$\frac{\sin\left(e+f\,x\right)^{2}-\frac{1}{3}}{a^{2}\,c^{2}\,f\sin\left(e+f\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (sin(e + f*x)^2 - 1/3)/(a^2*c^2*f*sin(e + f*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{4}\left(e+fx\right)-2\sec^{2}\left(e+fx\right)+1}\,dx}{a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c
**2)
```

3.49 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=80

$$\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}$$

```
[Out] 1/5*cot(f*x+e)^5/a^2/c^3/f+csc(f*x+e)/a^2/c^3/f-2/3*csc(f*x+e)^3/a^2/c^3/f+
1/5*csc(f*x+e)^5/a^2/c^3/f
```

Rubi [A] time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2606, 194, 2607, 30}

$$\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Cot[e + f*x]^5/(5*a^2*c^3*f) + Csc[e + f*x]/(a^2*c^3*f) - (2*Csc[e + f*x]^3
)/(3*a^2*c^3*f) + Csc[e + f*x]^5/(5*a^2*c^3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \left(a\cot^5(e+fx)\csc(e+fx)+a\cot^4(e+fx)\csc^2(e+fx)\right)\,d}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^2c^3} - \frac{\int \cot^4(e+fx)\csc^2(e+fx)\,d}{a^2c^3}$$

$$= -\frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(e+fx)\right)}{a^2c^3f} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x\right.}{a^2c^3f}$$

$$= \frac{\cot^5(e+fx)}{5a^2c^3f} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{a^2c^3f}$$

$$= \frac{\cot^5(e+fx)}{5a^2c^3f} + \frac{\csc(e+fx)}{a^2c^3f} - \frac{2\csc^3(e+fx)}{3a^2c^3f} + \frac{\csc^5(e+fx)}{5a^2c^3f}$$

Mathematica [A] time = 0.96, size = 147, normalized size = 1.84

$$-\frac{\csc(e)(534\sin(e+fx)-178\sin(2(e+fx))-178\sin(3(e+fx))+89\sin(4(e+fx))+40\sin(2e+fx)-168\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -1/1920*(Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]^3*(-200*Sin[e] + 104*Sin[f*
x] + 534*Sin[e + f*x] - 178*Sin[2*(e + f*x)] - 178*Sin[3*(e + f*x)] + 89*Si
n[4*(e + f*x)] + 40*Sin[2*e + f*x] - 168*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f
*x] + 72*Sin[2*e + 3*f*x] - 120*Sin[4*e + 3*f*x] + 24*Sin[3*e + 4*f*x]))/(a
^2*c^3*f)
```

fricas [A] time = 0.45, size = 109, normalized size = 1.36

$$\frac{3\,\cos\left(fx+e\right)^{4}+12\,\cos\left(fx+e\right)^{3}-12\,\cos\left(fx+e\right)^{2}-8\,\cos\left(fx+e\right)+8}{15\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+e\right)^{2}-a^{2}c^{3}f\cos\left(fx+e\right)+a^{2}c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(3*cos(f*x + e)^4 + 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 - 8*cos(f*x
+ e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f
*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.36, size = 101, normalized size = 1.26

$$\frac{\frac{90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-20\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3}{a^{2}c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}-\frac{5\left(a^{4}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-12\,a^{4}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{6}c^{9}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*((90*tan(1/2*f*x + 1/2*e)^4 - 20*tan(1/2*f*x + 1/2*e)^2 + 3)/(a^2*c^3
*tan(1/2*f*x + 1/2*e)^5) - 5*(a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 12*a^4*c^6*t
an(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

maple [A] time = 0.86, size = 76, normalized size = 0.95

$$\frac{-\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{4}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}+\frac{6}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{16f\,a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/16/f/a^2/c^3*(-1/3*tan(1/2*e+1/2*f*x)^3+4*tan(1/2*e+1/2*f*x)-4/3/tan(1/2*
e+1/2*f*x)^3+1/5/tan(1/2*e+1/2*f*x)^5+6/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.34, size = 121, normalized size = 1.51

$$\frac{\frac{5\left(\frac{12\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{a^{2}c^{3}}-\frac{\left(\frac{20\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{90\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-3\right)\left(\cos\left(fx+e\right)+1\right)^{5}}{a^{2}c^{3}\sin\left(fx+e\right)^{5}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(5*(12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^3) - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 90*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^
5))/f
```

mupad [B] time = 2.01, size = 76, normalized size = 0.95

$$\frac{-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+60\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+90\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+3}{240\,a^{2}\,c^{3}\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (90*tan(e/2 + (f*x)/2)^4 - 20*tan(e/2 + (f*x)/2)^2 + 60*tan(e/2 + (f*x)/2)^
6 - 5*tan(e/2 + (f*x)/2)^8 + 3)/(240*a^2*c^3*f*tan(e/2 + (f*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-\sec^{4}\left(e+fx\right)-2\sec^{3}\left(e+fx\right)+2\sec^{2}\left(e+fx\right)+\sec\left(e+fx\right)-1}\,dx}{a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)*
*3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

3.50 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}$$

```
[Out] -2/7*cot(f*x+e)^7/a^2/c^4/f+csc(f*x+e)/a^2/c^4/f-4/3*csc(f*x+e)^3/a^2/c^4/f
+csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Rubi [A] time = 0.19, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 270}

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (-2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + Csc[e + f*x]/(a^2*c^4*f) - (4*Csc[e + f
*x]^3)/(3*a^2*c^4*f) + Csc[e + f*x]^5/(a^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a
^2*c^4*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(a^2\cot^7(e+fx)\csc(e+fx)+2a^2\cot^6(e+fx)\csc^2(e+fx)\right.}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^2c^4} + \frac{\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{a^2c^4}$$

$$= -\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{a^2c^4f} - \frac{\text{Subst}\left(\int \left(-1+\right.\right.}{}$$

$$= -\frac{2\cot^7(e+fx)}{7a^2c^4f} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, \csc(e\right.}{a^2c^4f}$$

$$= -\frac{2\cot^7(e+fx)}{7a^2c^4f} + \frac{\csc(e+fx)}{a^2c^4f} - \frac{4\csc^3(e+fx)}{3a^2c^4f} + \frac{\csc^5(e+fx)}{a^2c^4f}$$

Mathematica [A] time = 0.93, size = 179, normalized size = 1.83

$$\frac{\csc(e)(-182\sin(e+fx)+104\sin(2(e+fx))+39\sin(3(e+fx))-52\sin(4(e+fx))+13\sin(5(e+fx))-56\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^4*Csc[e + f*x]^3*(42*Sin[e] - 28*Sin[f*x] - 182*Si
n[e + f*x] + 104*Sin[2*(e + f*x)] + 39*Sin[3*(e + f*x)] - 52*Sin[4*(e + f*x
)] + 13*Sin[5*(e + f*x)] - 56*Sin[2*e + f*x] + 76*Sin[e + 2*f*x] - 28*Sin[3
*e + 2*f*x] - 24*Sin[2*e + 3*f*x] + 42*Sin[4*e + 3*f*x] - 3*Sin[3*e + 4*f*x
] - 21*Sin[5*e + 4*f*x] + 6*Sin[4*e + 5*f*x]))/(1344*a^2*c^4*f)
```

fricas [A] time = 0.45, size = 120, normalized size = 1.22

$$\frac{6\cos\left(fx+e\right)^{5}+9\cos\left(fx+e\right)^{4}-24\cos\left(fx+e\right)^{3}+4\cos\left(fx+e\right)^{2}+16\cos\left(fx+e\right)-8}{21\left(a^{2}c^{4}f\cos\left(fx+e\right)^{4}-2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}+2\,a^{2}c^{4}f\cos\left(fx+e\right)-a^{2}c^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/21*(6*cos(f*x + e)^5 + 9*cos(f*x + e)^4 - 24*cos(f*x + e)^3 + 4*cos(f*x +
 e)^2 + 16*cos(f*x + e) - 8)/((a^2*c^4*f*cos(f*x + e)^4 - 2*a^2*c^4*f*cos(f
*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*sin(f*x + e))
```

giac [A] time = 0.44, size = 115, normalized size = 1.17

$$\frac{\frac{210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-3}{a^{2}c^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}}-\frac{7\left(a^{4}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-15\,a^{4}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{6}c^{12}}}{672\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/672*((210*tan(1/2*f*x + 1/2*e)^6 - 70*tan(1/2*f*x + 1/2*e)^4 + 21*tan(1/2
*f*x + 1/2*e)^2 - 3)/(a^2*c^4*tan(1/2*f*x + 1/2*e)^7) - 7*(a^4*c^8*tan(1/2*
f*x + 1/2*e)^3 - 15*a^4*c^8*tan(1/2*f*x + 1/2*e))/(a^6*c^12))/f
```

maple [A] time = 0.84, size = 87, normalized size = 0.89

$$\frac{-\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}-\frac{10}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{1}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}+\frac{10}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{32f\,a^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/32/f/a^2/c^4*(-1/3*tan(1/2*e+1/2*f*x)^3+5*tan(1/2*e+1/2*f*x)-1/7/tan(1/2*
e+1/2*f*x)^7-10/3/tan(1/2*e+1/2*f*x)^3+1/tan(1/2*e+1/2*f*x)^5+10/tan(1/2*e+
1/2*f*x))
```

maxima [A] time = 0.34, size = 140, normalized size = 1.43

$$\frac{\frac{7\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{70\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{210\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{a^2c^4\sin(fx+e)^7}}{672f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/672*(7*(15*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^4) + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 210*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(
cos(f*x + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

mupad [B] time = 2.67, size = 89, normalized size = 0.91

$$\frac{-7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+105\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+210\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-70\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-3}{672\,a^2\,c^4\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (21*tan(e/2 + (f*x)/2)^2 - 70*tan(e/2 + (f*x)/2)^4 + 210*tan(e/2 + (f*x)/2)
^6 + 105*tan(e/2 + (f*x)/2)^8 - 7*tan(e/2 + (f*x)/2)^10 - 3)/(672*a^2*c^4*f
*tan(e/2 + (f*x)/2)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec(e+fx)}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**
4 + 4*sec(e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**
4)
```

3.51 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=141

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}$$

```
[Out] 1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+csc(f*x+e)/a^2/c^5/f-
7/3*csc(f*x+e)^3/a^2/c^5/f+3*csc(f*x+e)^5/a^2/c^5/f-13/7*csc(f*x+e)^7/a^2/c
^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Rubi [A] time = 0.25, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2606, 194, 2607, 30, 270, 14}

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^9)/(9*a^2*c^5*f) + Csc[e + f
*x]/(a^2*c^5*f) - (7*Csc[e + f*x]^3)/(3*a^2*c^5*f) + (3*Csc[e + f*x]^5)/(a^
2*c^5*f) - (13*Csc[e + f*x]^7)/(7*a^2*c^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^
5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = -\frac{\int \left(a^3\cot^9(e+fx)\csc(e+fx)+3a^3\cot^8(e+fx)\csc^2(e+fx)\right.}{a}$$

$$= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^2c^5} - \frac{\int \cot^6(e+fx)\csc^4(e+fx)\,dx}{a^2c^5}$$

$$= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx, x, \csc(e+fx)\right)}{a^2c^5f} - \frac{\text{Subst}\left(\int x^6\left(1+x^2\right)\right.}{a^2c^5}$$

$$= \frac{\cot^9(e+fx)}{3a^2c^5f} + \frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)dx, x, \csc(e\right.}{a^2c^5f}$$

$$= \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{4\cot^9(e+fx)}{9a^2c^5f} + \frac{\csc(e+fx)}{a^2c^5f} - \frac{7\csc^3(e+fx)}{3a^2c^5f} +$$

Mathematica [A] time = 1.43, size = 211, normalized size = 1.50

$$-\frac{\csc(e)(36252\sin(e+fx)-27189\sin(2(e+fx))-2014\sin(3(e+fx))+12084\sin(4(e+fx))-6042\sin(5(e+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] -1/516096*(Csc[e]*Csc[(e + f*x)/2]^6*Csc[e + f*x]^3*(-9408*Sin[e] + 9792*Si
n[f*x] + 36252*Sin[e + f*x] - 27189*Sin[2*(e + f*x)] - 2014*Sin[3*(e + f*x)
] + 12084*Sin[4*(e + f*x)] - 6042*Sin[5*(e + f*x)] + 1007*Sin[6*(e + f*x)]
+ 12096*Sin[2*e + f*x] - 14400*Sin[e + 2*f*x] - 2016*Sin[3*e + 2*f*x] + 752
0*Sin[2*e + 3*f*x] - 8736*Sin[4*e + 3*f*x] + 1248*Sin[3*e + 4*f*x] + 6048*S
in[5*e + 4*f*x] - 1632*Sin[4*e + 5*f*x] - 2016*Sin[6*e + 5*f*x] + 608*Sin[5
*e + 6*f*x]))/(a^2*c^5*f)
```

fricas [A] time = 0.46, size = 163, normalized size = 1.16

$$\frac{19\cos\left(fx+e\right)^{6}+6\cos\left(fx+e\right)^{5}-66\cos\left(fx+e\right)^{4}+56\cos\left(fx+e\right)^{3}+24\cos\left(fx+e\right)^{2}-48\cdots}{63\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5}-3\,a^{2}c^{5}f\cos\left(fx+e\right)^{4}+2\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}+2\,a^{2}c^{5}f\cos\left(fx+e\right)^{2}-3\,a^{2}c^{5}f\cos\left(f\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(19*cos(f*x + e)^6 + 6*cos(f*x + e)^5 - 66*cos(f*x + e)^4 + 56*cos(f*x
 + e)^3 + 24*cos(f*x + e)^2 - 48*cos(f*x + e) + 16)/((a^2*c^5*f*cos(f*x + e
)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f
*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

giac [A] time = 1.38, size = 129, normalized size = 0.91

$$\frac{\frac{945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-420\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-54\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+7}{a^{2}c^{5}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}}-\frac{21\left(a^{4}c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-18\,a^{4}c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{6}c^{15}}}{4032\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

[Out] 1/4032*((945*tan(1/2*f*x + 1/2*e)^8 - 420*tan(1/2*f*x + 1/2*e)^6 + 189*tan(1/2*f*x + 1/2*e)^4 - 54*tan(1/2*f*x + 1/2*e)^2 + 7)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) - 21*(a^4*c^10*tan(1/2*f*x + 1/2*e)^3 - 18*a^4*c^10*tan(1/2*f*x + 1/2*e))/(a^6*c^15))/f

maple [A] time = 0.98, size = 102, normalized size = 0.72

$$\frac{-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{6}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}+\frac{1}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9}-\frac{20}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}+\frac{3}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}+\frac{15}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{64f\,a^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)

[Out] 1/64/f/a^2/c^5*(-1/3*tan(1/2*e+1/2*f*x)^3+6*tan(1/2*e+1/2*f*x)-6/7/tan(1/2*e+1/2*f*x)^7+1/9/tan(1/2*e+1/2*f*x)^9-20/3/tan(1/2*e+1/2*f*x)^3+3/tan(1/2*e+1/2*f*x)^5+15/tan(1/2*e+1/2*f*x))

maxima [A] time = 0.34, size = 161, normalized size = 1.14

$$\frac{\frac{21\left(\frac{18\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right)}{a^2c^5}-\frac{\left(\frac{54\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{189\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{420\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{945\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}-7\right)\left(\cos\left(fx+e\right)+1\right)^9}{a^2c^5\sin\left(fx+e\right)^9}}{4032\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] 1/4032*(21*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*c^5) - (54*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 945*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(f*x + e)^9))/f

mupad [B] time = 4.24, size = 102, normalized size = 0.72

$$\frac{-21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+378\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+945\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-420\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+189\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-54\tan}{4032\,a^2\,c^5\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (189*tan(e/2 + (f*x)/2)^4 - 54*tan(e/2 + (f*x)/2)^2 - 420*tan(e/2 + (f*x)/2
)^6 + 945*tan(e/2 + (f*x)/2)^8 + 378*tan(e/2 + (f*x)/2)^10 - 21*tan(e/2 + (
f*x)/2)^12 + 7)/(4032*a^2*c^5*f*tan(e/2 + (f*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec\left(e+fx\right)}{\sec^7\left(e+fx\right)-3\sec^6\left(e+fx\right)+\sec^5\left(e+fx\right)+5\sec^4\left(e+fx\right)-5\sec^3\left(e+fx\right)-\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx}{a^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)*
*5 + 5*sec(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*
x) - 1), x)/(a**2*c**5)
```

3.52 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{77c^6\tan^3(e+fx)}{5a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f}-\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{693c^6\tan(e+fx)\sec(e+fx)}{10a^3f}+\frac{66\tan(e+fx)}{5f\left(a^3\right.}$$

```
[Out] -231/2*c^6*arctanh(sin(f*x+e))/a^3/f+924/5*c^6*tan(f*x+e)/a^3/f-693/10*c^6*
sec(f*x+e)*tan(f*x+e)/a^3/f-22/15*c^2*(c-c*sec(f*x+e))^4*tan(f*x+e)/a/f/(a+
a*sec(f*x+e))^2+2/5*c*(c-c*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+66
/5*(c^2-c^2*sec(f*x+e))^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+77/5*c^6*tan(f*
x+e)^3/a^3/f
```

Rubi [A] time = 0.34, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3791, 3770, 3767, 8, 3768}

$$\frac{77c^6\tan^3(e+fx)}{5a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f}-\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{693c^6\tan(e+fx)\sec(e+fx)}{10a^3f}+\frac{66\tan(e+fx)}{5f\left(a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-231*c^6*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (924*c^6*Tan[e + f*x])/(5*a^3*
f) - (693*c^6*Sec[e + f*x]*Tan[e + f*x])/(10*a^3*f) - (22*c^2*(c - c*Sec[e
+ f*x])^4*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e
 + f*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (66*(c^2 - c^2*Sec[
e + f*x])^3*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) + (77*c^6*Tan[e +
f*x]^3)/(5*a^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(11c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx}{5f(a+a\sec(e+fx))^3}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx}{5f(a+a\sec(e+fx))^3}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx}{5f(a+a\sec(e+fx))^3}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx}{5f(a+a\sec(e+fx))^3}$$

$$= -\frac{231c^6\tanh^{-1}(\sin(e+fx))}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f} - \frac{22c^2(c-}{15a}$$

$$= -\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f} + \frac{924c^6\tan(e+fx)}{5a^3f} - \frac{693c^6\sec(e+fx)\tan}{10a^3f}$$

Mathematica [A] time = 2.17, size = 406, normalized size = 1.89

$$\frac{c^6\cos\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\left(\sec\left(\frac{e}{2}\right)\sec(e)\left(-130340\sin\left(e-\frac{fx}{2}\right)+75600\sin\left(e+\frac{fx}{2}\right)-120176\sin\left(2e+\frac{fx}{2}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^6*Cos[(e + f*x)/2]*Sec[e + f*x]^3*(887040*Cos[(e + f*x)/2]^5*(Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
+ Sec[e/2]*Sec[e]*Sec[e + f*x]^3*(-65436*Sin[(f*x)/2] + 127498*Sin[(3*f*x)/
2] - 130340*Sin[e - (f*x)/2] + 75600*Sin[e + (f*x)/2] - 120176*Sin[2*e + (f
*x)/2] - 34230*Sin[e + (3*f*x)/2] + 82278*Sin[2*e + (3*f*x)/2] - 79450*Sin[
3*e + (3*f*x)/2] + 91670*Sin[e + (5*f*x)/2] - 14730*Sin[2*e + (5*f*x)/2] +
61920*Sin[3*e + (5*f*x)/2] - 44480*Sin[4*e + (5*f*x)/2] + 53593*Sin[2*e + (
7*f*x)/2] - 1735*Sin[3*e + (7*f*x)/2] + 38123*Sin[4*e + (7*f*x)/2] - 17205*
Sin[5*e + (7*f*x)/2] + 23735*Sin[3*e + (9*f*x)/2] + 2455*Sin[4*e + (9*f*x)/
2] + 17785*Sin[5*e + (9*f*x)/2] - 3495*Sin[6*e + (9*f*x)/2] + 5446*Sin[4*e
```

```
+ (11*f*x)/2] + 1190*Sin[5*e + (11*f*x)/2] + 4256*Sin[6*e + (11*f*x)/2])))/
(960*a^3*f*(1 + Sec[e + f*x])^3)
```

fricas [A] time = 0.48, size = 263, normalized size = 1.22

$$-\frac{3465\left(c^6\cos\left(fx+e\right)^6+3\,c^6\cos\left(fx+e\right)^5+3\,c^6\cos\left(fx+e\right)^4+c^6\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-34}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/60*(3465*(c^6*cos(f*x + e)^6 + 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)
^4 + c^6*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 3465*(c^6*cos(f*x + e)^6 +
 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)^4 + c^6*cos(f*x + e)^3)*log(-sin
(f*x + e) + 1) - 2*(5446*c^6*cos(f*x + e)^5 + 12843*c^6*cos(f*x + e)^4 + 87
11*c^6*cos(f*x + e)^3 + 815*c^6*cos(f*x + e)^2 - 105*c^6*cos(f*x + e) + 10*
c^6)*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f
*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((8/5*tan((f*x+exp(1))/2)^5*c^6*a^12+32/3*tan((f*x+exp(1))/2)^3
*c^6*a^12+80*tan((f*x+exp(1))/2)*c^6*a^12)/a^15+(-267*tan((f*x+exp(1))/2)^5
*c^6+472*tan((f*x+exp(1))/2)^3*c^6-213*tan((f*x+exp(1))/2)*c^6)*1/6/a^3/(ta
n((f*x+exp(1))/2)^2-1)^3+231*c^6*1/4/a^3*ln(abs(tan((f*x+exp(1))/2)-1))-231
*c^6*1/4/a^3*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.82, size = 256, normalized size = 1.19

$$\frac{16c^6\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5f\,a^3}+\frac{64c^6\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^3}+\frac{160c^6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^3}-\frac{c^6}{3f\,a^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3}-\frac{5c^6}{f\,a^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 16/5/f*c^6/a^3*tan(1/2*e+1/2*f*x)^5+64/3/f*c^6/a^3*tan(1/2*e+1/2*f*x)^3+160
/f*c^6/a^3*tan(1/2*e+1/2*f*x)-1/3/f*c^6/a^3/(tan(1/2*e+1/2*f*x)-1)^3-5/f*c^
6/a^3/(tan(1/2*e+1/2*f*x)-1)^2-89/2/f*c^6/a^3/(tan(1/2*e+1/2*f*x)-1)+231/2/
f*c^6/a^3*ln(tan(1/2*e+1/2*f*x)-1)-1/3/f*c^6/a^3/(tan(1/2*e+1/2*f*x)+1)^3+5
/f*c^6/a^3/(tan(1/2*e+1/2*f*x)+1)^2-89/2/f*c^6/a^3/(tan(1/2*e+1/2*f*x)+1)-2
31/2/f*c^6/a^3*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.38, size = 935, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^6*(60*(5*sin(f*x + e)/
(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/
a^3) + 45*c^6*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6
0*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a^3) + 20*c^6*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 18*c^6*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3)/f
```

mupad [B] time = 1.65, size = 193, normalized size = 0.90

$$\frac{160\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{89\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{472\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+71\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}+\frac{64\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (160*c^6*tan(e/2 + (f*x)/2))/(a^3*f) - (89*c^6*tan(e/2 + (f*x)/2)^5 - (472*
c^6*tan(e/2 + (f*x)/2)^3)/3 + 71*c^6*tan(e/2 + (f*x)/2))/(f*(3*a^3*tan(e/2
+ (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a^3)
) + (64*c^6*tan(e/2 + (f*x)/2)^3)/(3*a^3*f) + (16*c^6*tan(e/2 + (f*x)/2)^5)
/(5*a^3*f) - (231*c^6*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^6\left(\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{6\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{15\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**6*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-20*sec(e +
f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + In
tegral(15*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-6*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*
x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**7/(sec(e + f*x)**3
 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.53 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{42c^5\tan(e+fx)}{a^3f}-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{21c^5\tan(e+fx)\sec(e+fx)}{2a^3f}+\frac{42c\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{5f\left(a^3\sec(e+fx)+a^3\right)}$$

```
[Out] -63/2*c^5*arctanh(sin(f*x+e))/a^3/f+42*c^5*tan(f*x+e)/a^3/f-21/2*c^5*sec(f*
x+e)*tan(f*x+e)/a^3/f-6/5*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*
x+e))^2+2/5*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+42/5*c*(c^
2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] time = 0.31, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$\frac{42c^5\tan(e+fx)}{a^3f}-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{21c^5\tan(e+fx)\sec(e+fx)}{2a^3f}+\frac{42c\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{5f\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-63*c^5*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (42*c^5*Tan[e + f*x])/(a^3*f) -
 (21*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^3*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e + f*x])
^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (42*c*(c^2 - c^2*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(9c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

$$= \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx}{5af(a+a\sec(e+fx))^2}$$

$$= \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx}{5af(a+a\sec(e+fx))^2}$$

$$= -\frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f} + \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - $$

$$= -\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f} + \frac{42c^5\tan(e+fx)}{a^3f} - \frac{21c^5\sec(e+fx)\tan(e}{2a^3f}$$

Mathematica [A] time = 1.41, size = 380, normalized size = 1.97

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(\sec\left(\frac{e}{2}\right)\sec(e)\left(7351\sin\left(e-\frac{fx}{2}\right)-5271\sin\left(e+\frac{fx}{2}\right)+5545\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*(c - c*Sec[e + f*x])^5*(-40320*Cos[e +
 f*x]^2*Cot[(e + f*x)/2]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Csc[(e + f*x)/2]^5*Sec[e/2]*Sec[e]*
(3465*Sin[(f*x)/2] - 6115*Sin[(3*f*x)/2] + 7351*Sin[e - (f*x)/2] - 5271*Sin
[e + (f*x)/2] + 5545*Sin[2*e + (f*x)/2] + 2205*Sin[e + (3*f*x)/2] - 4515*Si
n[2*e + (3*f*x)/2] + 3805*Sin[3*e + (3*f*x)/2] - 4407*Sin[e + (5*f*x)/2] +
585*Sin[2*e + (5*f*x)/2] - 3447*Sin[3*e + (5*f*x)/2] + 1545*Sin[4*e + (5*f*
x)/2] - 2155*Sin[2*e + (7*f*x)/2] - 75*Sin[3*e + (7*f*x)/2] - 1755*Sin[4*e
+ (7*f*x)/2] + 325*Sin[5*e + (7*f*x)/2] - 496*Sin[3*e + (9*f*x)/2] - 80*Sin
[4*e + (9*f*x)/2] - 416*Sin[5*e + (9*f*x)/2])))/(5120*a^3*f*(1 + Sec[e + f*
x])^3)
```

fricas [A] time = 0.47, size = 250, normalized size = 1.30

$$-\frac{315\left(c^5\cos\left(fx+e\right)^5+3\,c^5\cos\left(fx+e\right)^4+3\,c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-315}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/20*(315*(c^5*cos(f*x + e)^5 + 3*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^
3 + c^5*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 3
*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f
*x + e) + 1) - 2*(496*c^5*cos(f*x + e)^4 + 1163*c^5*cos(f*x + e)^3 + 801*c^
5*cos(f*x + e)^2 + 65*c^5*cos(f*x + e) - 5*c^5)*sin(f*x + e))/(a^3*f*cos(f*
x + e)^5 + 3*a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x
+ e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*((-4/5*tan((f*x+exp(1))/2)^5*c^5*a^12-4*tan((f*x+exp(1))/2)^3*
c^5*a^12-24*tan((f*x+exp(1))/2)*c^5*a^12)/a^15-(-17*tan((f*x+exp(1))/2)^3*c
^5+15*tan((f*x+exp(1))/2)*c^5)*1/2/a^3/(tan((f*x+exp(1))/2)^2-1)^2-63*c^5*1
/4/a^3*ln(abs(tan((f*x+exp(1))/2)-1))+63*c^5*1/4/a^3*ln(abs(tan((f*x+exp(1)
)/2)+1)))
```

maple [A] time = 0.80, size = 208, normalized size = 1.08

$$\frac{8c^5\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5f\,a^3}+\frac{8c^5\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f\,a^3}+\frac{48c^5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^3}-\frac{c^5}{2f\,a^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}-\frac{17c^5}{2f\,a^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 8/5/f*c^5/a^3*tan(1/2*e+1/2*f*x)^5+8/f*c^5/a^3*tan(1/2*e+1/2*f*x)^3+48/f*c^
5/a^3*tan(1/2*e+1/2*f*x)-1/2/f*c^5/a^3/(tan(1/2*e+1/2*f*x)-1)^2-17/2/f*c^5/
a^3/(tan(1/2*e+1/2*f*x)-1)+63/2/f*c^5/a^3*ln(tan(1/2*e+1/2*f*x)-1)+1/2/f*c^
5/a^3/(tan(1/2*e+1/2*f*x)+1)^2-17/2/f*c^5/a^3/(tan(1/2*e+1/2*f*x)+1)-63/2/f
*c^5/a^3*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.37, size = 680, normalized size = 3.52

$$c^5\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^3-\frac{2a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^5*(40*sin(f*x + e)/((a^3 - a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^
3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*((105*sin(f*x
 + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*(15*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^5*(15*sin(f*x + e)/(cos(f*x
+ e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5)/a^3 - 15*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.64, size = 159, normalized size = 0.82

$$\frac{48\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{17\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^3\right)}+\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{a^3\,f}+\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (48*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (17*c^5*tan(e/2 + (f*x)/2)^3 - 15*c^5
*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^4 - 2*a^3*tan(e/2 + (f*x)/2
)^2 + a^3)) + (8*c^5*tan(e/2 + (f*x)/2)^3)/(a^3*f) + (8*c^5*tan(e/2 + (f*x)
/2)^5)/(5*a^3*f) - (63*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^5\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{5\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}dx+\int\left(-\frac{10\,\mathrm{se}}{\sec^3\left(e+fx\right)+3\sec^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(5*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e +
 f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + I
ntegral(-5*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.54 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{7c^4\tan(e+fx)}{a^3f}-\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{14\tan(e+fx)\left(c^4-c^4\sec(e+fx)\right)}{3f\left(a^3\sec(e+fx)+a^3\right)}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+f\right.}{15af(a\sec(e+fx)+a)^2}$$

```
[Out] -7*c^4*arctanh(sin(f*x+e))/a^3/f+7*c^4*tan(f*x+e)/a^3/f+2/5*c*(c-c*sec(f*x+
e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-14/15*(c^2-c^2*sec(f*x+e))^2*tan(f*x+
e)/a/f/(a+a*sec(f*x+e))^2+14/3*(c^4-c^4*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*s
ec(f*x+e))
```

Rubi [A] time = 0.28, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$\frac{7c^4\tan(e+fx)}{a^3f}-\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{14\tan(e+fx)\left(c^4-c^4\sec(e+fx)\right)}{3f\left(a^3\sec(e+fx)+a^3\right)}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+f\right.}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-7*c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) + (7*c^4*Tan[e + f*x])/(a^3*f) + (2*
c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (14*(
c^2 - c^2*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (
14*(c^4 - c^4*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(7c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+f}{15af(a+a\sec(e+fx))^2}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+f}{15af(a+a\sec(e+fx))^2}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+f}{15af(a+a\sec(e+fx))^2}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2\right.}{}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{7c^4\tan(e+fx)}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\,\mathrm{ta}}{5f(a+a\sec(e+f}$$

Mathematica [B] time = 6.32, size = 826, normalized size = 5.04

$$\frac{2\cos(e+fx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)\sec\left(\frac{e}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)\csc^7\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f(\sec(e+fx)a+a)^3} + \frac{2\cos(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c}{5f(\sec(e-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (7*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*Log[Cos[e/2 + (f*
x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x
])^3) - (7*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*Log[Cos[e
/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec
[e + f*x])^3) + (76*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^5*Csc[e/2 + (f*x)/2]^3*
Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(15*f*(a + a*Sec[e + f*x])^3)
 + (8*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^3*Csc[e/2 + (f*x)/2]^5*Sec[e/2]*(c -
c*Sec[e + f*x])^4*Sin[(f*x)/2])/(15*f*(a + a*Sec[e + f*x])^3) + (2*Cos[e +
f*x]*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]^7*Sec[e/2]*(c - c*Sec[e + f*x])^
4*Sin[(f*x)/2])/(5*f*(a + a*Sec[e + f*x])^3) + (Cos[e + f*x]*Cot[e/2 + (f*x
)/2]^6*Csc[e/2 + (f*x)/2]^2*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(2*f*(a +
a*Sec[e + f*x])^3*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*
x)/2])) + (Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*(c - c*Se
c[e + f*x])^4*Sin[(f*x)/2])/(2*f*(a + a*Sec[e + f*x])^3*(Cos[e/2] + Sin[e/2
])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])) + (8*Cos[e + f*x]*Cot[e/2 + (
f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4*Tan[e/2])/(15*f*(a +
a*Sec[e + f*x])^3) + (2*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2
]^6*(c - c*Sec[e + f*x])^4*Tan[e/2])/(5*f*(a + a*Sec[e + f*x])^3)
```

fricas [A] time = 0.48, size = 231, normalized size = 1.41

$$-\frac{105\left(c^4\cos\left(fx+e\right)^4+3\,c^4\cos\left(fx+e\right)^3+3\,c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c\right.}{30\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(105*(c^4*cos(f*x + e)^4 + 3*c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^
2 + c^4*cos(f*x + e))*log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 3*c
^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*log(-sin(f*x +
 e) + 1) - 2*(167*c^4*cos(f*x + e)^3 + 381*c^4*cos(f*x + e)^2 + 277*c^4*cos
(f*x + e) + 15*c^4)*sin(f*x + e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x +
 e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((2/5*tan((f*x+exp(1))/2)^5*c^4*a^12+4/3*tan((f*x+exp(1))/2)^3*
c^4*a^12+6*tan((f*x+exp(1))/2)*c^4*a^12)/a^15-tan((f*x+exp(1))/2)*c^4/a^3/(
tan((f*x+exp(1))/2)^2-1)+7*c^4*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)-1))-7*c^4
*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.68, size = 160, normalized size = 0.98

$$\frac{4c^4\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5f\,a^3}+\frac{8c^4\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^3}+\frac{12c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^3}-\frac{c^4}{f\,a^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}+\frac{7c^4\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 4/5/f*c^4/a^3*tan(1/2*e+1/2*f*x)^5+8/3/f*c^4/a^3*tan(1/2*e+1/2*f*x)^3+12/f*
c^4/a^3*tan(1/2*e+1/2*f*x)-1/f*c^4/a^3/(tan(1/2*e+1/2*f*x)-1)+7/f*c^4/a^3*l
n(tan(1/2*e+1/2*f*x)-1)-1/f*c^4/a^3/(tan(1/2*e+1/2*f*x)+1)-7/f*c^4/a^3*ln(t
an(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.36, size = 470, normalized size = 2.87

$$3\,c^4\left(\frac{40\,\sin\left(fx+e\right)}{\left(a^3-\frac{a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)\left(\cos\left(fx+e\right)+1\right)}+\frac{\frac{85\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*c^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) + 4*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) + 6*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) +
```

```
10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 12*c^4*(5*s
in(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/
f
```

mupad [B] time = 1.63, size = 126, normalized size = 0.77

$$\frac{12\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}+\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}+\frac{4\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{14\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}-\frac{2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (12*c^4*tan(e/2 + (f*x)/2))/(a^3*f) + (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^3*f
) + (4*c^4*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) - (14*c^4*atanh(tan(e/2 + (f*x)/
2)))/(a^3*f) - (2*c^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^2 - a^
3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)\cdots}\cdots\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-4*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x
)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

3.55 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{2c^3\tan(e+fx)}{f\left(a^3\sec(e+fx)+a^3\right)}-\frac{2\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec}{5f(a\sec(e+fx)+}$$

```
[Out] -c^3*arctanh(sin(f*x+e))/a^3/f+2*c^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+2/5*
c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-2/3*(c^3-c^3*sec(f*x+e
))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.21, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{2c^3\tan(e+fx)}{f\left(a^3\sec(e+fx)+a^3\right)}-\frac{2\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec}{5f(a\sec(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*ArcTanh[Sin[e + f*x]])/(a^3*f)) + (2*c^3*Tan[e + f*x])/(f*(a^3 + a^3
*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec
[e + f*x])^3) - (2*(c^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*a*f*(a + a*Sec
[e + f*x])^2)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}dx}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2\left(c^3-c^3\sec(e+fx)\right)\tan(e+fx)}{3af(a+a\sec(e+fx))^2} +$$

$$= \frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2\left(c^3-c^3\right.}{3af(}$$

$$= -\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx)}{5f(a+a\sec(}$$

Mathematica [A] time = 0.12, size = 139, normalized size = 1.06

$$-\frac{c^3\left(-\frac{26\tan\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)}{5f} + \frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(\right.\right.}{}\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]]/f - (26*Tan[(e + f*x)/2])/(15*f) + (2*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/(15*f) - (2*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])/(5*f
)))/a^3)
```

fricas [A] time = 0.48, size = 192, normalized size = 1.47

$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+3\,c^3\cos\left(fx+e\right)^2+3\,c^3\cos\left(fx+e\right)+c^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+\right.}{30\left(a^3f\cos\left(fx+e\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*log(sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2
 + 3*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 4*(13*c^3*cos(f*x + e
```

```
)^2 + 24*c^3*cos(f*x + e) + 23*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3
*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(-c^3*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)-1))+c^3*1/2/a^3*ln(ab
s(tan((f*x+exp(1))/2)+1))+(-1/5*tan((f*x+exp(1))/2)^5*c^3*a^12-1/3*tan((f*x
+exp(1))/2)^3*c^3*a^12-tan((f*x+exp(1))/2)*c^3*a^12)/a^15)
```

maple [A] time = 0.83, size = 111, normalized size = 0.85

$$\frac{2c^3\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5f\,a^3}+\frac{2c^3\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^3}+\frac{2c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^3}+\frac{c^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,a^3}-\frac{c^3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 2/5/f*c^3/a^3*tan(1/2*e+1/2*f*x)^5+2/3/f*c^3/a^3*tan(1/2*e+1/2*f*x)^3+2/f*c
^3/a^3*tan(1/2*e+1/2*f*x)+1/f*c^3/a^3*ln(tan(1/2*e+1/2*f*x)-1)-1/f*c^3/a^3*
ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.35, size = 304, normalized size = 2.32

$$\frac{c^3\left(\frac{\frac{105\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{20\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}{a^3}-\frac{60\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^3}\right)+\frac{3c^3\left(\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\mathrm{s}}{(\cos}\right.}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x
```

```
 + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
- 1)/a^3) + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^3*(15*
sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^3*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.65, size = 61, normalized size = 0.47

$$\frac{2\,c^3\,\left(15\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-15\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right)}{15\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (2*c^3*(15*tan(e/2 + (f*x)/2) - 15*atanh(tan(e/2 + (f*x)/2)) + 5*tan(e/2 +
(f*x)/2)^3 + 3*tan(e/2 + (f*x)/2)^5))/(15*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^3}{\sec^3\left(e+fx\right)+3\sec^2\left(\right.}\right.\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f
*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.56 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=38

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] 1/5*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

Mathematica [A] time = 0.13, size = 25, normalized size = 0.66

$$\frac{c^2\tan^5\left(\frac{1}{2}(e+fx)\right)}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*Tan[(e + f*x)/2]^5)/(5*a^3*f)
```

fricas [B] time = 0.42, size = 82, normalized size = 2.16

$$\frac{\left(c^2\cos\left(fx+e\right)^2 - 2\,c^2\cos\left(fx+e\right) + c^2\right)\sin\left(fx+e\right)}{5\left(a^3f\cos\left(fx+e\right)^3 + 3\,a^3f\cos\left(fx+e\right)^2 + 3\,a^3f\cos\left(fx+e\right) + a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) + c^2)*sin(f*x + e)/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

giac [A] time = 1.80, size = 23, normalized size = 0.61

$$\frac{c^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^5}{5\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*c^2*tan(1/2*f*x + 1/2*e)^5/(a^3*f)
```

maple [A] time = 0.84, size = 23, normalized size = 0.61

$$\frac{c^2\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/5/f*c^2/a^3*tan(1/2*e+1/2*f*x)^5
```

maxima [B] time = 0.34, size = 185, normalized size = 4.87

$$\frac{\frac{c^2\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3} + \frac{c^2\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3} - \frac{6\,c^2\left(\frac{5\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.59, size = 22, normalized size = 0.58

$$\frac{c^2 \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^2*tan(e/2 + (f*x)/2)^5)/(5*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int \left(-\frac{2\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx + \int \frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+f\right.}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-2*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
*3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.57 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] 1/5*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(c-c*sec(f*x+e))*
tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - c*
Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a}$$
$$= \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}$$

Mathematica [A] time = 0.35, size = 87, normalized size = 1.14

$$\frac{c\sec\left(\frac{e}{2}\right)\left(-15\sin\left(e+\frac{fx}{2}\right)+5\sin\left(e+\frac{3fx}{2}\right)-15\sin\left(2e+\frac{3fx}{2}\right)+4\sin\left(2e+\frac{5fx}{2}\right)+25\sin\left(\frac{fx}{2}\right)\right)\sec^5\left(\frac{1}{2}(e+fx\right.}{240a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^5*(25*Sin[(f*x)/2] - 15*Sin[e + (f*x)/2] + 5*S
in[e + (3*f*x)/2] - 15*Sin[2*e + (3*f*x)/2] + 4*Sin[2*e + (5*f*x)/2]))/(240
*a^3*f)
```

fricas [A] time = 0.43, size = 79, normalized size = 1.04

$$\frac{\left(4\,c\cos\left(fx+e\right)^2-3\,c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*c*cos(f*x + e)^2 - 3*c*cos(f*x + e) - c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

giac [A] time = 0.40, size = 39, normalized size = 0.51

$$\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{30\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 - 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

maple [A] time = 0.80, size = 37, normalized size = 0.49

$$\frac{c\left(\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}\right)}{2f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f*c/a^3*(1/5*tan(1/2*e+1/2*f*x)^5-1/3*tan(1/2*e+1/2*f*x)^3)
```

maxima [A] time = 0.34, size = 115, normalized size = 1.51

$$\frac{\frac{c\left(\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}-\frac{3c\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.58, size = 35, normalized size = 0.46

$$\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-5\right)}{30\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 - 5))/(30*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.58 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=78

$$-\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{\csc(e+fx)}{a^3cf}$$

```
[Out] -2/5*cot(f*x+e)^5/a^3/c/f+csc(f*x+e)/a^3/c/f-csc(f*x+e)^3/a^3/c/f+2/5*csc(f
*x+e)^5/a^3/c/f
```

Rubi [A] time = 0.18, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 14}

$$-\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{\csc(e+fx)}{a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] (-2*Cot[e + f*x]^5)/(5*a^3*c*f) + Csc[e + f*x]/(a^3*c*f) - Csc[e + f*x]^3/(
a^3*c*f) + (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \left(c^2\cot^5(e+fx)\csc(e+fx)-2c^2\cot^4(e+fx)\csc^2(e+fx)\right.}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c} - \frac{\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a^3c}$$

$$= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \csc(e+fx)\right)}{a^3cf} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)\right.}{a^3}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \csc(e+fx)\right)}{a^3cf} +$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\csc(e+fx)}{a^3cf} - \frac{\csc^3(e+fx)}{a^3cf} + \frac{2\csc^5(e+fx)}{5a^3cf}$$

Mathematica [A] time = 0.82, size = 109, normalized size = 1.40

$$-\frac{\csc(e)\sin^4\left(\frac{1}{2}(e+fx)\right)(65\sin(e+fx)+52\sin(2(e+fx))+13\sin(3(e+fx))-40\sin(2e+fx)-12\sin(e+2}{20a^3cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/20*(Csc[e]*Csc[e + f*x]^5*Sin[(e + f*x)/2]^4*(-40*Sin[e] + 65*Sin[e + f*
x] + 52*Sin[2*(e + f*x)] + 13*Sin[3*(e + f*x)] - 40*Sin[2*e + f*x] - 12*Sin
[e + 2*f*x] - 20*Sin[3*e + 2*f*x] - 8*Sin[2*e + 3*f*x]))/(a^3*c*f)
```

fricas [A] time = 0.47, size = 76, normalized size = 0.97

$$-\frac{2 \cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-4 \cos\left(fx+e\right)-2}{5 \left(a^{3} c f \cos\left(fx+e\right)^{2}+2 a^{3} c f \cos\left(fx+e\right)+a^{3} c f\right) \sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/5*(2*cos(f*x + e)^3 - cos(f*x + e)^2 - 4*cos(f*x + e) - 2)/((a^3*c*f*cos
(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

giac [A] time = 0.37, size = 91, normalized size = 1.17

$$\frac{\frac{5}{a^{3} c \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\frac{a^{12} c^{4} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{5}-5 a^{12} c^{4} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3}+15 a^{12} c^{4} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}{a^{15} c^{5}}}{40 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/40*(5/(a^3*c*tan(1/2*f*x + 1/2*e)) + (a^12*c^4*tan(1/2*f*x + 1/2*e)^5 - 5
*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e))/(a^15*
c^5))/f
```

maple [A] time = 0.70, size = 61, normalized size = 0.78

$$\frac{\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)+3 \tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{1}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{8 f a^{3} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/8/f/a^3/c*(1/5*tan(1/2*e+1/2*f*x)^5-tan(1/2*e+1/2*f*x)^3+3*tan(1/2*e+1/2*
f*x)+1/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.33, size = 95, normalized size = 1.22

$$\frac{\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}+\frac{5(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{40f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/40*((15*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) + 5*(cos(f*x + e) + 1
)/(a^3*c*sin(f*x + e)))/f
```

mupad [B] time = 1.62, size = 74, normalized size = 0.95

$$-\frac{16\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6-28\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4+8\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}{40\,a^3\,c\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

```
[Out] -(8*cos(e/2 + (f*x)/2)^2 - 28*cos(e/2 + (f*x)/2)^4 + 16*cos(e/2 + (f*x)/2)^
6 - 1)/(40*a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec(e+fx)}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x
) - 1), x)/(a**3*c)
```

3.59 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}$$

```
[Out] -1/5*cot(f*x+e)^5/a^3/c^2/f+csc(f*x+e)/a^3/c^2/f-2/3*csc(f*x+e)^3/a^3/c^2/f
+1/5*csc(f*x+e)^5/a^3/c^2/f
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2606, 194, 2607, 30}

$$-\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -Cot[e + f*x]^5/(5*a^3*c^2*f) + Csc[e + f*x]/(a^3*c^2*f) - (2*Csc[e + f*x]^
3)/(3*a^3*c^2*f) + Csc[e + f*x]^5/(5*a^3*c^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\int \left(c\cot^5(e+fx)\csc(e+fx)-c\cot^4(e+fx)\csc^2(e+fx)\right)\,dx}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^2} + \frac{\int \cot^4(e+fx)\csc^2(e+fx)\,d}{a^3c^2}$$

$$= \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(e+fx)\right)}{a^3c^2 f} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x,\right.}{a^3c^2 f}$$

$$= -\frac{\cot^5(e+fx)}{5a^3c^2 f} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{a^3c^2 f}$$

$$= -\frac{\cot^5(e+fx)}{5a^3c^2 f} + \frac{\csc(e+fx)}{a^3c^2 f} - \frac{2\csc^3(e+fx)}{3a^3c^2 f} + \frac{\csc^5(e+fx)}{5a^3c^2 f}$$

Mathematica [A] time = 0.82, size = 147, normalized size = 1.84

$$\frac{\csc(e)\sin^2\left(\frac{1}{2}(e+fx)\right)(-534\sin(e+fx)-178\sin(2(e+fx))+178\sin(3(e+fx))+89\sin(4(e+fx))+40\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^5*Sin[(e + f*x)/2]^2*(200*Sin[e] + 104*Sin[f*x] - 534*
Sin[e + f*x] - 178*Sin[2*(e + f*x)] + 178*Sin[3*(e + f*x)] + 89*Sin[4*(e +
f*x)] + 40*Sin[2*e + f*x] + 168*Sin[e + 2*f*x] - 120*Sin[3*e + 2*f*x] + 72*
Sin[2*e + 3*f*x] - 120*Sin[4*e + 3*f*x] - 24*Sin[3*e + 4*f*x]))/(480*a^3*c^
2*f)
```

fricas [A] time = 0.44, size = 109, normalized size = 1.36

$$-\frac{3\cos\left(fx+e\right)^{4}-12\cos\left(fx+e\right)^{3}-12\cos\left(fx+e\right)^{2}+8\cos\left(fx+e\right)+8}{15\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}+a^{3}c^{2}f\cos\left(fx+e\right)^{2}-a^{3}c^{2}f\cos\left(fx+e\right)-a^{3}c^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 + 8*cos(f*x
 + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*
f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

giac [A] time = 0.39, size = 108, normalized size = 1.35

$$\frac{\frac{5\left(12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)}{a^{3}c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}}+\frac{3a^{12}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-20a^{12}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+90a^{12}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{10}}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/240*(5*(12*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2*tan(1/2*f*x + 1/2*e)^3) +
 (3*a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 20*a^12*c^8*tan(1/2*f*x + 1/2*e)^3 +
90*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

maple [A] time = 0.87, size = 76, normalized size = 0.95

$$\frac{\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{16f\,a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/16/f/a^3/c^2*(1/5*tan(1/2*e+1/2*f*x)^5-4/3*tan(1/2*e+1/2*f*x)^3+6*tan(1/2
*e+1/2*f*x)-1/3/tan(1/2*e+1/2*f*x)^3+4/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.34, size = 120, normalized size = 1.50

$$\frac{\frac{\frac{90\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{20\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}{a^{3}c^{2}}+\frac{5\left(\frac{12\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-1\right)\left(\cos\left(fx+e\right)+1\right)^{3}}{a^{3}c^{2}\sin\left(fx+e\right)^{3}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*((90*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) + 5*(12*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e
)^3))/f
```

mupad [B] time = 1.71, size = 111, normalized size = 1.39

$$\frac{48\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8-192\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6+168\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4-32\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3}{240\,a^3\,c^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (168*cos(e/2 + (f*x)/2)^4 - 32*cos(e/2 + (f*x)/2)^2 - 192*cos(e/2 + (f*x)/2
)^6 + 48*cos(e/2 + (f*x)/2)^8 + 3)/(240*a^3*c^2*f*(cos(e/2 + (f*x)/2)^5*sin
(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)^7*sin(e/2 + (f*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)+\sec^4\left(e+fx\right)-2\sec^3\left(e+fx\right)-2\sec^2\left(e+fx\right)+\sec\left(e+fx\right)+1}\,dx}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**
3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

3.60 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\csc^5(e+fx)}{5a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc(e+fx)}{a^3c^3f}$$

```
[Out] csc(f*x+e)/a^3/c^3/f-2/3*csc(f*x+e)^3/a^3/c^3/f+1/5*csc(f*x+e)^5/a^3/c^3/f
```

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2606, 194}

$$\frac{\csc^5(e+fx)}{5a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Csc[e + f*x]/(a^3*c^3*f) - (2*Csc[e + f*x]^3)/(3*a^3*c^3*f) + Csc[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \csc(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, \csc(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\csc(e+fx)}{a^3c^3f} - \frac{2\csc^3(e+fx)}{3a^3c^3f} + \frac{\csc^5(e+fx)}{5a^3c^3f}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.85

$$-\frac{-\frac{\csc^5(e+fx)}{5f}+\frac{2\csc^3(e+fx)}{3f}-\frac{\csc(e+fx)}{f}}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -((-(Csc[e + f*x]/f) + (2*Csc[e + f*x]^3)/(3*f) - Csc[e + f*x]^5/(5*f))/(a^
3*c^3))
```

fricas [A] time = 0.43, size = 76, normalized size = 1.29

$$\frac{15\cos\left(fx+e\right)^4-20\cos\left(fx+e\right)^2+8}{15\left(a^3c^3f\cos\left(fx+e\right)^4-2a^3c^3f\cos\left(fx+e\right)^2+a^3c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)/((a^3*c^3*f*cos(f*x + e)^4
 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.70, size = 44, normalized size = 0.75

$$\frac{15\sin\left(fx+e\right)^4-10\sin\left(fx+e\right)^2+3}{15a^3c^3f\sin\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)^{3}\left(c-c\sec\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

```
[Out] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

maxima [A] time = 0.38, size = 41, normalized size = 0.69

$$\frac{15\,\sin\left(fx+e\right)^{4}-10\,\sin\left(fx+e\right)^{2}+3}{15\,a^{3}c^{3}f\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

mupad [B] time = 1.63, size = 38, normalized size = 0.64

$$\frac{\sin\left(e+f\,x\right)^{4}-\frac{2\sin\left(e+f\,x\right)^{2}}{3}+\frac{1}{5}}{a^{3}\,c^{3}\,f\sin\left(e+f\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (sin(e + f*x)^4 - (2*sin(e + f*x)^2)/3 + 1/5)/(a^3*c^3*f*sin(e + f*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{6}\left(e+fx\right)-3\sec^{4}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)-1}\,dx}{a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x
)**2 - 1), x)/(a**3*c**3)
```

3.61 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=99

$$-\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}$$

```
[Out] -1/7*cot(f*x+e)^7/a^3/c^4/f+csc(f*x+e)/a^3/c^4/f-csc(f*x+e)^3/a^3/c^4/f+3/5
*csc(f*x+e)^5/a^3/c^4/f-1/7*csc(f*x+e)^7/a^3/c^4/f
```

Rubi [A] time = 0.15, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2606, 194, 2607, 30}

$$-\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -Cot[e + f*x]^7/(7*a^3*c^4*f) + Csc[e + f*x]/(a^3*c^4*f) - Csc[e + f*x]^3/(
a^3*c^4*f) + (3*Csc[e + f*x]^5)/(5*a^3*c^4*f) - Csc[e + f*x]^7/(7*a^3*c^4*f
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(a\cot^7(e+fx)\csc(e+fx)+a\cot^6(e+fx)\csc^2(e+fx)\right)dx}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^3c^4} + \frac{\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^3c^4}$$

$$= \frac{\text{Subst}\left(\int x^6\,dx, x, -\cot(e+fx)\right)}{a^3c^4f} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, c\right.}{a^3c^4f}$$

$$= -\frac{\cot^7(e+fx)}{7a^3c^4f} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx, x, \csc(e+\right.}{a^3c^4f}$$

$$= -\frac{\cot^7(e+fx)}{7a^3c^4f} + \frac{\csc(e+fx)}{a^3c^4f} - \frac{\csc^3(e+fx)}{a^3c^4f} + \frac{3\csc^5(e+fx)}{5a^3c^4f} -$$

Mathematica [B] time = 1.41, size = 211, normalized size = 2.13

$$\frac{\csc(e)(-7620\sin(e+fx)+1905\sin(2(e+fx))+3810\sin(3(e+fx))-1524\sin(4(e+fx))-762\sin(5(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]^5*(2912*Sin[e] + 416*Sin[f*x] - 762
0*Sin[e + f*x] + 1905*Sin[2*(e + f*x)] + 3810*Sin[3*(e + f*x)] - 1524*Sin[4
*(e + f*x)] - 762*Sin[5*(e + f*x)] + 381*Sin[6*(e + f*x)] - 2016*Sin[2*e +
f*x] + 2080*Sin[e + 2*f*x] - 1680*Sin[3*e + 2*f*x] + 240*Sin[2*e + 3*f*x] +
 560*Sin[4*e + 3*f*x] - 880*Sin[3*e + 4*f*x] + 560*Sin[5*e + 4*f*x] + 400*S
```

```
in[4*e + 5*f*x] - 560*Sin[6*e + 5*f*x] + 80*Sin[5*e + 6*f*x]))/(35840*a^3*c
^4*f)
```

fricas [A] time = 0.42, size = 163, normalized size = 1.65

$$\frac{5\cos\left(fx+e\right)^{6}+30\cos\left(fx+e\right)^{5}-30\cos\left(fx+e\right)^{4}-40\cos\left(fx+e\right)^{3}+40\cos\left(fx+e\right)^{2}+16\cos}{35\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}-a^{3}c^{4}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{4}f\cos\left(fx+e\right)^{3}+2\,a^{3}c^{4}f\cos\left(fx+e\right)^{2}+a^{3}c^{4}f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(5*cos(f*x + e)^6 + 30*cos(f*x + e)^5 - 30*cos(f*x + e)^4 - 40*cos(f*x
 + e)^3 + 40*cos(f*x + e)^2 + 16*cos(f*x + e) - 16)/((a^3*c^4*f*cos(f*x + e
)^5 - a^3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*c
os(f*x + e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

giac [A] time = 1.01, size = 135, normalized size = 1.36

$$\frac{\frac{700\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-175\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+42\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-5}{a^{3}c^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}}+\frac{7\left(a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-10\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+75\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}c^{20}}}{2240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/2240*((700*tan(1/2*f*x + 1/2*e)^6 - 175*tan(1/2*f*x + 1/2*e)^4 + 42*tan(1
/2*f*x + 1/2*e)^2 - 5)/(a^3*c^4*tan(1/2*f*x + 1/2*e)^7) + 7*(a^12*c^16*tan(
1/2*f*x + 1/2*e)^5 - 10*a^12*c^16*tan(1/2*f*x + 1/2*e)^3 + 75*a^12*c^16*tan
(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

maple [A] time = 0.93, size = 102, normalized size = 1.03

$$\frac{\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-2\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}-\frac{5}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{6}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}+\frac{20}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{64f\,a^{3}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/64/f/a^3/c^4*(1/5*tan(1/2*e+1/2*f*x)^5-2*tan(1/2*e+1/2*f*x)^3+15*tan(1/2*
e+1/2*f*x)-1/7/tan(1/2*e+1/2*f*x)^7-5/tan(1/2*e+1/2*f*x)^3+6/5/tan(1/2*e+1/
2*f*x)^5+20/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.35, size = 159, normalized size = 1.61

$$\frac{\frac{7\left(\frac{75\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}+\frac{\left(\frac{42\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{175\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{700\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{a^3c^4\sin(fx+e)^7}}{2240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/2240*(7*(75*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) + (42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 700
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin
(f*x + e)^7))/f
```

mupad [B] time = 2.54, size = 129, normalized size = 1.30

$$\frac{\left(2\sin\left(\frac{e}{4}+\frac{f\,x}{4}\right)^2-1\right)\left(\frac{235\sin\left(e+f\,x\right)^2}{16}-\frac{45\sin\left(2\,e+2\,f\,x\right)^2}{8}+\frac{19\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{2}+\frac{5\sin\left(3\,e+3\,f\,x\right)^2}{16}-\frac{5\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2}{4}+\frac{15\sin\left(\frac{5\,e}{2}+5\right.}{4}\right.}{2240\,a^3\,c^4\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

```
[Out] ((2*sin(e/4 + (f*x)/4)^2 - 1)*((19*sin(e/2 + (f*x)/2)^2)/2 - (45*sin(2*e +
2*f*x)^2)/8 + (5*sin(3*e + 3*f*x)^2)/16 - (5*sin((3*e)/2 + (3*f*x)/2)^2)/4
+ (15*sin((5*e)/2 + (5*f*x)/2)^2)/4 + (235*sin(e + f*x)^2)/16 - 5))/(2240*a
^3*c^4*f*sin(e/2 + (f*x)/2)^7*(sin(e/2 + (f*x)/2)^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(e+fx\right)}{\sec^7\left(e+fx\right)-\sec^6\left(e+fx\right)-3\sec^5\left(e+fx\right)+3\sec^4\left(e+fx\right)+3\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**
5 + 3*sec(e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x
) + 1), x)/(a**3*c**4)
```

3.62 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=120

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}$$

```
[Out] 2/9*cot(f*x+e)^9/a^3/c^5/f+csc(f*x+e)/a^3/c^5/f-5/3*csc(f*x+e)^3/a^3/c^5/f+
9/5*csc(f*x+e)^5/a^3/c^5/f-csc(f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/
f
```

Rubi [A] time = 0.20, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 270}

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (2*Cot[e + f*x]^9)/(9*a^3*c^5*f) + Csc[e + f*x]/(a^3*c^5*f) - (5*Csc[e + f*
x]^3)/(3*a^3*c^5*f) + (9*Csc[e + f*x]^5)/(5*a^3*c^5*f) - Csc[e + f*x]^7/(a^
3*c^5*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx &= -\frac{\int \left(a^2\cot^9(e+fx)\csc(e+fx)+2a^2\cot^8(e+fx)\csc^2(e+fx)\right.}{a^5c^5}\\ &= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^3c^5}-\frac{\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^3c^5}\\ &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(e+fx)\right)}{a^3c^5f}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)\right.}{a^3}\\ &= \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(\right.}{a^3c^5f}\\ &= \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}\end{aligned}$$

Mathematica [B] time = 1.61, size = 257, normalized size = 2.14

$$-\frac{\csc(e)(76455\sin(e+fx)-33980\sin(2(e+fx))-32281\sin(3(e+fx))+27184\sin(4(e+fx))+1699\sin(5(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] -1/184320*(Csc[e]*Sec[e + f*x]^7*(-33024*Sin[e] + 6144*Sin[f*x] + 76455*Sin
[e + f*x] - 33980*Sin[2*(e + f*x)] - 32281*Sin[3*(e + f*x)] + 27184*Sin[4*(
e + f*x)] + 1699*Sin[5*(e + f*x)] - 6796*Sin[6*(e + f*x)] + 1699*Sin[7*(e +
 f*x)] + 22656*Sin[2*e + f*x] - 17216*Sin[e + 2*f*x] + 4416*Sin[3*e + 2*f*x
] + 3200*Sin[2*e + 3*f*x] - 15360*Sin[4*e + 3*f*x] + 12160*Sin[3*e + 4*f*x]
 - 1920*Sin[5*e + 4*f*x] - 5120*Sin[4*e + 5*f*x] + 5760*Sin[6*e + 5*f*x] +
320*Sin[5*e + 6*f*x] - 2880*Sin[7*e + 6*f*x] + 640*Sin[6*e + 7*f*x])*Tan[e
+ f*x])/(a^3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

fricas [A] time = 0.45, size = 190, normalized size = 1.58

$$\frac{10\cos\left(fx+e\right)^{7}+25\cos\left(fx+e\right)^{6}-60\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{4}+80\cos\left(fx+e\right)^{3}-24}{45\left(a^{3}c^{5}f\cos\left(fx+e\right)^{6}-2a^{3}c^{5}f\cos\left(fx+e\right)^{5}-a^{3}c^{5}f\cos\left(fx+e\right)^{4}+4a^{3}c^{5}f\cos\left(fx+e\right)^{3}-a^{3}c^{5}f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/45*(10*cos(f*x + e)^7 + 25*cos(f*x + e)^6 - 60*cos(f*x + e)^5 - 10*cos(f*
x + e)^4 + 80*cos(f*x + e)^3 - 24*cos(f*x + e)^2 - 32*cos(f*x + e) + 16)/((
a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^3*c^5*f*cos(f*x +
 e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e)^2 - 2*a^3*c^5*f
*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

giac [A] time = 0.55, size = 150, normalized size = 1.25

$$\frac{\frac{1575\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-525\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-45\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+5}{a^{3}c^{5}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}}+\frac{3\left(3a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-35a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}\right.}{a^{15}c^{25}}}{5760f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/5760*((1575*tan(1/2*f*x + 1/2*e)^8 - 525*tan(1/2*f*x + 1/2*e)^6 + 189*tan
(1/2*f*x + 1/2*e)^4 - 45*tan(1/2*f*x + 1/2*e)^2 + 5)/(a^3*c^5*tan(1/2*f*x +
 1/2*e)^9) + 3*(3*a^12*c^20*tan(1/2*f*x + 1/2*e)^5 - 35*a^12*c^20*tan(1/2*f
*x + 1/2*e)^3 + 315*a^12*c^20*tan(1/2*f*x + 1/2*e))/(a^15*c^25))/f
```

maple [A] time = 0.97, size = 115, normalized size = 0.96

$$\frac{\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-\frac{7\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\frac{1}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}-\frac{35}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}+\frac{21}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}+\frac{35}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}}{128f\,a^{3}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/128/f/a^3/c^5*(1/5*tan(1/2*e+1/2*f*x)^5-7/3*tan(1/2*e+1/2*f*x)^3+21*tan(1
/2*e+1/2*f*x)-1/tan(1/2*e+1/2*f*x)^7+1/9/tan(1/2*e+1/2*f*x)^9-35/3/tan(1/2*
e+1/2*f*x)^3+21/5/tan(1/2*e+1/2*f*x)^5+35/tan(1/2*e+1/2*f*x))
```

maxima [A] time = 0.34, size = 181, normalized size = 1.51

$$\frac{\frac{3\left(\frac{315\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{35\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}c^{5}}-\frac{\left(\frac{45\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{189\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{525\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{1575\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{9}}{a^{3}c^{5}\sin\left(fx+e\right)^{9}}}{5760\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/5760*(3*(315*sin(f*x + e)/(cos(f*x + e) + 1) - 35*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - (45*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
525*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1575*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

mupad [B] time = 2.88, size = 109, normalized size = 0.91

$$\frac{\frac{145\cos\left(3e+3fx\right)}{32}-\frac{169\cos\left(2e+2fx\right)}{32}-\frac{129\cos\left(e+fx\right)}{32}+\frac{55\cos\left(4e+4fx\right)}{16}-\frac{85\cos\left(5e+5fx\right)}{32}+\frac{25\cos\left(6e+6fx\right)}{32}+\frac{5\cos\left(7e+7fx\right)}{32}+}{5760\,a^{3}\,c^{5}\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

```
[Out] ((145*cos(3*e + 3*f*x))/32 - (169*cos(2*e + 2*f*x))/32 - (129*cos(e + f*x))
/32 + (55*cos(4*e + 4*f*x))/16 - (85*cos(5*e + 5*f*x))/32 + (25*cos(6*e + 6
```

```
*f*x))/32 + (5*cos(7*e + 7*f*x))/32 + 129/16)/(5760*a^3*c^5*f*cos(e/2 + (f*
x)/2)^5*sin(e/2 + (f*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec(e+fx)}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x
)**6 + 6*sec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e
+ f*x) - 1), x)/(a**3*c**5)
```

3.63 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=162

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{8\csc^3(e+f}{3a^3c^6f}$$

```
[Out] -1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*x+e)^11/a^3/c^6/f+csc(f*x+e)/a^3/c^6
/f-8/3*csc(f*x+e)^3/a^3/c^6/f+22/5*csc(f*x+e)^5/a^3/c^6/f-4*csc(f*x+e)^7/a^
3/c^6/f+17/9*csc(f*x+e)^9/a^3/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Rubi [A] time = 0.26, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2606, 194, 2607, 30, 270, 14}

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{8\csc^3(e+f}{3a^3c^6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] -Cot[e + f*x]^9/(9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + Csc[e
 + f*x]/(a^3*c^6*f) - (8*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (22*Csc[e + f*x]^5)
/(5*a^3*c^6*f) - (4*Csc[e + f*x]^7)/(a^3*c^6*f) + (17*Csc[e + f*x]^9)/(9*a^
3*c^6*f) - (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \left(a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3\cot^{10}(e+fx)\csc^2(e+fx)\right.}{}$$

$$= \frac{\int \cot^{11}(e+fx)\csc(e+fx)\,dx}{a^3c^6} + \frac{\int \cot^8(e+fx)\csc^4(e+fx)\,dx}{a^3c^6}$$

$$= -\frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^5 dx, x, \csc(e+fx)\right)}{a^3c^6f} + \frac{\mathrm{Subst}\left(\int x^8\left(1+x^2\right.\right.}{a}$$

$$= -\frac{3\cot^{11}(e+fx)}{11a^3c^6f} - \frac{\mathrm{Subst}\left(\int \left(-1+5x^2-10x^4+10x^6-5x^8+\right.\right.}{a^3c^6f}$$

$$= -\frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{4\cot^{11}(e+fx)}{11a^3c^6f} + \frac{\csc(e+fx)}{a^3c^6f} - \frac{8\csc^3(e+fx}{3a^3c^6f}$$

Mathematica [A] time = 2.32, size = 289, normalized size = 1.78

$$\frac{\csc(e)(-3440690\sin(e+fx)+2064414\sin(2(e+fx))+1063486\sin(3(e+fx))-1563950\sin(4(e+fx))+312\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] (Csc[e]*Sec[e + f*x]^8*(1119360*Sin[e] - 260480*Sin[f*x] - 3440690*Sin[e +
f*x] + 2064414*Sin[2*(e + f*x)] + 1063486*Sin[3*(e + f*x)] - 1563950*Sin[4*
(e + f*x)] + 312790*Sin[5*(e + f*x)] + 312790*Sin[6*(e + f*x)] - 187674*Sin
[7*(e + f*x)] + 31279*Sin[8*(e + f*x)] - 1499520*Sin[2*e + f*x] + 1051776*S
in[e + 2*f*x] + 4224*Sin[3*e + 2*f*x] - 85376*Sin[2*e + 3*f*x] + 629376*Sin
[4*e + 3*f*x] - 483200*Sin[3*e + 4*f*x] - 316800*Sin[5*e + 4*f*x] + 392320*
Sin[4*e + 5*f*x] - 232320*Sin[6*e + 5*f*x] - 30080*Sin[5*e + 6*f*x] + 19008
0*Sin[7*e + 6*f*x] - 32640*Sin[6*e + 7*f*x] - 63360*Sin[8*e + 7*f*x] + 1600
0*Sin[7*e + 8*f*x])*Tan[e + f*x])/(8110080*a^3*c^6*f*(-1 + Sec[e + f*x])^6*
(1 + Sec[e + f*x])^3)
```

fricas [A] time = 0.45, size = 217, normalized size = 1.34

$$\frac{125\cos\left(fx+e\right)^{8}+120\cos\left(fx+e\right)^{7}-680\cos\left(fx+e\right)^{6}+400\cos\left(fx+e\right)^{5}+720\cos\left(fx+e\right)^{4}-\cdots}{495\left(a^{3}c^{6}f\cos\left(fx+e\right)^{7}-3a^{3}c^{6}f\cos\left(fx+e\right)^{6}+a^{3}c^{6}f\cos\left(fx+e\right)^{5}+5a^{3}c^{6}f\cos\left(fx+e\right)^{4}-5a^{3}c^{6}f\cos\left(fx\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/495*(125*cos(f*x + e)^8 + 120*cos(f*x + e)^7 - 680*cos(f*x + e)^6 + 400*c
os(f*x + e)^5 + 720*cos(f*x + e)^4 - 832*cos(f*x + e)^3 - 64*cos(f*x + e)^2
 + 384*cos(f*x + e) - 128)/((a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x
 + e)^6 + a^3*c^6*f*cos(f*x + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6
*f*cos(f*x + e)^3 - a^3*c^6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a
^3*c^6*f)*sin(f*x + e))
```

giac [A] time = 0.53, size = 164, normalized size = 1.01

$$\frac{\frac{27720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-11550\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}+5544\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-1980\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+440\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-45}{a^{3}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}}+\frac{33\left(3a^{12}c^{24}\tan\left(\frac{1}{2}fx+\frac{1}{2}\cdots\right.\right.}{\cdots}}{126720f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/126720*((27720*tan(1/2*f*x + 1/2*e)^10 - 11550*tan(1/2*f*x + 1/2*e)^8 + 5
544*tan(1/2*f*x + 1/2*e)^6 - 1980*tan(1/2*f*x + 1/2*e)^4 + 440*tan(1/2*f*x
+ 1/2*e)^2 - 45)/(a^3*c^6*tan(1/2*f*x + 1/2*e)^11) + 33*(3*a^12*c^24*tan(1/
2*f*x + 1/2*e)^5 - 40*a^12*c^24*tan(1/2*f*x + 1/2*e)^3 + 420*a^12*c^24*tan(
1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

maple [A] time = 1.00, size = 128, normalized size = 0.79

$$\frac{\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5} - \frac{8\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3} + 28\tan\left(\frac{e}{2}+\frac{fx}{2}\right) - \frac{1}{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}} - \frac{4}{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}} + \frac{8}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}} - \frac{70}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}} + \frac{56}{5\tan\left(\frac{e}{2}+\right.}}{256 f a^3 c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

```
[Out] 1/256/f/a^3/c^6*(1/5*tan(1/2*e+1/2*f*x)^5-8/3*tan(1/2*e+1/2*f*x)^3+28*tan(1
/2*e+1/2*f*x)-1/11/tan(1/2*e+1/2*f*x)^11-4/tan(1/2*e+1/2*f*x)^7+8/9/tan(1/2
*e+1/2*f*x)^9-70/3/tan(1/2*e+1/2*f*x)^3+56/5/tan(1/2*e+1/2*f*x)^5+56/tan(1/
2*e+1/2*f*x))
```

maxima [A] time = 0.34, size = 200, normalized size = 1.23

$$\frac{\frac{33\left(\frac{420\sin(fx+e)}{\cos(fx+e)+1} - \frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6} + \frac{\left(\frac{440\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1980\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{5544\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{11550\sin(fx+e)^8}{(\cos(fx+e)+1)^8} + \frac{27720\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}} - 45\right)(\cos(}{a^3c^6\sin(fx+e)^{11}}}{126720 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/126720*(33*(420*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) + (440*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1980*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 5544*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 11550*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 27720*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 45)*(cos(f*x
 + e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

mupad [B] time = 3.11, size = 120, normalized size = 0.74

$$-\frac{\frac{605\cos\left(e+f\,x\right)}{8}+\frac{1023\cos\left(2\,e+2\,f\,x\right)}{16}-\frac{349\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{325\cos\left(4\,e+4\,f\,x\right)}{32}+\frac{305\cos\left(5\,e+5\,f\,x\right)}{8}-\frac{215\cos\left(6\,e+6\,f\,x\right)}{16}+\frac{15\cos\left(7\,e+7\,f\,x\right)}{8}}{126720\,a^3\,c^6\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -((605*cos(e + f*x))/8 + (1023*cos(2*e + 2*f*x))/16 - (349*cos(3*e + 3*f*x)
)/8 - (325*cos(4*e + 4*f*x))/32 + (305*cos(5*e + 5*f*x))/8 - (215*cos(6*e +
 6*f*x))/16 + (15*cos(7*e + 7*f*x))/8 + (125*cos(8*e + 8*f*x))/128 - 8745/1
28)/(126720*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(e+fx\right)}{\sec^9\left(e+fx\right)-3\sec^8\left(e+fx\right)+8\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)-6\sec^4\left(e+fx\right)+8\sec^3\left(e+fx\right)-3\sec\left(e+fx\right)+1}\,dx}{a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)
**6 - 6*sec(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e +
 f*x) + 1), x)/(a**3*c**6)
```

3.64 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=163

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{105f}-\frac{8c^2\tan(e+fx)}{}$$

```
[Out] -8/21*c^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/9*c*(a+a*s
ec(f*x+e))*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/315*c^4*(a+a*sec(f*x+e))
*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/105*c^3*(a+a*sec(f*x+e))*(c-c*sec(f
*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.28, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3955, 3953}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{105f}-\frac{8c^2\tan(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]
]) - (64*c^3*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(1
05*f) - (8*c^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(21*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(9*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
```

```
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+f}{9f}$$

$$= -\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+}{21f}$$

$$= -\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,\tan(e+f}{105f}$$

$$= -\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\,s}{}$$

Mathematica [A] time = 0.91, size = 86, normalized size = 0.53

$$\frac{ac^3\cos^2\left(\frac{1}{2}(e+fx)\right)(1617\cos(e+fx)-642\cos(2(e+fx))+319\cos(3(e+fx))-782)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*c^3*Cos[(e + f*x)/2]^2*(-782 + 1617*Cos[e + f*x] - 642*Cos[2*(e + f*x)]
+ 319*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e +
f*x]])/(315*f)
```

fricas [A] time = 0.46, size = 119, normalized size = 0.73

$$\frac{2\left(319\,ac^3\cos\left(fx+e\right)^5+317\,ac^3\cos\left(fx+e\right)^4-158\,ac^3\cos\left(fx+e\right)^3-26\,ac^3\cos\left(fx+e\right)^2+95\,ac^3\cos\left(fx+\right.\right.}{315\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
fricas")
```

```
[Out] 2/315*(319*a*c^3*cos(f*x + e)^5 + 317*a*c^3*cos(f*x + e)^4 - 158*a*c^3*cos(
f*x + e)^3 - 26*a*c^3*cos(f*x + e)^2 + 95*a*c^3*cos(f*x + e) - 35*a*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

giac [A] time = 3.36, size = 111, normalized size = 0.68

$$\frac{32\sqrt{2}\left(105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{3}c^{2}+189\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{3}+135\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{4}+35\right.}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
giac")
```

```
[Out] 32/315*sqrt(2)*(105*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^2 + 189*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c^3 + 135*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 35*c^5)
*a*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

maple [A] time = 1.22, size = 83, normalized size = 0.51

$$\frac{2a\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\left(\sin^{3}\left(fx+e\right)\right)\left(319\left(\cos^{3}\left(fx+e\right)\right)-321\left(\cos^{2}\left(fx+e\right)\right)+165\cos\left(fx+e\right)-35\right)}{315f\left(-1+\cos\left(fx+e\right)\right)^{5}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/315*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*sin(f*x+e)^3*(319*cos(f*x+e)
^3-321*cos(f*x+e)^2+165*cos(f*x+e)-35)/(-1+cos(f*x+e))^5/cos(f*x+e)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [B] time = 9.13, size = 483, normalized size = 2.96

$$\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a\,c^3\,2\mathrm{i}}{f}+\frac{a\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,638\mathrm{i}}{315\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a\,c^3\,32\mathrm{i}}{9\,f}+\frac{a\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}+\frac{\sqrt{c-\frac{}{\mathrm{e}^{-e\,1\mathrm{i}}}}}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*2i)/
f + (a*c^3*exp(e*1i + f*x*1i)*638i)/(315*f)))/(exp(e*1i + f*x*1i) - 1) - ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*32i)/(
9*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(ex
p(e*2i + f*x*2i) + 1)^4) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a*c^3*96i)/(7*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(63*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*64i)/(5*f) - (a*c^3*ex
p(e*1i + f*x*1i)*736i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*
2i) + 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2
)*((a*c^3*8i)/(3*f) - (a*c^3*exp(e*1i + f*x*1i)*1256i)/(315*f)))/((exp(e*1i
 + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.65 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=122

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{35f}-\frac{2c\tan(e+fx)(a$$

```
[Out] -2/7*c*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/105*c^3*(a+a
*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/35*c^2*(a+a*sec(f*x+e))
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3955, 3953}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{35f}-\frac{2c\tan(e+fx)(a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]]
) - (16*c^2*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(35
*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(7
*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+f$$
$$= -\frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,\tan(e+f}{35f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\,\mathrm{se}}{}$$

Mathematica [A] time = 0.49, size = 76, normalized size = 0.62

$$\frac{2ac^2\cos^2\left(\frac{1}{2}(e+fx)\right)(-108\cos(e+fx)+71\cos(2(e+fx))+101)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx}}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a*c^2*Cos[(e + f*x)/2]^2*(101 - 108*Cos[e + f*x] + 71*Cos[2*(e + f*x)])*
Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(105*f)
```

fricas [A] time = 0.45, size = 105, normalized size = 0.86

$$\frac{2\left(71\,ac^2\cos\left(fx+e\right)^4+88\,ac^2\cos\left(fx+e\right)^3-22\,ac^2\cos\left(fx+e\right)^2-24\,ac^2\cos\left(fx+e\right)+15\,ac^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{105\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] 2/105*(71*a*c^2*cos(f*x + e)^4 + 88*a*c^2*cos(f*x + e)^3 - 22*a*c^2*cos(f*x
 + e)^2 - 24*a*c^2*cos(f*x + e) + 15*a*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

giac [A] time = 3.41, size = 86, normalized size = 0.70

$$\frac{16\,\sqrt{2}\left(35\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{2}+42\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{3}+15\,c^{4}\right)ac^{2}}{105\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 16/105*sqrt(2)*(35*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 + 42*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*c^3 + 15*c^4)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*
f)
```

maple [A] time = 1.18, size = 73, normalized size = 0.60

$$\frac{2a\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\left(\sin^{3}\left(fx+e\right)\right)\left(71\left(\cos^{2}\left(fx+e\right)\right)-54\cos\left(fx+e\right)+15\right)}{105f\left(-1+\cos\left(fx+e\right)\right)^{4}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/105*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*sin(f*x+e)^3*(71*cos(f*x+e)^
2-54*cos(f*x+e)+15)/(-1+cos(f*x+e))^4/cos(f*x+e)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [B] time = 6.00, size = 384, normalized size = 3.15

$$\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^2\,2\mathrm{i}}{f}+\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,142\mathrm{i}}{105\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}+\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^2\,16\mathrm{i}}{7\,f}-\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}-\sqrt{c-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*2i)/
f + (a*c^2*exp(e*1i + f*x*1i)*142i)/(105*f)))/(exp(e*1i + f*x*1i) - 1) + ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*16i)/(
7*f) - (a*c^2*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(ex
p(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a*c^2*8i)/(5*f) - (a*c^2*exp(e*1i + f*x*1i)*184i)/(35*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) - ((c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*4i)/(3*f) + (a*c^2*exp
(e*1i + f*x*1i)*244i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2
i) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.66 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

```
[Out] -8/15*c^2*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/5*c*(a+a*s
ec(f*x+e))*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f*x]])
- (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5f}$$

$$= -\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e}{}$$

Mathematica [A] time = 0.28, size = 64, normalized size = 0.79

$$\frac{4ac\cos^2\left(\frac{1}{2}(e+fx)\right)(7\cos(e+fx)-3)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*a*c*Cos[(e + f*x)/2]^2*(-3 + 7*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e + f*
x]^2*Sqrt[c - c*Sec[e + f*x]])/(15*f)
```

fricas [A] time = 0.43, size = 82, normalized size = 1.01

$$\frac{2\left(7\,ac\cos\left(fx+e\right)^3+11\,ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)-3\,ac\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] 2/15*(7*a*c*cos(f*x + e)^3 + 11*a*c*cos(f*x + e)^2 + a*c*cos(f*x + e) - 3*a
*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

giac [A] time = 2.54, size = 58, normalized size = 0.72

$$\frac{8\sqrt{2}\left(5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+3\,c^4\right)a}{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 8/15*sqrt(2)*(5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + 3*c^4)*a/((c*tan(1/2*f
*x + 1/2*e)^2 - c)^(5/2)*f)
```

maple [A] time = 1.21, size = 63, normalized size = 0.78

$$\frac{2a\left(7\cos\left(fx+e\right)-3\right)\left(\sin^{3}\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{15f\left(-1+\cos\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/15*a/f*(7*cos(f*x+e)-3)*sin(f*x+e)^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/(-1+cos(f*x+e))^3/cos(f*x+e)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.36, size = 120, normalized size = 1.48

$$-\frac{2\,a\,c\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^{3}\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(7+7\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-6\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\right)}{15\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] -(2*a*c*(exp(e*1i + f*x*1i)*1i + 1i)^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp
(e*1i + f*x*1i)/2))^(1/2)*(7*exp(e*2i + f*x*2i) - 6*exp(e*1i + f*x*1i) + 7)
)/(15*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{3}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-c*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x))
```

3.67 $\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=39

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/3*c*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.18, size = 51, normalized size = 1.31

$$\frac{4a\cos^2\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a*Cos[(e + f*x)/2]^2*Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*
x]])/(3*f)
```

fricas [A] time = 0.46, size = 65, normalized size = 1.67

$$\frac{2\left(a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)+a\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] 2/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

giac [A] time = 2.39, size = 32, normalized size = 0.82

$$\frac{4\sqrt{2}\,ac^{2}}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] 4/3*sqrt(2)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*f)
```

maple [A] time = 1.34, size = 53, normalized size = 1.36

$$\frac{2a\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sin^{3}\left(fx+e\right)\right)}{3f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/3*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^3/cos(f*x+e)/(-1+co
s(f*x+e))^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.75, size = 87, normalized size = 2.23

$$\frac{2\,a\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{3\,f\,\left(8\,\cos\left(2\,e+2\,f\,x\right)-12\,\cos\left(e+f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a*(c - c/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(3
*f*(8*cos(2*e + 2*f*x) - 12*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(4*e + 4
*f*x) + 7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(sqrt(-c*s
ec(e + f*x) + c)*sec(e + f*x)**2, x))
```

3.68 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}$$

```
[Out] -2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)/
f/c^(1/2)+2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3956, 3795, 203}

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x
]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
```

```
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx$$

$$= \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{(4a)\,\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{f}$$

$$= -\frac{2\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f} + \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.64, size = 132, normalized size = 1.71

$$-\frac{i\sqrt{2}\,a\left(-1+e^{i(e+fx)}\right)\left(\sqrt{2}\,\left(1+e^{i(e+fx)}\right)-2\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)\right)}{f\left(1+e^{2i(e+fx)}\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*a*(-1 + E^(I*(e + f*x)))*(Sqrt[2]*(1 + E^(I*(e + f*x))) - 2*S
qrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1
+ E^((2*I)*(e + f*x))])]))/((1 + E^((2*I)*(e + f*x)))*f*Sqrt[c - c*Sec[e +
f*x]])
```

fricas [A] time = 0.52, size = 272, normalized size = 3.53

$$\left[\frac{\sqrt{2}\,ac\sqrt{-\frac{1}{c}}\,\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\,\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\left(a\cos\left(fx+\right.\right.}{cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*sin
(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(a*cos(f*x +
 e) + a)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e)), 2*(sq
rt(2)*a*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(
f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - (a*cos(f*x + e) + a)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((i*a*
sqrt(2)*atan(-i)-a*sqrt(2))/sqrt(-c)*sign(tan((f*x+exp(1))/2))+2*a*(-1/2*sq
rt(2)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(c)/sign(tan((f*x+e
xp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)-1/sqrt(2)/sqrt(c*tan((f*x+exp(1))/
2)^2-c)/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [A] time = 1.60, size = 85, normalized size = 1.10

$$-\frac{2a\left(\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}-1\right)\sin\left(fx+e\right)}{f\sqrt{\frac{c\left(-1+\cos(fx+e)\right)}{\cos(fx+e)}}\,\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*a/f*(arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)-1)*sin(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x
+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f
*x)**2/sqrt(-c*sec(e + f*x) + c), x))
```

3.69 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] 1/2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(3/2)
/f*2^(1/2)-a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3957, 3795, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]])/(Sqrt
[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
```

```
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a\int\frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}\\
&= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}+\frac{a\,\mathrm{Subst}\left(\int\frac{1}{2c+x^2}\,dx,x,\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{cf}\\
&= \frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.15, size = 246, normalized size = 3.24

$$\frac{a\left(\frac{i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^3\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{3/2}}+8\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^3\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)-8\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\sin^3\cdots\right.}{2cf(\sec(e+fx)-\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*((I*Sqrt[2]*(-1 + E^(I*(e + f*x)))^3*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*I)*(e + f*x)))^(3/2) - 4*Cs
c[e/2]*Sec[e + f*x]^2*Sin[(f*x)/2]*Sin[(e + f*x)/2] + 4*Cot[e/2]*Sec[e + f*
x]^2*Sin[(e + f*x)/2]^2 - 8*Cos[e/2]*Cos[(f*x)/2]*Sec[e + f*x]^2*Sin[(e + f
*x)/2]^3 + 8*Sec[e + f*x]^2*Sin[e/2]*Sin[(f*x)/2]*Sin[(e + f*x)/2]^3))/(2*c
*f*(-1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.53, size = 342, normalized size = 4.50

$$\left[\frac{\sqrt{2}\left(ac\cos\left(fx+e\right)-ac\right)\sqrt{-\frac{1}{c}}\,\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}+\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)}{4\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*c*cos(f*x + e) - a*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) +
(3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f
*x + e) + 4*(a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), -1/2*(sqrt(2)*(a
*c*cos(f*x + e) - a*c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sqrt(c) - 2*(a*cos(f*x
 + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*
cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*a*(1/2*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))+1/
2*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)/c/tan((f*x+exp(1))/2)^2)/sqrt(2)/c^2/si
gn(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 1.38, size = 164, normalized size = 2.16

$$\frac{2a\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\cos\left(fx+e\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}+\cos\left(fx+e\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)+\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}-a\right.}{f\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{3}\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 2*a/f*(-1+cos(f*x+e))^2*(cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+co
s(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)-arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+co
s(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqr
t(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**2/(-c*sqrt(-c*sec(e +
f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

3.70 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f}+\frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] 1/16*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(5/2
)/f*2^(1/2)-1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+1/8*a*tan(f*x+e)/c/f/
(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3957, 3796, 3795, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f}+\frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*Sq
rt[2]*c^(5/2)*f) - (a*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2)) + (a*T
an[e + f*x])/(8*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
```

```
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}$$

$$= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,d}{16c^2}$$

$$= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2c+x^2}\,d\right.}{8}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx}{8cf(c-c\sec(e+}$$

Mathematica [C] time = 1.12, size = 309, normalized size = 2.73

$$a\left(-\frac{i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^5\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{5/2}} + 48\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^5\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx) - 48\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)s\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(((-I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^5*ArcTanh[(1 + E^(I*(e + f*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*I)*(e + f*x)))^(5/2) + 1
6*Csc[e/2]*Sec[e + f*x]^3*Sin[(f*x)/2]*Sin[(e + f*x)/2] - 16*Cot[e/2]*Sec[e
 + f*x]^3*Sin[(e + f*x)/2]^2 - 56*Csc[e/2]*Sec[e + f*x]^3*Sin[(f*x)/2]*Sin[
(e + f*x)/2]^3 + 56*Cot[e/2]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^4 - 48*Cos[e/2
]*Cos[(f*x)/2]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^5 + 48*Sec[e + f*x]^3*Sin[e/
2]*Sin[(f*x)/2]*Sin[(e + f*x)/2]^5))/(16*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[c
 - c*Sec[e + f*x]])
```

fricas [A] time = 0.51, size = 405, normalized size = 3.58

$$\left[-\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^2-2a\cos\left(fx+e\right)+a\right)\sqrt{-c}\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-\left(3c\cos\left(fx+e\right)+c\right)s\cdots}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\cdots\right)}{32\left(c^3f\cos\left(fx+e\right)^2-2c^3f\cos\left(fx\cdots\right)\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-c)*log(-(2*
sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e)) - (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*si
n(f*x + e)))*sin(f*x + e) - 4*(3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), -1/16*(sqrt(2)*(a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(
3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3
*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/2*(1/8*(a*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))
/2)^2-c)-a*c^2*sqrt(c*tan((f*x+exp(1))/2)^2-c))/(c*tan((f*x+exp(1))/2)^2)^2
+1/8*a*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c)))/sqrt(2)/c^3/s
ign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 1.42, size = 308, normalized size = 2.73

$$-\frac{a\left(-1+\cos\left(fx+e\right)\right)^{3}\left(\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+4\cos\left(fx+e\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}-\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2*a/f*(-1+cos(f*x+e))^3*(cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/
2)+4*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-cos(f*x+e)^2*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)-cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2))+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+2*cos(f*x+e)*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)+2*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-arctan(1/(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 -
2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) +
 c)), x) + Integral(sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e +
 f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec
(e + f*x) + c)), x))
```

3.71 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{231f}-\frac{8c^2\tan(e+fx}{}$$

```
[Out] -8/33*c^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/11*c*(a+
a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/1155*c^4*(a+a*sec(f
*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/231*c^3*(a+a*sec(f*x+e))^2*
(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.45, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{231f}-\frac{8c^2\tan(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(231*f) - (8*c^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(33*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)*Ta
n[e + f*x])/(11*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
```

```
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e}{11f}$$
$$= -\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e}{33f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e}{231f}$$
$$= -\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+}{}$$

Mathematica [A] time = 1.64, size = 88, normalized size = 0.51

$$\frac{2a^2c^3\cos^4\left(\frac{1}{2}(e+fx)\right)(3419\cos(e+fx)-1510\cos(2(e+fx))+533\cos(3(e+fx))-1930)\cot\left(\frac{1}{2}(e+fx)\right)\text{se}}{1155f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*c^3*Cos[(e + f*x)/2]^4*(-1930 + 3419*Cos[e + f*x] - 1510*Cos[2*(e +
f*x)] + 533*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Se
c[e + f*x]])/(1155*f)
```

fricas [A] time = 0.45, size = 147, normalized size = 0.86

$$\frac{2\left(533\,a^2c^3\cos\left(fx+e\right)^6+844\,a^2c^3\cos\left(fx+e\right)^5-211\,a^2c^3\cos\left(fx+e\right)^4-472\,a^2c^3\cos\left(fx+e\right)^3+295\,a^2c^3\,\mathrm{c}\right.}{1155\,f\cos\left(fx+e\right)^5\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/1155*(533*a^2*c^3*cos(f*x + e)^6 + 844*a^2*c^3*cos(f*x + e)^5 - 211*a^2*c
^3*cos(f*x + e)^4 - 472*a^2*c^3*cos(f*x + e)^3 + 295*a^2*c^3*cos(f*x + e)^2
 + 140*a^2*c^3*cos(f*x + e) - 105*a^2*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

giac [A] time = 4.14, size = 113, normalized size = 0.66

$$-\frac{64\sqrt{2}\left(231\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^3+495\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^4+385\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+10}{1155\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{11}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] -64/1155*sqrt(2)*(231*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 495*(c*tan(1/2
*f*x + 1/2*e)^2 - c)^2*c^4 + 385*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 105*c
^6)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2)*f)
```

maple [A] time = 1.65, size = 85, normalized size = 0.50

$$-\frac{2a^2\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\left(\sin^5\left(fx+e\right)\right)\left(533\left(\cos^3\left(fx+e\right)\right)-755\left(\cos^2\left(fx+e\right)\right)+455\cos\left(fx+e\right)-105\right)}{1155f\left(-1+\cos\left(fx+e\right)\right)^6\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -2/1155*a^2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*sin(f*x+e)^5*(533*cos(f*
x+e)^3-755*cos(f*x+e)^2+455*cos(f*x+e)-105)/(-1+cos(f*x+e))^6/cos(f*x+e)^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.40, size = 606, normalized size = 3.54

$$\frac{\left(\frac{a^2\,c^3\,2\mathrm{i}}{f}+\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1066\mathrm{i}}{1155\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}+\frac{\left(\frac{a^2\,c^3\,64\mathrm{i}}{11\,f}-\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,64\mathrm{i}}{11\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5}-\frac{\left(\frac{a^2\,c^3}{3 \cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] (((a^2*c^3*2i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*1066i)/(1155*f))*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^2*c^3*64i)/(11*f) - (a^2*c^3*exp(e*1i + f*x*1i)*64i)/(11*f))*(c -
c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*
1i) - 1)*(exp(e*2i + f*x*2i) + 1)^5) - (((a^2*c^3*32i)/(3*f) - (a^2*c^3*exp
(e*1i + f*x*1i)*608i)/(33*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^4) - (
((a^2*c^3*4i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*2932i)/(1155*f))*(c - c/(exp(
- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1
)*(exp(e*2i + f*x*2i) + 1)) + (((a^2*c^3*16i)/(5*f) + (a^2*c^3*exp(e*1i + f
*x*1i)*4272i)/(385*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^2*c
^3*32i)/(7*f) - (a^2*c^3*exp(e*1i + f*x*1i)*4640i)/(231*f))*(c - c/(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.72 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2}{315f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{63f}-\frac{2c\tan(e+fx)(a$$

```
[Out] -2/9*c*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/315*c^3*(a
+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/63*c^2*(a+a*sec(f*x
+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.33, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2}{315f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{63f}-\frac{2c\tan(e+fx)(a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(63*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(9*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e}{9f}$$
$$= -\frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e}{63f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+}{}$$

Mathematica [A] time = 1.24, size = 78, normalized size = 0.61

$$\frac{4a^2c^2\cos^4\left(\frac{1}{2}(e+fx)\right)(-220\cos(e+fx)+107\cos(2(e+fx))+177)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e}}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*a^2*c^2*Cos[(e + f*x)/2]^4*(177 - 220*Cos[e + f*x] + 107*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(315*f)
```

fricas [A] time = 0.44, size = 131, normalized size = 1.02

$$\frac{2\left(107\,a^2c^2\cos\left(fx+e\right)^5+211\,a^2c^2\cos\left(fx+e\right)^4+26\,a^2c^2\cos\left(fx+e\right)^3-118\,a^2c^2\cos\left(fx+e\right)^2-5\,a^2c^2\cos(}{315\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/315*(107*a^2*c^2*cos(f*x + e)^5 + 211*a^2*c^2*cos(f*x + e)^4 + 26*a^2*c^2
*cos(f*x + e)^3 - 118*a^2*c^2*cos(f*x + e)^2 - 5*a^2*c^2*cos(f*x + e) + 35*
a^2*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x
+ e))
```

giac [A] time = 2.74, size = 88, normalized size = 0.69

$$-\frac{32 \sqrt{2} \left(63 \left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right)^2 c^3 + 90 \left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right) c^4 + 35 c^5\right) a^2 c^2}{315 \left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right)^{\frac{9}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -32/315*sqrt(2)*(63*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^3 + 90*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^4 + 35*c^5)*a^2*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/
2)*f)
```

maple [A] time = 1.64, size = 75, normalized size = 0.59

$$-\frac{2a^2 \left(107 \left(\cos^2\left(f x + e\right)\right) - 110 \cos\left(f x + e\right) + 35\right) \left(\sin^5\left(f x + e\right)\right) \left(\frac{c\left(-1+\cos\left(f x+e\right)\right)}{\cos\left(f x+e\right)}\right)^{\frac{5}{2}}}{315 f \left(-1 + \cos\left(f x + e\right)\right)^5 \cos\left(f x + e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/315*a^2/f*(107*cos(f*x+e)^2-110*cos(f*x+e)+35)*sin(f*x+e)^5*(c*(-1+cos(f
*x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^5/cos(f*x+e)^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.99, size = 503, normalized size = 3.93

$$\frac{\left(\frac{a^2\, c^2\, 2i}{f} + \frac{a^2\, c^2\, e^{e\, 1i + f\, x\, 1i}\, 214i}{315\, f}\right) \sqrt{c - \frac{c}{\frac{e^{-e\, 1i - f\, x\, 1i}}{2} + \frac{e^{e\, 1i + f\, x\, 1i}}{2}}}}{e^{e\, 1i + f\, x\, 1i} - 1} + \frac{\left(\frac{a^2\, c^2\, 32i}{9\, f} + \frac{a^2\, c^2\, e^{e\, 1i + f\, x\, 1i}\, 32i}{9\, f}\right) \sqrt{c - \frac{c}{\frac{e^{-e\, 1i - f\, x\, 1i}}{2} + \frac{e^{e\, 1i + f\, x\, 1i}}{2}}}}{\left(e^{e\, 1i + f\, x\, 1i} - 1\right) \left(e^{e\, 2i + f\, x\, 2i} + 1\right)^4} - \frac{\left(\frac{a^2\, c^2\, 64}{7\, f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (((a^2*c^2*2i)/f + (a^2*c^2*exp(e*1i + f*x*1i)*214i)/(315*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1)
+ (((a^2*c^2*32i)/(9*f) + (a^2*c^2*exp(e*1i + f*x*1i)*32i)/(9*f))*(c - c/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i)
- 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^2*c^2*64i)/(7*f) + (a^2*c^2*exp(e*1
i + f*x*1i)*320i)/(63*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) + (((a^
2*c^2*48i)/(5*f) + (a^2*c^2*exp(e*1i + f*x*1i)*368i)/(105*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^2) - (((a^2*c^2*16i)/(3*f) + (a^2*c^2*exp(e*1i +
f*x*1i)*208i)/(315*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int\left(-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^3\left(e+fx\right)\right)dx+\int c^2\sqrt{-c\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-
2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(c**2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**5, x))
```

3.73 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=85

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

```
[Out] -8/35*c^2*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/7*c*(a+a
*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.21, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(7*f
)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,\tan(e+}{7f}$$

$$= -\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\,\mathrm{se}}{}$$

Mathematica [A] time = 0.78, size = 66, normalized size = 0.78

$$\frac{8a^2c\cos^4\left(\frac{1}{2}(e+fx)\right)(9\cos(e+fx)-5)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c*Cos[(e + f*x)/2]^4*(-5 + 9*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e +
f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(35*f)
```

fricas [A] time = 0.44, size = 105, normalized size = 1.24

$$\frac{2\left(9\,a^2c\cos\left(fx+e\right)^4+22\,a^2c\cos\left(fx+e\right)^3+12\,a^2c\cos\left(fx+e\right)^2-6\,a^2c\cos\left(fx+e\right)-5\,a^2c\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{35\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/35*(9*a^2*c*cos(f*x + e)^4 + 22*a^2*c*cos(f*x + e)^3 + 12*a^2*c*cos(f*x +
 e)^2 - 6*a^2*c*cos(f*x + e) - 5*a^2*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

giac [A] time = 3.40, size = 60, normalized size = 0.71

$$-\frac{16\sqrt{2}\left(7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^4+5\,c^5\right)a^2}{35\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -16/35*sqrt(2)*(7*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 5*c^5)*a^2/((c*tan(1
/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

maple [A] time = 1.58, size = 65, normalized size = 0.76

$$-\frac{2a^{2}\left(9\cos\left(fx+e\right)-5\right)\left(\sin^{5}\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{35f\left(-1+\cos\left(fx+e\right)\right)^{4}\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/35*a^2/f*(9*cos(f*x+e)-5)*sin(f*x+e)^5*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)/(-1+cos(f*x+e))^4/cos(f*x+e)^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 6.14, size = 384, normalized size = 4.52

$$\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a^2\,c\,2i}{f}+\frac{a^2\,c\,e^{e\,1i+f\,x\,1i}\,18i}{35\,f}\right)}{e^{e\,1i+f\,x\,1i}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a^2\,c\,16i}{7\,f}-\frac{a^2\,c\,e^{e\,1i+f\,x\,1i}\,16i}{7\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3}-\frac{\sqrt{c-\frac{}{\frac{e^{-e\,1i-}}{2}}}}{\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*2i)/
f + (a^2*c*exp(e*1i + f*x*1i)*18i)/(35*f)))/(exp(e*1i + f*x*1i) - 1) - ((c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*16i)/(7*
```

```
f) - (a^2*c*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(
e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*((a^2*c*4i)/f - (a^2*c*exp(e*1i + f*x*1i)*44i)/(35*f)))/((exp(
e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*24i)/(5*f) - (a^2*c*exp(e*1i +
f*x*1i)*72i)/(35*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)dx+\int\left(-c\sqrt{-c\sec\left(e+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(c*sq
rt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-c*sqrt(-c*sec(e + f
*x) + c)*sec(e + f*x)**3, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e
+ f*x)**4, x))
```

3.74 $\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/5*c*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.44, size = 55, normalized size = 1.34

$$\frac{8a^2\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (8*a^2*Cos[(e + f*x)/2]^4*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e
+ f*x]])/(5*f)
```

fricas [B] time = 0.45, size = 84, normalized size = 2.05

$$\frac{2\left(a^{2}\cos\left(fx+e\right)^{3}+3\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)+a^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

giac [A] time = 2.82, size = 34, normalized size = 0.83

$$-\frac{8\,\sqrt{2}\,a^{2}c^{3}}{5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -8/5*sqrt(2)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*f)
```

maple [A] time = 1.80, size = 55, normalized size = 1.34

$$-\frac{2a^{2}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sin^{5}\left(fx+e\right)\right)}{5f\cos\left(fx+e\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/5*a^2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^5/cos(f*x+e)^2/(
-1+cos(f*x+e))^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.73, size = 93, normalized size = 2.27

$$\frac{2\,a^2\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)^5\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{5\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a^2*(exp(e*1i + f*x*1i)*1i + 1i)^5*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^(1/2))/(5*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i)
 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int 2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)dx+\int\sqrt{-c\sec\left(e+fx\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**3, x))
```

3.75 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3cf}+\frac{16a^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/f/c^(1/2)+16/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/3*a^2*(c-c*sec(f
*x+e))^(1/2)*tan(f*x+e)/c/f
```

Rubi [A] time = 0.21, antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3956, 3795, 203}

$$-\frac{4\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-4*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (4*a^2*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
```

```
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+f}{\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + \left(4a^2\right)\int -$$

$$= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{\left(8a^2\right)\text{Subs}}{}$$

$$= -\frac{4\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f} + \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\left(a^2+a^2\text{se}\right.}{3f\sqrt{c}}$$

Mathematica [C] time = 0.93, size = 173, normalized size = 1.48

$$\frac{4a^2e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+7)\right.}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^2*Sec[e + f*x]*((-3*Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 +
 E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/E^((I/2)*(e + f
*x)) + Cos[(e + f*x)/2]*(7 + Sec[e + f*x]))*(Cos[(e + f*x)/2] + I*Sin[(e +
f*x)/2])*Sin[(e + f*x)/2])/(3*E^((I/2)*(e + f*x))*f*Sqrt[c - c*Sec[e + f*x]
])
```

fricas [A] time = 0.53, size = 343, normalized size = 2.93

$$\left[\frac{2\left(3\sqrt{2}a^2c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)\right.}{3cf\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a^2*c*sqrt(-1/c)*cos(f*x + e)*log(-(2*sqrt(2)*(cos(f*x + e)
^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*
cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x
+ e) - (7*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)*sin(f*x + e)), 2/3*(6*sqrt(2)*a^2*
sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - (7*a^2*cos(f*x + e)^2
+ 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*c
os(f*x + e)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((12*i
*a^2*atan(-i)-16*a^2)/3/sqrt(-2*c)*sign(tan((f*x+exp(1))/2))-4*a^2*(1/2*sqr
t(2)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(c)/sign(tan((f*x+ex
p(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)+1/3*(3*(c*tan((f*x+exp(1))/2)^2-c)-
c)/sqrt(c*tan((f*x+exp(1))/2)^2-c)/(c*tan((f*x+exp(1))/2)^2-c)/sqrt(2)/sign
(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [A] time = 1.75, size = 145, normalized size = 1.24

$$\frac{2a^{2}\left(3\cos\left(fx+e\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)}\right)^{\frac{3}{2}}+3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)}\right)^{\frac{3}{2}}+7\cos\left(fx\right.\right.}{3f\cos\left(fx+e\right)^{2}\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/3*a^2/f*(3*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+7*cos(f*x+e)+1)*sin(f*x+e)/cos(
f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{2}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{2}}{\cos\left(e+f\,x\right)\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(2*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**3/sqrt(-
c*sec(e + f*x) + c), x))
```

3.76 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] 3*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/c^(3/2)/f-2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-2*a^2*tan(f*x+e)/c/f/(
c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3956, 3795, 203}

$$\frac{3\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (3*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*
x]])])/(c^(3/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[
e + f*x])^(3/2)) - (3*a^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
```

```
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{\left(3a^2\right)\int \cdots}{\cdots}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} + \frac{\left(6a^2\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{3\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \cdots$$

Mathematica [C] time = 1.53, size = 184, normalized size = 1.63

$$\frac{a^2e^{-2i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(4\left(1+e^{3i(e+fx)}\right)-3\sqrt{2}\left(-1+e^{i(e+fx)}\right)^2\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+f\cdots}}{\sqrt{2}\sqrt{1+e^{2i(\cdots}}}\right.\cdots}{2cf(\sec(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (a^2*(4*(1 + E^((3*I)*(e + f*x))) - 3*Sqrt[2]*(-1 + E^(I*(e + f*x)))^2*Sqrt
[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E
^((2*I)*(e + f*x))])])*Sec[e + f*x]^2*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2
])*Sin[(e + f*x)/2])/(2*c*E^((2*I)*(e + f*x))*f*(-1 + Sec[e + f*x])*Sqrt[c
- c*Sec[e + f*x]])
```

fricas [A] time = 0.52, size = 372, normalized size = 3.29

$$\left[\frac{3\sqrt{2}\left(a^2c\cos\left(fx+e\right)-a^2c\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}+\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx\right.}{2\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1
/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))
*sin(f*x + e) + 4*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
, -(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sq
rt(c) - 2*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)-2/f*2*a^2*(1/2*(-3*(c*tan((f*x+exp(1))/2)^2-c)-2*c)/s
```

```
qrt(2)/c/(sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+c*sqr
t(c*tan((f*x+exp(1))/2)^2-c))/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1
))/2)^2-1)-3/2*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(2)/sqrt(c
)/c/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 1.70, size = 144, normalized size = 1.27

$$\frac{a^{2}\left(3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)-3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,-4\cos\left(fx+\right.}{f\cos\left(fx+e\right)^{2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] a^2/f*(3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*cos(f*x+e)-3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-4*cos(f*x+e)+2)*sin(f*x+e)/cos(f*x+
e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{2}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{2\sec^{2}\left(e+\right.}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(2*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(s
ec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e
 + f*x) + c)), x))
```

3.77 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{5a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -3/8*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(5
/2)/f*2^(1/2)-a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+5/4*a^2*tan(f*x+e)/c/
f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.24, antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3957, 3795, 203}

$$-\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-3*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(2*f*(c - c
*Sec[e + f*x])^(5/2)) + (3*a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(3
/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
```

```
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{\left(3a^2\right)\ldots}{\ldots}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} - \frac{\left(3a^2\right)\ldots}{\ldots}$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} - \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{\ldots}{4cf\ldots}$$

Mathematica [C] time = 2.63, size = 359, normalized size = 3.07

$$-\frac{a^2\csc\left(\frac{e}{2}\right)e^{-\frac{1}{2}i(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\,(\sec(e+fx)+1)^2\left(3\sin\left(\frac{e}{2}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -1/4*(a^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])
^2*(((-1 + E^(I*e))*(Cos[(f*x)/2] + I*Sin[(f*x)/2])*((-9*I)*E^(I*e)*(1 + E^
(I*e))*Cos[(f*x)/2] + I*(1 + E^((3*I)*e))*Cos[(3*f*x)/2] - 9*E^(I*e)*Sin[(f
*x)/2] + 9*E^((2*I)*e)*Sin[(f*x)/2] + Sin[(3*f*x)/2] - E^((3*I)*e)*Sin[(3*f
*x)/2]))/(16*E^(((3*I)/2)*e)*Sqrt[Sec[e + f*x]]) + 3*Sqrt[E^(I*(e + f*x))/(
1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(
e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sin[e/2]*Sin[(e + f*x)/
2]^4)*Tan[(e + f*x)/2])/(c^2*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^2*Sq
rt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.56, size = 429, normalized size = 3.67

$$\left[-\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^2 - 2a^2\cos\left(fx+e\right) + a^2\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)\right.}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{16\left(c^3f\cos\left(fx+e\right)^2-2c^3f\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x +
e)^2 - 5*a^2*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f
*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/8*(3*sqrt(
2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(c)*arctan(sqrt(2)*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))
*sin(f*x + e) - 2*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x + e)^2 - 5*a^2*cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2
*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*a^2*(3/8*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))
+1/8*(3*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+5*c^2
*sqrt(c*tan((f*x+exp(1))/2)^2-c))/(c*tan((f*x+exp(1))/2)^2)^2)/sqrt(2)/c^3/
sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 1.90, size = 230, normalized size = 1.97

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)^3\left(\left(\cos^2\left(fx+e\right)\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}+3\left(\cos^2\left(fx+e\right)\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)-4\cos\left(fx+\right.\right.}{f\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -a^2/f*(-1+cos(f*x+e))^3*(cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
+3*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-4*cos(f*x+e)
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-6*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2))-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*arctan(1/(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/s
in(f*x+e)^5/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(2*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**3/(c**2*sqrt(-c*sec(e + f
*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) +
c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

3.78 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)\right.}{3f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/32*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(
7/2)/f*2^(1/2)-1/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
+1/4*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)-1/16*a^2*tan(f*x+e)/c^2/f/(c
-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3796, 3795, 203}

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)\right.}{3f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -(a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*
Sec[e + f*x])^(7/2)) + (a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(5/2)
) - (a^2*Tan[e + f*x])/(16*c^2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx}{2c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} + \frac{a^2\int}{}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{}{16c^2f}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{}{16c^2f}$$

$$= -\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} - \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{}{4cf}$$

Mathematica [C] time = 5.90, size = 398, normalized size = 2.43

$$\frac{a^2\csc\left(\frac{e}{2}\right)e^{-\frac{1}{2}i(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(\sec(e+fx)+1)^2\left(e^{\frac{ie}{2}}\sqrt{\sec(e+fx)}\left(e^{\frac{ifx}{2}}\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),
x]
```

```
[Out] (a^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^2*(-
3*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*
x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]
*Sin[e/2]*Sin[(e + f*x)/2]^6 + E^((I/2)*e)*Sqrt[Sec[e + f*x]]*(-4*I + (4*I)
*E^(I*f*x) - (E^((I/2)*f*x)*Cos[e/2]*(-57 + 36*Cos[e + f*x] - 43*Cos[2*(e +
 f*x)])*Sin[(e + f*x)/2])/8 - (7*E^((I/2)*f*x)*Csc[(f*x)/2]*Sin[e]*Sin[f*x]
*Sin[(e + f*x)/2]^6)/2 + E^((I/2)*f*x)*Sin[(f*x)/2]*Sin[(e + f*x)/2]^2*(34
- 43*Sin[(e + f*x)/2]^2 + 14*Sin[e/2]^2*Sin[(e + f*x)/2]^4)))*Tan[(e + f*x)
/2])/(24*c^3*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e +
 f*x]])
```

fricas [A] time = 0.58, size = 517, normalized size = 3.15

$$\left[-\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^3 - 3a^2\cos\left(fx+e\right)^2 + 3a^2\cos\left(fx+e\right) - a^2\right)\sqrt{-c}\,\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\cdots}{\cos\cdots}}\cdots}{\left(\cos\left(fx+e\right)-1\cdots\right)}\cdots\right.}{192\left(c^4 f\cos\left(fx+e\right)^3 - \cdots\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f
*x + e) - a^2)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f
*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*(7*a^2*cos(f*x
 + e)^4 + 29*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*
f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/96*(3*sqr
t(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)
*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e
)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*(7*a^2*cos(f*x + e)^4 + 29*a^2*c
os(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/2*(1/48*(-8*a^2*c^2*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x
+exp(1))/2)^2-c)-3*a^2*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1)
)/2)^2-c)^2+3*a^2*c^3*sqrt(c*tan((f*x+exp(1))/2)^2-c))/(c*tan((f*x+exp(1))/
2)^2)^3-1/16*a^2*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c)))/sqr
t(2)/c^4/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 2.46, size = 402, normalized size = 2.45

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)^4\left(5\left(\cos^3\left(fx+e\right)\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+15\left(\cos^2\left(fx+e\right)\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+3\left(\cos^3\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/6*a^2/f*(-1+cos(f*x+e))^4*(5*cos(f*x+e)^3*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(3/2)+15*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+3*cos(f*x+e)^3*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*cos(f*x+e)^3*arctan(1/(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2))+27*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-
9*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-9*cos(f*x+e)^2*arctan(1
/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+17*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3
/2)+9*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+9*cos(f*x+e)*arctan(1
/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)-3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos
(f*x+e))^(7/2)/sin(f*x+e)^7/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{-c^3\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^3\left(e+fx\right)+3c^3\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)-3c^3\sqrt{-c\sec\left(e+fx\right)+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**
3 + 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e
 + f*x) + c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(
2*sec(e + f*x)**2/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3
*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) +
c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*
x)**3/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x))
```

3.79 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^3}{3003f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{429f}-\frac{24c^2\tan(e+$$

```
[Out] -24/143*c^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/13*c*(
a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/3003*c^4*(a+a*sec
(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/429*c^3*(a+a*sec(f*x+e))^
3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.44, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^3}{3003f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{429f}-\frac{24c^2\tan(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(429*f) - (24*c^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[
e + f*x])/(143*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)*
Tan[e + f*x])/(13*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
```

```
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+}{13f}$$

$$= -\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e}{143f}$$

$$= -\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\tan(e+}{429f}$$

$$= -\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a}{}$$

Mathematica [A] time = 2.54, size = 88, normalized size = 0.51

$$\frac{4a^3c^3\cos^6\left(\frac{1}{2}(e+fx)\right)(6285\cos(e+fx)-2842\cos(2(e+fx))+835\cos(3(e+fx))-3766)\cot\left(\frac{1}{2}(e+fx)\right)\sec}{3003f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (4*a^3*c^3*Cos[(e + f*x)/2]^6*(-3766 + 6285*Cos[e + f*x] - 2842*Cos[2*(e +
f*x)] + 835*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^6*Sqrt[c - c*Se
c[e + f*x]])/(3003*f)
```

fricas [A] time = 0.47, size = 163, normalized size = 0.95

$$\frac{2\left(835\,a^3c^3\cos\left(fx+e\right)^7+1919\,a^3c^3\cos\left(fx+e\right)^6+271\,a^3c^3\cos\left(fx+e\right)^5-1637\,a^3c^3\cos\left(fx+e\right)^4-103\,a^3c^3\right.}{3003\,f\cos\left(fx+e\right)^6\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/3003*(835*a^3*c^3*cos(f*x + e)^7 + 1919*a^3*c^3*cos(f*x + e)^6 + 271*a^3*
c^3*cos(f*x + e)^5 - 1637*a^3*c^3*cos(f*x + e)^4 - 103*a^3*c^3*cos(f*x + e)
^3 + 973*a^3*c^3*cos(f*x + e)^2 + 21*a^3*c^3*cos(f*x + e) - 231*a^3*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^6*sin(f*x + e))
```

giac [A] time = 4.90, size = 113, normalized size = 0.66

$$\frac{128\,\sqrt{2}\left(429\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{3}c^{4}+1001\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{5}+819\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{6}+\right.}{3003\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{13}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] 128/3003*sqrt(2)*(429*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 1001*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2*c^5 + 819*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 231*
c^7)*a^3*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(13/2)*f)
```

maple [A] time = 1.96, size = 85, normalized size = 0.50

$$\frac{2a^{3}\left(835\left(\cos^{3}\left(fx+e\right)\right)-1421\left(\cos^{2}\left(fx+e\right)\right)+945\cos\left(fx+e\right)-231\right)\left(\sin^{7}\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}{3003f\left(-1+\cos\left(fx+e\right)\right)^{7}\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/3003*a^3/f*(835*cos(f*x+e)^3-1421*cos(f*x+e)^2+945*cos(f*x+e)-231)*sin(f*
x+e)^7*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/(-1+cos(f*x+e))^7/cos(f*x+e)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.67, size = 710, normalized size = 4.15

$$\frac{\left(\frac{a^3\,c^3\,2\mathrm{i}}{f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1670\mathrm{i}}{3003\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}+\frac{\left(\frac{a^3\,c^3\,128\mathrm{i}}{13\,f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,128\mathrm{i}}{13\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^6}-\frac{\left(\frac{a^3\,c}{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] (((a^3*c^3*2i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*1670i)/(3003*f))*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^3*c^3*128i)/(13*f) + (a^3*c^3*exp(e*1i + f*x*1i)*128i)/(13*f))*(c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*
x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^6) - (((a^3*c^3*384i)/(11*f) + (a^3*c^3
*exp(e*1i + f*x*1i)*3456i)/(143*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^
5) - (((a^3*c^3*8i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*2168i)/(3003*f))*(c - c
/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1
i) - 1)*(exp(e*2i + f*x*2i) + 1)) + (((a^3*c^3*24i)/f + (a^3*c^3*exp(e*1i +
 f*x*1i)*5464i)/(1001*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^
3*c^3*160i)/(3*f) + (a^3*c^3*exp(e*1i + f*x*1i)*11360i)/(429*f))*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) -
 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^3*320i)/(7*f) + (a^3*c^3*exp(e*1
i + f*x*1i)*46400i)/(3003*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.80 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3}{693f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{99f}-\frac{2c\tan(e+fx)}{}$$

```
[Out] -2/11*c*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/693*c^3*(
a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/99*c^2*(a+a*sec(f*
x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.32, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{99f}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3}{693f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(99*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(11*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+}{11f}$$

$$= -\frac{16c^2(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\tan(e+}{99f}$$

$$= -\frac{64c^3(a+a\sec(e+fx))^3\tan(e+fx)}{693f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a}{}$$

Mathematica [A] time = 1.56, size = 78, normalized size = 0.61

$$\frac{8a^3c^2\cos^6\left(\frac{1}{2}(e+fx)\right)(-364\cos(e+fx)+151\cos(2(e+fx))+277)\cot\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sqrt{c-c\sec(e+}}{693f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (8*a^3*c^2*Cos[(e + f*x)/2]^6*(277 - 364*Cos[e + f*x] + 151*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Sec[e + f*x]])/(693*f)
```

fricas [A] time = 0.43, size = 147, normalized size = 1.15

$$\frac{2\left(151\,a^3c^2\cos\left(fx+e\right)^6+422\,a^3c^2\cos\left(fx+e\right)^5+241\,a^3c^2\cos\left(fx+e\right)^4-236\,a^3c^2\cos\left(fx+e\right)^3-199\,a^3c^2\,\mathrm{co}}{693\,f\cos\left(fx+e\right)^5\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/693*(151*a^3*c^2*cos(f*x + e)^6 + 422*a^3*c^2*cos(f*x + e)^5 + 241*a^3*c^
2*cos(f*x + e)^4 - 236*a^3*c^2*cos(f*x + e)^3 - 199*a^3*c^2*cos(f*x + e)^2
+ 70*a^3*c^2*cos(f*x + e) + 63*a^3*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

giac [A] time = 3.83, size = 88, normalized size = 0.69

$$\frac{64 \sqrt{2} \left(99 \left(c \tan \left(\frac{1}{2} f x + \frac{1}{2} e\right)^{2} - c\right)^{2} c^{4} + 154 \left(c \tan \left(\frac{1}{2} f x + \frac{1}{2} e\right)^{2} - c\right) c^{5} + 63 c^{6}\right) a^{3} c^{2}}{693 \left(c \tan \left(\frac{1}{2} f x + \frac{1}{2} e\right)^{2} - c\right)^{\frac{11}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 64/693*sqrt(2)*(99*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 154*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^5 + 63*c^6)*a^3*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11
/2)*f)
```

maple [A] time = 1.84, size = 75, normalized size = 0.59

$$\frac{2a^{3} \left(151 \left(\cos^{2}\left(f x + e\right)\right) - 182 \cos\left(f x + e\right) + 63\right) \left(\sin^{7}\left(f x + e\right)\right) \left(\frac{c\left(-1+\cos\left(f x+e\right)\right)}{\cos\left(f x+e\right)}\right)^{\frac{5}{2}}}{693 f \left(-1 + \cos\left(f x + e\right)\right)^{6} \cos\left(f x + e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/693*a^3/f*(151*cos(f*x+e)^2-182*cos(f*x+e)+63)*sin(f*x+e)^7*(c*(-1+cos(f*
x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^6/cos(f*x+e)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 13.71, size = 607, normalized size = 4.74

$$\frac{\left(\frac{a^{3} c^{2} 2i}{f} + \frac{a^{3} c^{2} e^{e 1i + f x 1i} 302i}{693 f}\right) \sqrt{c - \frac{c}{\frac{e^{-e 1i - f x 1i}}{2} + \frac{e^{e 1i + f x 1i}}{2}}}}{e^{e 1i + f x 1i} - 1} - \frac{\left(\frac{a^{3} c^{2} 64i}{11 f} - \frac{a^{3} c^{2} e^{e 1i + f x 1i} 64i}{11 f}\right) \sqrt{c - \frac{c}{\frac{e^{-e 1i - f x 1i}}{2} + \frac{e^{e 1i + f x 1i}}{2}}}}{\left(e^{e 1i + f x 1i} - 1\right) \left(e^{e 2i + f x 2i} + 1\right)^{5}} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (((a^3*c^2*2i)/f + (a^3*c^2*exp(e*1i + f*x*1i)*302i)/(693*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1)
- (((a^3*c^2*64i)/(11*f) - (a^3*c^2*exp(e*1i + f*x*1i)*64i)/(11*f))*(c - c/
(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i
) - 1)*(exp(e*2i + f*x*2i) + 1)^5) + (((a^3*c^2*16i)/f - (a^3*c^2*exp(e*1i
+ f*x*1i)*944i)/(231*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i
)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^3
*c^2*160i)/(9*f) - (a^3*c^2*exp(e*1i + f*x*1i)*1120i)/(99*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^2*20i)/(3*f) - (a^3*c^2*exp(e*1i +
f*x*1i)*844i)/(693*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) - (((a^3*c^2
*160i)/(7*f) - (a^3*c^2*exp(e*1i + f*x*1i)*6880i)/(693*f))*(c - c/(exp(- e*
1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(e
xp(e*2i + f*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.81 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=85

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

```
[Out] -8/63*c^2*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/9*c*(a+a
*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.21, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(9*f
)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\tan(e+f}{9f}$$

$$= -\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(}{}$$

Mathematica [A] time = 1.11, size = 66, normalized size = 0.78

$$\frac{16a^3c\cos^6\left(\frac{1}{2}(e+fx)\right)(11\cos(e+fx)-7)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{63f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (16*a^3*c*Cos[(e + f*x)/2]^6*(-7 + 11*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e
+ f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(63*f)
```

fricas [A] time = 0.43, size = 119, normalized size = 1.40

$$\frac{2\left(11\,a^3c\cos\left(fx+e\right)^5+37\,a^3c\cos\left(fx+e\right)^4+38\,a^3c\cos\left(fx+e\right)^3+2\,a^3c\cos\left(fx+e\right)^2-17\,a^3c\cos\left(fx+e\right)-}{63\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/63*(11*a^3*c*cos(f*x + e)^5 + 37*a^3*c*cos(f*x + e)^4 + 38*a^3*c*cos(f*x
+ e)^3 + 2*a^3*c*cos(f*x + e)^2 - 17*a^3*c*cos(f*x + e) - 7*a^3*c)*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

giac [A] time = 3.13, size = 60, normalized size = 0.71

$$\frac{32\sqrt{2}\left(9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+7\,c^6\right)a^3}{63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 32/63*sqrt(2)*(9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 7*c^6)*a^3/((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

maple [A] time = 1.80, size = 65, normalized size = 0.76

$$\frac{2a^3\left(11\cos\left(fx+e\right)-7\right)\left(\sin^7\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{63f\left(-1+\cos\left(fx+e\right)\right)^5\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/63*a^3/f*(11*cos(f*x+e)-7)*sin(f*x+e)^7*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)/(-1+cos(f*x+e))^5/cos(f*x+e)^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 9.19, size = 471, normalized size = 5.54

$$\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a^3\,c\,2i}{f}+\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\,22i}{63\,f}\right)}{e^{e\,1i+f\,x\,1i}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a^3\,c\,32i}{9\,f}+\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\,32i}{9\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4}-\frac{\sqrt{c-\frac{c}{e^{-e\,1\cdots}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*2i)/
f + (a^3*c*exp(e*1i + f*x*1i)*22i)/(63*f)))/(exp(e*1i + f*x*1i) - 1) - ((c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(9*
```

```
f) + (a^3*c*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(
e*2i + f*x*2i) + 1)^4) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*((a^3*c*8i)/(3*f) - (a^3*c*exp(e*1i + f*x*1i)*200i)/(63*f)))/(
(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i -
f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(7*f) + (a^3*c*exp(e*
1i + f*x*1i)*608i)/(63*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) +
 1)^3) - (a^3*c*exp(e*1i + f*x*1i)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*160i)/(21*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*
2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int 2c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)dx+\int\left(-2c\sqrt{-c\sec\left(e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*c*
sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-2*c*sqrt(-c*sec(e
 + f*x) + c)*sec(e + f*x)**4, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**5, x))
```

3.82 $\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/7*c*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.71, size = 55, normalized size = 1.34

$$\frac{16a^3\cos^6\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (16*a^3*Cos[(e + f*x)/2]^6*Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e
 + f*x]])/(7*f)
```

fricas [B] time = 0.42, size = 97, normalized size = 2.37

$$\frac{2\left(a^3\cos\left(fx+e\right)^4+4\,a^3\cos\left(fx+e\right)^3+6\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+a^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{7\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x +
 e)^3*sin(f*x + e))
```

giac [A] time = 3.02, size = 34, normalized size = 0.83

$$\frac{16\,\sqrt{2}\,a^3c^4}{7\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 16/7*sqrt(2)*a^3*c^4/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

maple [A] time = 1.98, size = 55, normalized size = 1.34

$$\frac{2a^3\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sin^7\left(fx+e\right)\right)}{7f\cos\left(fx+e\right)^3\left(-1+\cos\left(fx+e\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/7*a^3/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^7/cos(f*x+e)^3/(-
1+cos(f*x+e))^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.62, size = 375, normalized size = 9.15

$$\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,2\mathrm{i}}{f}+\frac{a^3\,e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}}{7\,f}\right)}{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,8\mathrm{i}}{f}+\frac{a^3\,e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,8\mathrm{i}}{7\,f}\right)}{\left(e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(e^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\dots}}}{\left(e^{e\,1\mathrm{i}+f\,x}\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*2i)/f
+ (a^3*exp(e*1i + f*x*1i)*2i)/(7*f)))/(exp(e*1i + f*x*1i) - 1) - ((c - c/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*8i)/f + (a^3*exp
(e*1i + f*x*1i)*8i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) +
 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a
^3*4i)/f + (a^3*exp(e*1i + f*x*1i)*36i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a^3*16i)/(7*f) - (a^3*exp(e*1i + f*x*1i)*16i)/(7*f)))/((ex
p(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int 3\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)dx+\int 3\sqrt{-c\sec\left(e+fx\dots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(3*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(3*sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x)**3, x) + Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
**4, x))
```

3.83 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=164

$$-\frac{8\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{4\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\sqrt{c-c\sec(e+fx)}}+\frac{2a\tan(e+fx)(a\,\mathrm{se}}{5f\sqrt{c-c\,\mathrm{sec}}}$$

```
[Out] -8*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/f/c^(1/2)+8*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+2/5*a*(a+a*sec(f*x+e)
)^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)
/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3956, 3795, 203}

$$-\frac{8\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{4\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\sqrt{c-c\sec(e+fx)}}+\frac{2a\tan(e+fx)(a\,\mathrm{se}}{5f\sqrt{c-c\,\mathrm{sec}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-8*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (8*a^3*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]]) + (
4*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+$$

$$= \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4\left(a^3+a^3\sec(e+fx)\right)\tan(e+f}{3f\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4\left(a^3+a^3\right.}{3f}$$

$$= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4\left(a^3+a^3\right.}{3f}$$

$$= -\frac{8\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f} + \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\,\mathrm{se}}{5f\sqrt{}}$$

Mathematica [C] time = 1.53, size = 185, normalized size = 1.13

$$\frac{4a^3e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(3\sec^2(e+fx)\right.\right.}{15f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Sec[e + f*x]*((-30*Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1
+ E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/E^((I/2)*(e +
f*x)) + Cos[(e + f*x)/2]*(73 + 16*Sec[e + f*x] + 3*Sec[e + f*x]^2))*(Cos[(e
 + f*x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/(15*E^((I/2)*(e + f*x))*
f*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.51, size = 377, normalized size = 2.30

$$\left[\frac{2\left(30\sqrt{2}a^3c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)^2\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+\right.\right.}{15cf\cos\left(fx+e\right)^2\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*a^3*c*sqrt(-1/c)*cos(f*x + e)^2*log(-(2*sqrt(2)*(cos(f*x
+ e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) -
 (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(
f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos(f*x
+ e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)^2*
sin(f*x + e)), 2/15*(60*sqrt(2)*a^3*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x
+ e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)^2
*sin(f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos
(f*x + e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x +
e)^2*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((60*i
*a^3*sqrt(2)*atan(-i)-92*a^3*sqrt(2))/15/sqrt(-c)*sign(tan((f*x+exp(1))/2))
+8*a^3*(-1/2*sqrt(2)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(c)/
sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)-1/15/sqrt(c*tan((f*
x+exp(1))/2)^2-c)/(c*tan((f*x+exp(1))/2)^2-c)^2*(-5*c*(c*tan((f*x+exp(1))/2
)^2-c)+3*c^2+15*(c*tan((f*x+exp(1))/2)^2-c)^2)/sqrt(2)/sign(tan((f*x+exp(1)
)/2))/sign(tan((f*x+exp(1))/2)^2-1)))
```

maple [A] time = 1.91, size = 206, normalized size = 1.26

$$-\frac{2a^3\left(15\left(\cos^2\left(fx+e\right)\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)}\right)^{\frac{5}{2}}+30\cos\left(fx+e\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-\frac{2\cos(}{1+\cos}\right.\right.}{15f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/15*a^3/f*(15*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+30*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+15*arctan(1/(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-73
*cos(f*x+e)^2-16*cos(f*x+e)-3)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/c
os(f*x+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^3\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^3}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{3\sec^3\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^4\left(e+\right.}{\sqrt{-c\sec\left(e+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(e + f*x)**3/sqrt
(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**4/sqrt(-c*sec(e + f*x) +
 c), x))
```

3.84 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{10\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\,\mathrm{s}}{f(c-c\sec(}$$

```
[Out] 10*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/c^(3/2)/f-a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-10*a^3
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)-5/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/
c/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3956, 3795, 203}

$$\frac{10\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\,\mathrm{s}}{f(c-c\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (10*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(c^(3/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec
[e + f*x])^(3/2)) - (10*a^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]]) -
(5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5\left(a^3+a^3\right.}{3cf}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5\left(a^3+a^3\right.}{3cf}$$

$$= \frac{10\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - $$

Mathematica [C] time = 3.10, size = 324, normalized size = 1.93

$$-\frac{a^3\csc\left(\frac{e}{2}\right)e^{-\frac{1}{2}i(e+fx)}\tan^3\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(-\frac{i\left(-1+e^{ie}\right)e^{\frac{ifx}{2}}\left(-24e^{i(e+fx)}+34e^{2i(e+fx)}-24e^{3i(e+f}\right.}{2\left(-1+e^{i(e+fx)}\right)^2\left(1+e^{2i(}\right.}\right.}{3cf(\sec(e+fx)-1)\sec^{\frac{3}{2}}(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] -1/3*(a^3*Csc[e/2]*Sec[(e + f*x)/2]^2*(1 + Sec[e + f*x])^3*(((-1/2*I)*E^((I
/2)*f*x)*(-1 + E^(I*e))*(19 - 24*E^(I*(e + f*x)) + 34*E^((2*I)*(e + f*x)) -
 24*E^((3*I)*(e + f*x)) + 19*E^((4*I)*(e + f*x)))*Sqrt[Sec[e + f*x]])/((-1
+ E^(I*(e + f*x)))^2*(1 + E^((2*I)*(e + f*x)))) - 15*Sqrt[E^(I*(e + f*x))/(
1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(
e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sec[(e + f*x)/2]*Sin[e/
2])*Tan[(e + f*x)/2]^3)/(c*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])*Sec[e
+ f*x]^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.54, size = 432, normalized size = 2.57

$$\left[\frac{15\sqrt{2}\left(a^3 c\cos\left(fx+e\right)^2 - a^3 c\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{c}}\,\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}+\left(3\cos\left(fx+e\right)+1\right)\sin\cdots}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\cdots\right.}{3\left(c^2 f\cos\left(fx+e\right)^2 - c^2 f\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*sqrt(-1/c)*log
((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sqrt(-1/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1
)*sin(f*x + e)))*sin(f*x + e) + 2*(19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x +
e)^2 - 13*a^3*cos(f*x + e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/
((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e)), -2/3*(15*sqrt(2
)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*arctan(sqrt(2)*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)
/sqrt(c) - (19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x + e)^2 - 13*a^3*cos(f*x +
 e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e)^2
- c^2*f*cos(f*x + e))*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*4*a^3*(1/3*(6*(c*tan((f*x+exp(1))/2)^
2-c)-c)/sqrt(2)/sqrt(c*tan((f*x+exp(1))/2)^2-c)/c/(c*tan((f*x+exp(1))/2)^2-
c)/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)+5/2*atan(sqrt(c*
tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(2)/sqrt(c)/c/sign(tan((f*x+exp(1))/2
))/sign(tan((f*x+exp(1))/2)^2-1)+1/2*sqrt(c*tan((f*x+exp(1))/2)^2-c)/c/tan(
(f*x+exp(1))/2)^2/sqrt(2)/c/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))
/2)^2-1))
```

maple [A] time = 1.73, size = 157, normalized size = 0.93

$$-\frac{a^{3}\left(15\left(\cos^{2}\left(fx+e\right)\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}-15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+38\left(c\right.\right.}{3f\cos\left(fx+e\right)^{3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/3*a^3/f*(15*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+38*cos(f*x+e)^2-24*cos(f*x
+e)-2)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{3}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{3\sec^2\left(e+\right.}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(3
*sec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec
(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(-c*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

3.85 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f}+\frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\,\mathrm{se}\ldots}{2f(c-c\sec(\ldots}$$

```
[Out] -15/4*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(
5/2)/f*2^(1/2)-1/2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
+5/4*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+15/4*a^3*ta
n(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3956, 3795, 203}

$$-\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f}+\frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\,\mathrm{se}\ldots}{2f(c-c\sec(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-15*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(2*f*(c -
c*Sec[e + f*x])^(5/2)) + (5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(4*c*f*(
c - c*Sec[e + f*x])^(3/2)) + (15*a^3*Tan[e + f*x])/(4*c^2*f*Sqrt[c - c*Sec[
e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(5a)\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+f}{4cf(c-c\sec(e+fx))^{3/2}}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+f}{4cf(c-c\sec(e+fx))^{3/2}}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+f}{4cf(c-c\sec(e+fx))^{3/2}}$$

$$= -\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5}{}$$

Mathematica [C] time = 4.57, size = 263, normalized size = 1.51

$$-\frac{a^3e^{-\frac{1}{2}i(2e+fx)}\tan^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(120e^{\frac{ie}{2}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1}{\sqrt{2}}\right.\right.}{32c^2f(\sec(e+fx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -1/32*(a^3*Sec[(e + f*x)/2]*(1 + Sec[e + f*x])^3*(120*E^((I/2)*e)*Sqrt[E^(I
*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTan
h[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] + (25*Cos[
(3*(e + f*x))/2] - 9*Cos[(5*(e + f*x))/2])*Csc[(e + f*x)/2]^4*Sqrt[Sec[e +
f*x]]*(Cos[e + (f*x)/2] + I*Sin[e + (f*x)/2]))*Tan[(e + f*x)/2]^5)/(c^2*E^(
(I/2)*(2*e + f*x))*f*(-1 + Sec[e + f*x])^2*Sqrt[Sec[e + f*x]]*Sqrt[c - c*Se
c[e + f*x]])
```

fricas [A] time = 0.58, size = 441, normalized size = 2.53

$$\left[-\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^2-2a^3\cos\left(fx+e\right)+a^3\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)\cdots\right.}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{8\left(c^3f\cos\left(fx+e\right)^2-2c^3f\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
+ e)^2 - 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1
/4*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(c)*arct
an(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*si
n(f*x + e)))*sin(f*x + e) - 2*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c
^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*a^3*(1/33554432*(-14680064*sqrt(2)*
sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)-18874368*sqrt(2
)*c*sqrt(c*tan((f*x+exp(1))/2)^2-c))/c^2/(c*tan((f*x+exp(1))/2)^2)^2/sign(t
an((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1)-15/8/sqrt(c)/c^2*atan(sqr
t(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(2)/sign(tan((f*x+exp(1))/2))/sig
n(tan((f*x+exp(1))/2)^2-1)-1/sqrt(2)/c^2/sqrt(c*tan((f*x+exp(1))/2)^2-c)/si
gn(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1))

maple [A] time = 1.88, size = 206, normalized size = 1.18

$$-\frac{a^3\left(15\left(\cos^2\left(fx+e\right)\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}-30\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}\right)\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}\cos\left(f\right.\right.}{4f\cos\left(fx+e\right)^3\left(\frac{c(}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x)

[Out] -1/4*a^3/f*(15*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-30*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)-18*cos(f*x+e)^2
+15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)+34*cos(f*x+e)-8)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(3*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**3/(c**2*sqrt(-c*sec(e +
 f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
+ c**2*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(c**2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

3.86 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=142

$$\frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}}+\frac{32c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5af}+\frac{12c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af}+\frac{2c\tan(e+$$

```
[Out] 12/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^(5/2)*t
an(f*x+e)/f/(a+a*sec(f*x+e))+128/5*c^4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2
)+32/5*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f
```

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3954, 3793, 3792}

$$\frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}}+\frac{32c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5af}+\frac{12c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af}+\frac{2c\tan(e+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (128*c^4*Tan[e + f*x])/(5*a*f*Sqrt[c - c*Sec[e + f*x]]) + (32*c^3*Sqrt[c -
c*Sec[e + f*x]]*Tan[e + f*x])/(5*a*f) + (12*c^2*(c - c*Sec[e + f*x])^(3/2)*
Tan[e + f*x])/(5*a*f) + (2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(a
 + a*Sec[e + f*x]))
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
```

```
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx &= \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(6c)\int \sec(e+fx)(c-c\sec(e+f\\ &= \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+\\ &= \frac{32c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5af} + \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+\\ &= \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5af} + \frac{12c^2(c\end{aligned}$$

Mathematica [A] time = 0.74, size = 86, normalized size = 0.61

$$-\frac{c^3(245\cos(e+fx)+86\cos(2(e+fx))+91\cos(3(e+fx))+90)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx}}{10af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -1/10*(c^3*(90 + 245*Cos[e + f*x] + 86*Cos[2*(e + f*x)] + 91*Cos[3*(e + f*x
)])*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos
[e + f*x]))
```

fricas [A] time = 0.44, size = 88, normalized size = 0.62

$$-\frac{2\left(91\,c^3\cos\left(fx+e\right)^3+43\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+c^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\,af\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/5*(91*c^3*cos(f*x + e)^3 + 43*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) +
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)^2*sin(f*x +
e))
```

giac [A] time = 2.66, size = 112, normalized size = 0.79

$$-\frac{8\sqrt{2}c^3\left(\frac{5\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{a}-\frac{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^2+c^3}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}a}\right)}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -8/5*sqrt(2)*c^3*(5*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (15*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 + c^3)/((c*ta
n(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a))/f
```

maple [A] time = 1.76, size = 83, normalized size = 0.58

$$-\frac{2\left(91\left(\cos^3\left(fx+e\right)\right)+43\left(\cos^2\left(fx+e\right)\right)-7\cos\left(fx+e\right)+1\right)\cos\left(fx+e\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}{5af\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/5/a/f*(91*cos(f*x+e)^3+43*cos(f*x+e)^2-7*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1
+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)/(-1+cos(f*x+e))^3
```

maxima [A] time = 0.69, size = 163, normalized size = 1.15

$$\frac{8\left(16\sqrt{2}c^{\frac{7}{2}}-\frac{56\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{70\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{35\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{5\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}\right)}{5af\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 8/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8)/(a*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(7/2))
```

mupad [B] time = 6.33, size = 164, normalized size = 1.15

$$-\frac{2c^3\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(e^{e\,1i+f\,x\,1i}\,86i+e^{e\,2i+f\,x\,2i}\,245i+e^{e\,3i+f\,x\,3i}\,180i+e^{e\,4i+f\,x\,4i}\,245i+e^{e\,5i+f\,x\,5i}\,86i+e\right.}{5\,a\,f\left(e^{e\,2i+f\,x\,2i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(
e*1i + f*x*1i)*86i + exp(e*2i + f*x*2i)*245i + exp(e*3i + f*x*3i)*180i + ex
p(e*4i + f*x*4i)*245i + exp(e*5i + f*x*5i)*86i + exp(e*6i + f*x*6i)*91i + 9
1i))/(5*a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}$$

```
[Out] 2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))+32/3*c^3*tan(f*x+e
)/a/f/(c-c*sec(f*x+e))^(1/2)+8/3*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f
```

Rubi [A] time = 0.19, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3954, 3793, 3792}

$$\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (32*c^3*Tan[e + f*x])/(3*a*f*Sqrt[c - c*Sec[e + f*x]]) + (8*c^2*Sqrt[c - c*
Sec[e + f*x]]*Tan[e + f*x])/(3*a*f) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
```

```
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(4c)\int \sec(e+fx)(c-c\sec(e+f}{a}$$

$$= \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3af} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx}{f(a+a\sec(e+fx))}$$

$$= \frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}} + \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3af} + \frac{2c(c-c}{}$$

Mathematica [A] time = 0.44, size = 74, normalized size = 0.69

$$-\frac{c^2(20\cos(e+fx)+23\cos(2(e+fx))+21)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{3af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -1/3*(c^2*(21 + 20*Cos[e + f*x] + 23*Cos[2*(e + f*x)])*Cot[(e + f*x)/2]*Sec
[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos[e + f*x]))
```

fricas [A] time = 0.43, size = 77, normalized size = 0.71

$$-\frac{2\left(23\,c^2\cos\left(fx+e\right)^2+10\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/3*(23*c^2*cos(f*x + e)^2 + 10*c^2*cos(f*x + e) - c^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)*sin(f*x + e))
```

giac [A] time = 2.51, size = 87, normalized size = 0.81

$$-\frac{4\sqrt{2}c^{2}\left(\frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{a}-\frac{6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c+c^{2}}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}a}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -4/3*sqrt(2)*c^2*(3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (6*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a))/f
```

maple [A] time = 1.89, size = 73, normalized size = 0.68

$$-\frac{2\left(23\left(\cos^{2}\left(fx+e\right)\right)+10\cos\left(fx+e\right)-1\right)\cos\left(fx+e\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}{3af\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/3/a/f*(23*cos(f*x+e)^2+10*cos(f*x+e)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)/sin(f*x+e)/(-1+cos(f*x+e))^2
```

maxima [A] time = 0.64, size = 137, normalized size = 1.27

$$-\frac{4\left(8\sqrt{2}c^{\frac{5}{2}}-\frac{20\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{15\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{3\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{3af\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -4/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*sqrt(2)*
```

```
c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a*f*(sin(f*x + e)/(cos(f*x +
e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

mupad [B] time = 4.23, size = 125, normalized size = 1.16

$$\frac{2\,c^2\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)-44\,\sin\left(2\,e+2\,f\,x\right)+25\,\sin\left(3\,e+3\,f\,x\right)-26\,\sin\left(4\,e+4\,f\,x\right)+23\,\sin\left(\right.\right.}{3\,a\,f\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)+\cos\left(5\,e+5\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (2*c^2*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(2*sin(e + f*x) - 44*sin
(2*e + 2*f*x) + 25*sin(3*e + 3*f*x) - 26*sin(4*e + 4*f*x) + 23*sin(5*e + 5*
f*x)))/(3*a*f*(cos(3*e + 3*f*x) - 2*cos(e + f*x) - 2*cos(4*e + 4*f*x) + cos
(5*e + 5*f*x) + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int \frac{c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x
) + Integral(-2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x
) + 1), x) + Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3/(sec(e
 + f*x) + 1), x))/a
```

3.88 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

```
[Out] 4*c^2*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2)+2*c*(c-c*sec(f*x+e))^(1/2)*tan(
f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3792}

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c - c*Sec[e
 + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(2c)\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a}$$

$$= \frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [A] time = 0.24, size = 54, normalized size = 0.75

$$-\frac{2c(3\cos(e+fx)+1)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
[Out] (-2*c*(1 + 3*Cos[e + f*x])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a*f*
(1 + Cos[e + f*x]))
```

fricas [A] time = 0.43, size = 50, normalized size = 0.69

$$-\frac{2\left(3c\cos\left(fx+e\right)+c\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
[Out] -2*(3*c*cos(f*x + e) + c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*sin(
f*x + e))
```

giac [A] time = 2.73, size = 62, normalized size = 0.86

$$-\frac{2\sqrt{2}\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c}{a}-\frac{c^2}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*sqrt(2)*(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/a - c^2/(sqrt(c*tan(1/2*f*
x + 1/2*e)^2 - c)*a))/f
```

maple [A] time = 1.85, size = 63, normalized size = 0.88

$$-\frac{2\left(3\cos\left(fx+e\right)+1\right)\cos\left(fx+e\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{af\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/a/f*(3*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin
(f*x+e)/(-1+cos(f*x+e))
```

maxima [A] time = 0.86, size = 110, normalized size = 1.53

$$\frac{2\left(2\sqrt{2}c^{\frac{3}{2}}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{af\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a*f*(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

mupad [B] time = 2.37, size = 77, normalized size = 1.07

$$-\frac{c\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)+6\,\sin\left(2\,e+2\,f\,x\right)+2\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)}{a\,f\,\sin\left(2\,e+2\,f\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(c*(c - c/cos(e + f*x))^(1/2)*(2*sin(e + f*x) + 6*sin(2*e + 2*f*x) + 2*sin
(3*e + 3*f*x) + 3*sin(4*e + 4*f*x)))/(a*f*sin(2*e + 2*f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x) +
 Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x) + 1),
x))/a
```

3.89 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=39

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.14, size = 29, normalized size = 0.74

$$-\frac{2\cot(e+fx)\sqrt{c-c\sec(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*Cot[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f)
```

fricas [A] time = 0.44, size = 45, normalized size = 1.15

$$-\frac{2\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(a*f*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)-sqrt(2)*sqrt(c*tan(1/2*(f*x+exp(1)))^2-c)*sign(tan(1/2*(f*x+exp(1)
))^3+tan(1/2*(f*x+exp(1))))*sign(cos(f*x+exp(1)))/a/f
```

maple [A] time = 2.12, size = 43, normalized size = 1.10

$$-\frac{2\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)
```

maxima [B] time = 0.75, size = 84, normalized size = 2.15

$$-\frac{\sqrt{2}\sqrt{c}-\frac{\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}}{af\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -(sqrt(2)*sqrt(c) - sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/(a
*f*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e
) + 1) - 1))
```

mupad [B] time = 1.77, size = 40, normalized size = 1.03

$$-\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{a\,f\,{\sin\left(e+f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(sin(2*e + 2*f*x)*(c - c/cos(e + f*x))^(1/2))/(a*f*sin(e + f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x)/a
```

3.90 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}$$

```
[Out] -1/2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/f*2^(1
/2)/c^(1/2)+tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3960, 3795, 203}

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -(ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2
]*a*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
```

```
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{2a}$$

$$= \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, -\right.}{af}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+}}$$

Mathematica [C] time = 0.48, size = 155, normalized size = 1.74

$$-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(2\left(1+e^{2i(e+fx)}\right)-\sqrt{2}\left(1+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\right)}{2af\left(1+e^{2i(e+fx)}\right)^2(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((-1/2*I)*(-1 + E^((2*I)*(e + f*x)))*(2*(1 + E^((2*I)*(e + f*x))) - Sqrt[2]
*(1 + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e +
 f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]))/(a*(1 + E^((2*I)*(e + f*
x)))^2*f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.51, size = 269, normalized size = 3.02

$$\left[\frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-4\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{4\,acf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*s
in(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt
(2)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x
+ e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((i*atan(-i)-1)/2/a
/sqrt(-c)/sqrt(2)*sign(tan((f*x+exp(1))/2))+1/2*c*(sqrt(c*tan((f*x+exp(1))/
2)^2-c)/c-atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(c))/sqrt(2)/a/
c/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 2.16, size = 107, normalized size = 1.20

$$-\frac{\left(-1+\cos \left(f x+e\right)\right) \left(\sqrt{-\frac{2 \cos \left(f x+e\right)}{1+\cos \left(f x+e\right)}}+\arctan \left(\frac{1}{\sqrt{-\frac{2 \cos \left(f x+e\right)}{1+\cos \left(f x+e\right)}}}\right)\right)}{a f \sqrt{\frac{c \left(-1+\cos \left(f x+e\right)\right)}{\cos \left(f x+e\right)}}\, \sin \left(f x+e\right) \sqrt{-\frac{2 \cos \left(f x+e\right)}{1+\cos \left(f x+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/a/f*(-1+cos(f*x+e))*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+arctan(1/(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(
f*x+e)/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec
(e + f*x) + c)), x)/a
```

3.91 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}-\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -3/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/c^(3/2
)/f*2^(1/2)-3/4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(3/2)+tan(f*x+e)/f/(a+a*sec
(f*x+e))/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}-\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) - (3*Tan[e + f*x])/(4*a*f*(c - c*Sec[e + f*x])^(3/2)) +
 Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
```

```
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}}{2a}$$

$$= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+$$

$$= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f} - \frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{}{f(a}$$

Mathematica [C] time = 1.42, size = 183, normalized size = 1.50

$$\frac{e^{-\frac{1}{2}i(e+fx)}\csc(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8(\cos(e+fx)-3)+\frac{6\sqrt{2}e^{-i(e+fx)}\left(-1+e^{i(e+fx)}\right)^2\left(1+e^{i(e+fx}\right.}{\sqrt{1+e^{2i(e+fx}}}\right.}{32acf\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (((6*Sqrt[2]*(-1 + E^(I*(e + f*x)))^2*(1 + E^(I*(e + f*x)))*ArcTanh[(1 + E^
(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(E^(I*(e + f*x))*S
qrt[1 + E^((2*I)*(e + f*x))]) - 8*(-3 + Cos[e + f*x]))*Csc[e + f*x]*(Cos[(e
 + f*x)/2] + I*Sin[(e + f*x)/2]))/(32*a*c*E^((I/2)*(e + f*x))*f*Sqrt[c - c*
Sec[e + f*x]])
```

fricas [A] time = 0.51, size = 329, normalized size = 2.70

$$\left[-\frac{3\sqrt{2}\sqrt{-c}\left(\cos\left(fx+e\right)-1\right)\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right) - \cdots}{16\left(ac^{2}f\cos\left(fx+e\right)-ac^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) - 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/8*(3*sqrt(2)*sqr
t(c)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(cos(f*x + e)^2 -
 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^2*f*cos(f*x
 + e) - a*c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/4*(sqrt(c*tan((f*x+exp(1))/2)^2-c)-3/2*sqrt(c)*atan(sqrt(c*tan(
(f*x+exp(1))/2)^2-c)/sqrt(c))+1/2*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)/c/tan((
f*x+exp(1))/2)^2)/sqrt(2)/c^2/a/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp
(1))/2)^2-1)
```

maple [B] time = 2.14, size = 266, normalized size = 2.18

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\cos\left(fx+e\right)+\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{5}{2}}+\cos\left(fx+e\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}-\left(-\frac{2\cos(}{1+\cos(}\right.\right.}{2af\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/a/f*(-1+cos(f*x+e))^2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)+
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(3/2)-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-3*cos(f*x+e)*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)-3*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2))+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*arctan(1/(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqr
t(-c*sec(e + f*x) + c)), x)/a
```

3.92 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}-\frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}-\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c}$$

```
[Out] -15/64*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/c^(5
/2)/f*2^(1/2)-5/8*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(5/2)+tan(f*x+e)/f/(a+a*s
ec(f*x+e))/(c-c*sec(f*x+e))^(5/2)-15/32*tan(f*x+e)/a/c/f/(c-c*sec(f*x+e))^(
3/2)
```

Rubi [A] time = 0.26, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}-\frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}-\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-15*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) - (5*Tan[e + f*x])/(8*a*f*(c - c*Sec[e + f*x])^(5/2))
 + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) - (15*T
an[e + f*x])/(32*a*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} + \frac{5\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{5/2}}\,d}{2a}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+}$$

$$= -\frac{15\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f} - \frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{}{f(a+}$$

Mathematica [C] time = 2.93, size = 306, normalized size = 1.96

$$\frac{e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{7}{2}}(e+fx)\left(-\frac{1}{32}e^{-\frac{5}{2}i(e+fx)}\left(40e^{i(e+fx)}-51e^{2i(e+fx)}+80e^{3i(e+fx)}-51e^{4i(e}\right.\right.}{4ac^2f(\sec(e+fx)-}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e + f*x]^(7/2)*Sin[(e + f*x)/2]*(-1/32*((3 + 40*E^(I*
(e + f*x)) - 51*E^((2*I)*(e + f*x)) + 80*E^((3*I)*(e + f*x)) - 51*E^((4*I)*
(e + f*x)) + 40*E^((5*I)*(e + f*x)) + 3*E^((6*I)*(e + f*x)))*Sqrt[Sec[e + f
*x]])/E^(((5*I)/2)*(e + f*x)) - (15*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e +
 f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sin[(e + f*x)/2]^3*Sin[e + f*x])/2))/(4*
a*c^2*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])*Sqrt[c
 - c*Sec[e + f*x]])
```

fricas [A] time = 0.55, size = 401, normalized size = 2.57

$$\left[-\frac{15\,\sqrt{2}\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)+1\right)\sqrt{-c}\,\log\left(\frac{2\,\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3\,c\cos\left(fx+e\right)+c\right)\sin}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{128\left(ac^{3}f\cos\left(fx+e\right)^{2}-2\,ac^{3}f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/128*(15*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin
(f*x + e)))*sin(f*x + e) - 4*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^
2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/64*(15*sqrt(2)*(cos(
f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*
(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) +
a*c^3*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/8*(-15/8*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))+
1/8*(9*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+7*c^2*
sqrt(c*tan((f*x+exp(1))/2)^2-c))/(c*tan((f*x+exp(1))/2)^2)^2+sqrt(c*tan((f*
x+exp(1))/2)^2-c))/sqrt(2)/c^3/a/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+ex
p(1))/2)^2-1)
```

maple [B] time = 2.32, size = 471, normalized size = 3.02

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{3}\left(5\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\left(\cos^{2}\left(fx+e\right)\right)+4\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\cos\left(fx+e\right)-\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}+3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/a/f*(-1+cos(f*x+e))^3*(5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+
e)^2+4*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)-(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(7/2)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)^2-6*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)+3*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(5/2)-5*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+10*cos(f*x+e)*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+
15*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+15*cos(f*x+e)^2*arctan
(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-30*cos(f*x+e)*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)-30*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
))+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+15*arctan(1/(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(5/2)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec{\left(e+fx \right)}}{c^{2} \sqrt{-c \sec{\left(e+fx \right)}+c} \sec^{3}{\left(e+fx \right)} - c^{2} \sqrt{-c \sec{\left(e+fx \right)}+c} \sec^{2}{\left(e+fx \right)} - c^{2} \sqrt{-c \sec{\left(e+fx \right)}+c} \sec{\left(e+fx \right)} + c^{2} \sqrt{-c \sec{\left(e+fx \right)}+c}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - c**2*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x)/a
```

3.93 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3a^2f}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+\ldots}{3f(\ldots}$$

```
[Out] -4*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*c*(c-c*
sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-64/3*c^4*tan(f*x+e)/a^2/f
/(c-c*sec(f*x+e))^(1/2)-16/3*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a^2/f
```

Rubi [A] time = 0.32, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3954, 3793, 3792}

$$-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3a^2f}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+\ldots}{3f(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-64*c^4*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (16*c^3*Sqrt[c
- c*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f) - (4*c^2*(c - c*Sec[e + f*x])^(3/
2)*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^(
5/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
```

```
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx}{a}$$

$$= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{16c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3a^2f} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e}{f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= -\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{16c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3a^2f} - 4c^2$$

Mathematica [A] time = 0.94, size = 84, normalized size = 0.54

$$\frac{c^3(195\cos(e+fx)+138\cos(2(e+fx))+45\cos(3(e+fx))+134)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+f}}{6a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (c^3*(134 + 195*Cos[e + f*x] + 138*Cos[2*(e + f*x)] + 45*Cos[3*(e + f*x)])*
Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(6*a^2*f*(1 + Cos[e
 + f*x])^2)
```

fricas [A] time = 0.43, size = 103, normalized size = 0.66

$$\frac{2\left(45\,c^3\cos\left(fx+e\right)^3+69\,c^3\cos\left(fx+e\right)^2+15\,c^3\cos\left(fx+e\right)-c^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^2f\cos\left(fx+e\right)^2+a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(45*c^3*cos(f*x + e)^3 + 69*c^3*cos(f*x + e)^2 + 15*c^3*cos(f*x + e) -
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e)^2 + a^2*f
*cos(f*x + e))*sin(f*x + e))
```

giac [A] time = 3.38, size = 125, normalized size = 0.81

$$-\frac{4\sqrt{2}\,c^3\left(\frac{9\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+c^2}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^2}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^4c^2+9\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^4c^3}{a^6c^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -4/3*sqrt(2)*c^3*((9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f*
x + 1/2*e)^2 - c)^(3/2)*a^2) - ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c^
2 + 9*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^4*c^3)/(a^6*c^3))/f
```

maple [A] time = 1.63, size = 85, normalized size = 0.55

$$-\frac{2\left(3\cos\left(fx+e\right)+1\right)\left(15\left(\cos^2\left(fx+e\right)\right)+18\cos\left(fx+e\right)-1\right)\left(\cos^2\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}{3a^2f\sin\left(fx+e\right)^3\left(-1+\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/a^2/f*(3*cos(f*x+e)+1)*(15*cos(f*x+e)^2+18*cos(f*x+e)-1)*cos(f*x+e)^2*
(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)^3/(-1+cos(f*x+e))^2
```

maxima [A] time = 0.50, size = 188, normalized size = 1.21

$$-\frac{4\left(16\sqrt{2}\,c^{\frac{7}{2}}-\frac{56\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{70\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{35\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{4\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}+\frac{\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}\right)}{3\,a^2f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -4/3*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2
)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*sqrt(2)*c^(7/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 + sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(7/2))
```

mupad [B] time = 6.02, size = 188, normalized size = 1.21

$$\frac{2\,c^3\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,138\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,195\mathrm{i}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,268\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,195\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,138\mathrm{i}+\right.}{3\,a^2\,f\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e
*1i + f*x*1i)*138i + exp(e*2i + f*x*2i)*195i + exp(e*3i + f*x*3i)*268i + ex
p(e*4i + f*x*4i)*195i + exp(e*5i + f*x*5i)*138i + exp(e*6i + f*x*6i)*45i +
45i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1)^3*(exp(e*1i + f*x*1i) - exp(e*2i +
f*x*2i) + exp(e*3i + f*x*3i) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.94 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] 2/3*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-16/3*c^3*tan(f
*x+e)/a^2/f/(c-c*sec(f*x+e))^(1/2)-8/3*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e
)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] time = 0.27, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3792}

$$-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-16*c^3*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (8*c^2*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c
*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(4c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c}{}$$

Mathematica [A] time = 0.39, size = 68, normalized size = 0.55

$$\frac{c^2(36\cos(e+fx)+11\cos(2(e+fx))+17)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{3a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (c^2*(17 + 36*Cos[e + f*x] + 11*Cos[2*(e + f*x)])*Cot[(e + f*x)/2]*Sqrt[c -
 c*Sec[e + f*x]])/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

fricas [A] time = 0.45, size = 82, normalized size = 0.67

$$\frac{2\left(11\,c^2\cos\left(fx+e\right)^2+18\,c^2\cos\left(fx+e\right)+3\,c^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^2f\cos\left(fx+e\right)+a^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(11*c^2*cos(f*x + e)^2 + 18*c^2*cos(f*x + e) + 3*c^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

giac [A] time = 2.77, size = 102, normalized size = 0.83

$$-\frac{2\sqrt{2}\,c^2\left(\frac{3\,c}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^2} - \frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^4c^2+6\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^4c^3}{a^6c^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -2/3*sqrt(2)*c^2*(3*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2) - ((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c^2 + 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*
a^4*c^3)/(a^6*c^3))/f
```

maple [A] time = 1.67, size = 75, normalized size = 0.61

$$-\frac{2\left(11\left(\cos^2\left(fx+e\right)\right)+18\cos\left(fx+e\right)+3\right)\left(\cos^2\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}{3a^2f\sin\left(fx+e\right)^3\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/a^2/f*(11*cos(f*x+e)^2+18*cos(f*x+e)+3)*cos(f*x+e)^2*(c*(-1+cos(f*x+e)
)/cos(f*x+e))^(5/2)/sin(f*x+e)^3/(-1+cos(f*x+e))
```

maxima [A] time = 0.62, size = 163, normalized size = 1.33

$$\frac{2\left(8\sqrt{2}c^{\frac{5}{2}}-\frac{20\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{15\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{2\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}\right)}{3a^2f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*sqrt(2)*c
^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - sqrt(2)*c^(5/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

mupad [B] time = 5.54, size = 136, normalized size = 1.11

$$\frac{2\,c^2\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,36{}\mathrm{i}+{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,34{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,36{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,11{}\mathrm{i}+11{}\mathrm{i}\right)}{3\,a^2\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e
*1i + f*x*1i)*36i + exp(e*2i + f*x*2i)*34i + exp(e*3i + f*x*3i)*36i + exp(e
*4i + f*x*4i)*11i + 11i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x
*1i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4/3*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)+2/3*c*(c-
c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.22, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3953}

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]
]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx}{3a}$$
$$= -\frac{4c^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}}{3f(a+a\sec(e+$$

Mathematica [A] time = 0.26, size = 60, normalized size = 0.67

$$\frac{2c\cos(e+fx)(\cos(e+fx)+3)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{3a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (2*c*Cos[e + f*x]*(3 + Cos[e + f*x])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*
x]])/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

fricas [A] time = 0.45, size = 72, normalized size = 0.81

$$\frac{2\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^2f\cos\left(fx+e\right)+a^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

giac [A] time = 3.68, size = 62, normalized size = 0.70

$$\frac{\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}}{a^2}+\frac{3\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c}{a^2}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)/a^2 + 3*sqrt(2)*sqrt(c*ta
n(1/2*f*x + 1/2*e)^2 - c)*c/a^2)/f
```

maple [A] time = 1.66, size = 53, normalized size = 0.60

$$-\frac{2\left(\cos\left(fx+e\right)+3\right)\left(\cos^{2}\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{3a^{2}f\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/a^2/f*(cos(f*x+e)+3)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/sin(f*x+e)^3
```

maxima [A] time = 0.67, size = 110, normalized size = 1.24

$$-\frac{2\sqrt{2}c^{\frac{3}{2}}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}}{3a^{2}f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + sqrt(2)*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^2*f*(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(
3/2))
```

mupad [B] time = 5.23, size = 134, normalized size = 1.51

$$\frac{2c\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{3a^{2}f\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e*1
i + f*x*1i)*6i + exp(e*2i + f*x*2i)*2i + exp(e*3i + f*x*3i)*6i + exp(e*4i +
 f*x*4i)*1i + 1i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) +
1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec
(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/
(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.96 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2/3*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx=\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 1.34

$$-\frac{\cos^2(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{6a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/6*(Cos[e + f*x]^2*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e +
 f*x]])/(a^2*f)
```

fricas [A] time = 0.50, size = 60, normalized size = 1.46

$$-\frac{2\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -2/3*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/((a^2*f*cos(f*x
 + e) + a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/6*sqrt(2)*sqrt(c*tan(1/2*(f*x+exp(1)))^2-c)*(c*tan(1/2*(f*x+exp(1
)))^2-c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*sign(cos(f*x+e
xp(1)))/a^2/c/f
```

maple [A] time = 1.85, size = 53, normalized size = 1.29

$$\frac{2\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\cos^{2}\left(fx+e\right)\right)}{3a^{2}f\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 2/3/a^2/f*(-1+cos(f*x+e))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2
/sin(f*x+e)^3
```

maxima [B] time = 0.72, size = 109, normalized size = 2.66

$$-\frac{\sqrt{2}\sqrt{c} - \frac{2\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} + \frac{\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}{6\,a^2 f\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/6*(sqrt(2)*sqrt(c) - 2*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a^2*f*sqrt(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

mupad [B] time = 5.30, size = 94, normalized size = 2.29

$$\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^2\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] ((exp(e*2i + f*x*2i)*1i + 1i)^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2)*2i)/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x)/a**2
```

3.97 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{2f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/f*2^
(1/2)/c^(1/2)+1/3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/
2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3960, 3795, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{2f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec
[e + f*x]]) + Tan[e + f*x]/(2*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e +
 f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}}{2a}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(}{2f\left(a^2+a^2\sec(e+\right.}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(}{2f\left(a^2+a^2\sec(e+\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+}}$$

Mathematica [C] time = 2.08, size = 259, normalized size = 1.88

$$\frac{2e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{5}{2}}(e+fx)\left(\frac{1}{8}e^{-\frac{3}{2}i(e+fx)}\left(6e^{i(e+fx)}+10e^{2i(e+fx)}+6e^{3i(e+fx)}+5e^{4i(e+fx)}\right.\right.}{3a^2f(\sec(e+fx)+1)^2\sqrt{c-c\sec}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-3*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqr
t[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 +
E^((2*I)*(e + f*x))])]*Cos[(e + f*x)/2]^3 + ((5 + 6*E^(I*(e + f*x)) + 10*E^
((2*I)*(e + f*x)) + 6*E^((3*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x)))*Sqrt[Sec
[e + f*x]])/(8*E^(((3*I)/2)*(e + f*x)))))*Sec[e + f*x]^(5/2)*Sin[(e + f*x)/2
])/(3*a^2*E^((I/2)*(e + f*x))*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]])
```

fricas [A] time = 0.53, size = 331, normalized size = 2.40

$$\left[-\frac{3\sqrt{2}\sqrt{-c}\left(\cos\left(fx+e\right)+1\right)\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)}{24\left(a^{2}cf\cos\left(fx+e\right)+a^{2}cf\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) + 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(5*cos(f*x + e)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*
sqrt(c)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e
)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c*f*co
s(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((3*i*atan(-i)-4)/1
2/a^2/sqrt(-c)/sqrt(2)*sign(tan((f*x+exp(1))/2))+1/4*((-1/3*c^4*sqrt(c*tan(
(f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+c^5*sqrt(c*tan((f*x+exp(1)
)/2)^2-c))/c^6-atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(c))/sqrt(
2)/a^2/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 1.82, size = 131, normalized size = 0.95

$$\frac{\left(-1+\cos\left(fx+e\right)\right)\left(\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}-3\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}-3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)\right)}{6a^{2}f\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/6/a^2/f*(-1+cos(f*x+e))*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-3*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)-3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^{2}\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*sqrt(-c*sec(f*x + e) + c)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{2}\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)+2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + 2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

3.98 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{5\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{3f(a\sec}$$

```
[Out] -5/16*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/c^(
3/2)/f*2^(1/2)-5/8*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(3/2)+1/3*tan(f*x+e)/f
/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2)+5/6*tan(f*x+e)/f/(a^2+a^2*sec(f*
x+e))/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.34, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{5\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{3f(a\sec}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-5*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) - (5*Tan[e + f*x])/(8*a^2*f*(c - c*Sec[e + f*x])^(3/2
)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) +
 (5*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\int \frac{\sec}{(a+a\sec(e+fx)}}{}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{}{6f\left(a^2+a^2\sec\right.}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f} - \frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{}{3f}$$

Mathematica [C] time = 1.39, size = 365, normalized size = 2.16

$$-\frac{\frac{15i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^3\left(1+e^{i(e+fx)}\right)^4\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{7/2}} - 416\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^8(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\csc^4(2(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),
x]
```

```
[Out] (((-15*I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^3*(1 + E^(I*(e + f*x)))^4*ArcTanh[
(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*
I)*(e + f*x)))^(7/2) - 48*Cos[(e + f*x)/2]^4*Csc[e/2]*Sec[e + f*x]^4*Sin[(f
*x)/2]*Sin[(e + f*x)/2] + 48*Cos[(e + f*x)/2]^4*Cot[e/2]*Sec[e + f*x]^4*Sin
[(e + f*x)/2]^2 + 32*Cos[(e + f*x)/2]*Sec[e + f*x]^4*Sin[(e + f*x)/2]^3 + 4
16*Cos[e/2]*Cos[(f*x)/2]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^4*Sin[(e + f*x)/2]
^3 - 416*Csc[(e + f*x)/2]*Csc[2*(e + f*x)]^4*Sin[e/2]*Sin[(f*x)/2]*Sin[e +
f*x]^8 - 40*Sec[e + f*x]*Tan[e + f*x]^3)/(48*a^2*c*f*(-1 + Sec[e + f*x])*(1
 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.53, size = 369, normalized size = 2.18

$$\left[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right.}{96\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/96*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x +
e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*
c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*
x + e) + 4*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*
sin(f*x + e)), 1/48*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(c)*arctan(sqrt(2)
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)
))*sin(f*x + e) - 2*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^
2*c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/8*(-5/2*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))+(
-1/3*c^2*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+2*c^3*
sqrt(c*tan((f*x+exp(1))/2)^2-c))/c^3+1/2*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)/
c/tan((f*x+exp(1))/2)^2)/sqrt(2)/a^2/c^2/sign(tan((f*x+exp(1))/2))/sign(tan
((f*x+exp(1))/2)^2-1)
```

maple [B] time = 1.99, size = 320, normalized size = 1.89

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(3\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\cos\left(fx+e\right)+3\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}+3\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\cos\left(fx+e\right)-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/12/a^2/f*(-1+cos(f*x+e))^2*(3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f
*x+e)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)
))^(5/2)*cos(f*x+e)-3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-5*cos(f*x+e)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+15
*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+15*cos(f*x+e)*arctan(1/(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-
15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f
*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^{2}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(-c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c*sqr
t(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqrt(-c*sec(e + f*x) + c)*sec(e
+ f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

3.99 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{35\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{7\tan(\ldots}{6f\left(a^2\sec(e+fx)+a\ldots\right.}$$

```
[Out] -35/128*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/c
^(5/2)/f*2^(1/2)-35/48*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(5/2)+1/3*tan(f*x+
e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+7/6*tan(f*x+e)/f/(a^2+a^2*se
c(f*x+e))/(c-c*sec(f*x+e))^(5/2)-35/64*tan(f*x+e)/a^2/c/f/(c-c*sec(f*x+e))^
(3/2)
```

Rubi [A] time = 0.39, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{35\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{7\tan(\ldots}{6f\left(a^2\sec(e+fx)+a\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-35*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) - (35*Tan[e + f*x])/(48*a^2*f*(c - c*Sec[e + f*x])^
(5/2)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2
)) + (7*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5
/2)) - (35*Tan[e + f*x])/(64*a^2*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{7\int \frac{\sec(e}{(a+a\sec(e+fx))(}}{6}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{7}{6f\left(a^2+a^2\sec(e\right.}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec}$$

$$= -\frac{35\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f} - \frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{}{3}$$

Mathematica [C] time = 2.33, size = 434, normalized size = 2.14

$$\frac{\cot^4(e+fx)\left(\frac{105i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^5\left(1+e^{i(e+fx)}\right)^4\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{9/2}}-3648\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^9(e+fx)\csc\left(\frac{1}{2}(e+fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (Cot[e + f*x]^4*(((105*I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^5*(1 + E^(I*(e + f
*x)))^4*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))
])])/(1 + E^((2*I)*(e + f*x)))^(9/2) + 192*Cos[(e + f*x)/2]^4*Csc[e/2]*Sec[
e + f*x]^5*Sin[(f*x)/2]*Sin[(e + f*x)/2] - 192*Cos[(e + f*x)/2]^4*Cot[e/2]*
Sec[e + f*x]^5*Sin[(e + f*x)/2]^2 + 256*Cos[(e + f*x)/2]*Sec[e + f*x]^5*Sin
[(e + f*x)/2]^5 + 2752*Cos[e/2]*Cos[(f*x)/2]*Cos[(e + f*x)/2]^4*Sec[e + f*x
]^5*Sin[(e + f*x)/2]^5 - 13312*Csc[2*(e + f*x)]^5*Sin[(e + f*x)/2]^2*Sin[e
+ f*x]^8 - 3648*Csc[e/2]*Csc[(e + f*x)/2]*Csc[2*(e + f*x)]^5*Sin[(f*x)/2]*S
in[e + f*x]^9 - 5504*Csc[2*(e + f*x)]^5*Sin[e/2]*Sin[(f*x)/2]*Sin[(e + f*x)
/2]*Sin[e + f*x]^9 + 114*Cot[e/2]*Sec[e + f*x]*Tan[e + f*x]^4))/(384*a^2*c^
2*f*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.57, size = 487, normalized size = 2.40

$$\left[-\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^2-\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(\right.}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{768\left(a^2c^3f\cos\left(fx+e\right)^3-a^2c^3f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/768*(105*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(43*cos(f*x + e)^4 - 161*cos(f
```

```
*x + e)^3 - 35*cos(f*x + e)^2 + 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2
*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/384*(105*sqrt(2)*(cos(f*x
 + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f
*x + e) - 2*(43*cos(f*x + e)^4 - 161*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 1
05*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f
*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*
sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/16*(-35/8*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))
+1/8*(13*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+11*c
^2*sqrt(c*tan((f*x+exp(1))/2)^2-c))/(c*tan((f*x+exp(1))/2)^2)^2+(-1/3*c^2*s
qrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+3*c^3*sqrt(c*tan
((f*x+exp(1))/2)^2-c))/c^3)/sqrt(2)/a^2/c^3/sign(tan((f*x+exp(1))/2))/sign(
tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 2.03, size = 551, normalized size = 2.71

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{3}\left(21\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{9}{2}}\left(\cos^{2}\left(fx+e\right)\right)+12\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{9}{2}}\cos\left(fx+e\right)+15\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/48/a^2/f*(-1+cos(f*x+e))^3*(21*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*cos(f
*x+e)^2+12*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*cos(f*x+e)+15*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)^2-9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)
-30*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)-21*(-2*cos(f*x+e)/(1+co
```

```
s(f*x+e)))^(5/2)*cos(f*x+e)^2+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+42*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)+35*cos(f*x+e)^2*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(3/2)-21*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-70*cos(f*x+
e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-105*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)-105*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2))+35*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+210*cos(f*x+e)*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)+210*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))-105*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-105*arctan(1/(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+
e)^5/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^{2}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(5/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{4}\left(e+fx\right)-2c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)+c^{2}\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c
**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c**2*sqrt(-c*sec(e + f*x) +
 c)), x)/a**2
```

3.100 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=169

$$\frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}+\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+ }{5f(}$$

```
[Out] -4/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+2/5*c*(c-
c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+32/5*c^4*tan(f*x+e)/a^3
/f/(c-c*sec(f*x+e))^(1/2)+16/5*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a^3
+a^3*sec(f*x+e))
```

Rubi [A] time = 0.40, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3792}

$$\frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}+\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+ }{5f(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (32*c^4*Tan[e + f*x])/(5*a^3*f*Sqrt[c - c*Sec[e + f*x]]) + (16*c^3*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) - (4*c^2*(c -
 c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*
(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(6c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

$$= \frac{16c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+}{5af(a+a\sec(e+fx))^2}$$

$$= \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}} + \frac{16c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{4c^2(c}{}$$

Mathematica [A] time = 0.63, size = 78, normalized size = 0.46

$$-\frac{c^3(249\cos(e+fx)+110\cos(2(e+fx))+23\cos(3(e+fx))+130)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{10a^3f(\cos(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] -1/10*(c^3*(130 + 249*Cos[e + f*x] + 110*Cos[2*(e + f*x)] + 23*Cos[3*(e + f
*x)])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a^3*f*(1 + Cos[e + f*x])^
3)
```

fricas [A] time = 0.43, size = 109, normalized size = 0.64

$$-\frac{2\left(23\,c^3\cos\left(fx+e\right)^3+55\,c^3\cos\left(fx+e\right)^2+45\,c^3\cos\left(fx+e\right)+5\,c^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*(23*c^3*cos(f*x + e)^3 + 55*c^3*cos(f*x + e)^2 + 45*c^3*cos(f*x + e) +
 5*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*
a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

giac [A] time = 4.47, size = 130, normalized size = 0.77

$$\frac{2\,\sqrt{2}\,c^3\left(\frac{5\,c}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^3}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}a^{12}c^8+5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^{12}c^9+15\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^{12}c^{10}}{a^{15}c^{10}}\right)}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 2/5*sqrt(2)*c^3*(5*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^3) - ((c*tan(1/2
*f*x + 1/2*e)^2 - c)^(5/2)*a^12*c^8 + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2
)*a^12*c^9 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^12*c^10)/(a^15*c^10))/
f
```

maple [A] time = 1.81, size = 85, normalized size = 0.50

$$-\frac{2\left(23\left(\cos^3\left(fx+e\right)\right)+55\left(\cos^2\left(fx+e\right)\right)+45\cos\left(fx+e\right)+5\right)\left(\cos^3\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}{5a^3f\sin\left(fx+e\right)^5\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/5/a^3/f*(23*cos(f*x+e)^3+55*cos(f*x+e)^2+45*cos(f*x+e)+5)*cos(f*x+e)^3*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)^5/(-1+cos(f*x+e))
```

maxima [A] time = 0.44, size = 214, normalized size = 1.27

$$\frac{2\left(16\,\sqrt{2}\,c^{\frac{7}{2}}-\frac{56\,\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{70\,\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{35\,\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{5\,\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}-\frac{\sqrt{2}\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}+}{5\,a^3f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 2/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 + sqrt(2)*c^(7/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/(a^3*f*(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(cos(f*x + e) + 1) -
1)^(7/2))
```

mupad [B] time = 10.24, size = 492, normalized size = 2.91

$$-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{c^3\,46i}{5\,a^3\,f}+\frac{c^3\,e^{e\,1i+f\,x\,1i}\,4i}{a^3\,f}+\frac{c^3\,e^{e\,2i+f\,x\,2i}\,46i}{5\,a^3\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,\left(e^{e\,1i+f\,x\,1i}+1\right)}-\frac{c^3\,\left(e^{e\,2i+f\,x\,2i}+1\right)\,\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,16i}{5\,a^3\,f\,\left(e^{e\,1i+f\,x\,1i}-1\right)\,\left(e^{e\,1i+f\,x\,1i}+1\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2)*128i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^4) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*
1i + f*x*1i) + 1)^2) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f
*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*48i)/(5*a^3*f*(exp(e*1i + f*x*1i) -
 1)*(exp(e*1i + f*x*1i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*((c^3*46i)/(5*a^3*f) + (c^3*exp(e*1i + f*x*1i)*4i)/(a
^3*f) + (c^3*exp(e*2i + f*x*2i)*46i)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1)*
(exp(e*1i + f*x*1i) + 1)) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i + f*x*
1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=135

$$\frac{16c^3\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+$$

```
[Out] 2/5*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+16/15*c^3*tan(
f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)-8/15*c^2*(c-c*sec(f*x+
e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.35, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3953}

$$\frac{16c^3\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (16*c^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]]) - (8*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(15*a*f*(a + a*Sec[e +
f*x])^2) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e
+ f*x])^3)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
```

```
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(4c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+f}{5f(a+a\sec(e+fx))^3}$$

$$= \frac{16c^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\mathrm{t}}{15af(a+a\sec(e+}$$

Mathematica [A] time = 0.35, size = 74, normalized size = 0.55

$$-\frac{c^2\cos(e+fx)(20\cos(e+fx)+7\cos(2(e+fx))+37)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{15a^3f(\cos(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] -1/15*(c^2*Cos[e + f*x]*(37 + 20*Cos[e + f*x] + 7*Cos[2*(e + f*x)])*Cot[(e
+ f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a^3*f*(1 + Cos[e + f*x])^3)
```

fricas [A] time = 0.45, size = 104, normalized size = 0.77

$$-\frac{2\left(7c^2\cos\left(fx+e\right)^3+10c^2\cos\left(fx+e\right)^2+15c^2\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(a^3f\cos\left(fx+e\right)^2+2a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(7*c^2*cos(f*x + e)^3 + 10*c^2*cos(f*x + e)^2 + 15*c^2*cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*co
s(f*x + e) + a^3*f)*sin(f*x + e))
```

giac [A] time = 3.15, size = 93, normalized size = 0.69

$$-\frac{\frac{15\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,c^{2}}{a^{3}}+\frac{3\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{5}{2}}+10\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}c}{a^{3}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/15*(15*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2/a^3 + (3*sqrt(2)*(
c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 10*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2
- c)^(3/2)*c)/a^3)/f
```

maple [A] time = 1.82, size = 65, normalized size = 0.48

$$-\frac{2\left(7\left(\cos^{2}\left(fx+e\right)\right)+10\cos\left(fx+e\right)+15\right)\left(\cos^{3}\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}{15a^{3}f\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/15/a^3/f*(7*cos(f*x+e)^2+10*cos(f*x+e)+15)*cos(f*x+e)^3*(c*(-1+cos(f*x+e
))/cos(f*x+e))^(5/2)/sin(f*x+e)^5
```

maxima [A] time = 0.44, size = 189, normalized size = 1.40

$$-\frac{8\sqrt{2}c^{\frac{5}{2}}-\frac{20\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{15\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{5\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{5\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-\frac{3\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}}{15a^{3}f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/15*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sqrt(2)
*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(5/2)*sin(f*x +
```

```
e)^8/(cos(f*x + e) + 1)^8 - 3*sqrt(2)*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)^(5/2))
```

mupad [B] time = 7.31, size = 456, normalized size = 3.38

$$-\frac{c^2\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,14\mathrm{i}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}+\frac{c^2\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^2}-\frac{c^2\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+\dots}{15\,a^3\,f\,\left({\mathrm{e}}^{e}\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2)*16i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^2) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*14i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e
*1i + f*x*1i) + 1)) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*
x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*112i)/(15*a^3*f*(exp(e*1i + f*x*1i)
- 1)*(exp(e*1i + f*x*1i) + 1)^3) + (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i
 + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c^2*(exp(e*2i + f*x*2i) + 1)
*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*
f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -4/15*c^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+2/5*c*(c
-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A] time = 0.22, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3953}

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])
^3)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx}{5a}$$
$$= -\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\,\mathrm{ta}}{5f(a+a\sec(e+}$$

Mathematica [A] time = 0.32, size = 60, normalized size = 0.68

$$-\frac{2c(\cos(e+fx)-5)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{15a^3f(\sec(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] (-2*c*(-5 + Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e +
f*x]])/(15*a^3*f*(1 + Sec[e + f*x])^3)
```

fricas [A] time = 0.45, size = 88, normalized size = 1.00

$$-\frac{2\left(c\cos\left(fx+e\right)^3-5\,c\cos\left(fx+e\right)^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(c*cos(f*x + e)^3 - 5*c*cos(f*x + e)^2)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x +
 e))
```

giac [A] time = 3.49, size = 60, normalized size = 0.68

$$-\frac{\sqrt{2}\left(3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}c\right)}{30\,a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/30*sqrt(2)*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*(c*tan(1/2*f*x +
1/2*e)^2 - c)^(3/2)*c)/(a^3*c*f)
```

maple [A] time = 1.82, size = 63, normalized size = 0.72

$$-\frac{2\left(5+\cos^{2}\left(fx+e\right)-6\cos\left(fx+e\right)\right)\left(\cos^{3}\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{15a^{3}f\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/15/a^3/f*(5+cos(f*x+e)^2-6*cos(f*x+e))*cos(f*x+e)^3*(c*(-1+cos(f*x+e))/c
os(f*x+e))^(3/2)/sin(f*x+e)^5
```

maxima [B] time = 0.44, size = 163, normalized size = 1.85

$$-\frac{2\sqrt{2}\,c^{\frac{3}{2}}-\frac{3\sqrt{2}\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{3\sqrt{2}\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{7\sqrt{2}\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{3\sqrt{2}\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}}{30\,a^{3}f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/30*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*sqrt(2)*c
^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

mupad [B] time = 7.72, size = 446, normalized size = 5.07

$$-\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{15\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,28\mathrm{i}}{15\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^{2}}-\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right.}{15\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*28i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i)
+ 1)^2) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^(1/2)*2i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i +
 f*x*1i) + 1)) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2
 + exp(e*1i + f*x*1i)/2))^(1/2)*76i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(ex
p(e*1i + f*x*1i) + 1)^3) + (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i -
 f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*f*(exp(e*1i + f*x*1i)
 - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i
+ f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) +
 1), x))/a**3
```

3.103 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 1.34

$$-\frac{\cos^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{20a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -1/20*(Cos[e + f*x]^3*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^5*Sqrt[c - c*Sec[e
+ f*x]])/(a^3*f)
```

fricas [A] time = 0.45, size = 74, normalized size = 1.80

$$-\frac{2\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^3/((a^3*f*cos(f*x
 + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)-1/20*sqrt(2)*sqrt(c*tan(1/2*(f*x+exp(1)))^2-c)*(c*tan(1/2*(f*x+exp
(1)))^2-c)^2*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*sign(cos(f
*x+exp(1)))/a^3/c^2/f
```

maple [A] time = 1.91, size = 55, normalized size = 1.34

$$-\frac{2\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\cos^{3}\left(fx+e\right)\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}}{5a^{3}f\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/5/a^3/f*(-1+cos(f*x+e))^2*cos(f*x+e)^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1
/2)/sin(f*x+e)^5
```

maxima [B] time = 0.44, size = 136, normalized size = 3.32

$$-\frac{\sqrt{2}\sqrt{c} - \frac{3\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + \frac{3\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}} - \frac{\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}}{20\,a^{3}f\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/20*(sqrt(2)*sqrt(c) - 3*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - sqrt(2)*sqrt(
c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^3*f*sqrt(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

mupad [B] time = 7.59, size = 441, normalized size = 10.76

$$-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{5\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}+\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^{2}}-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\cdots}}{5\,a^{3}\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)
^2) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2)*2i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f
*x*1i) + 1)^3) + ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
 exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e
*1i + f*x*1i) + 1)^4) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*
1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*
(exp(e*1i + f*x*1i) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x)/a**3
```

3.104 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}+\frac{\tan(e+fx)}{4f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{6af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+}$$

```
[Out] -1/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/f*2^
(1/2)/c^(1/2)+1/5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)+1/
6*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+e)/f
/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3960, 3795, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}+\frac{\tan(e+fx)}{4f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{6af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec
[e + f*x]]) + Tan[e + f*x]/(6*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e +
 f*x]]) + Tan[e + f*x]/(4*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\,\mathrm{s}}}}{2a}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(}{6af(a+a\sec(e+f}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(}{6af(a+a\sec(e+f}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(}{6af(a+a\sec(e+f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+}}$$

Mathematica [C] time = 1.53, size = 225, normalized size = 1.24

$$\frac{2e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{7}{2}}(e+fx)\left(\frac{e^{\frac{1}{2}i(e+fx)}(80\cos(e+fx)+37\cos(2(e+fx))+67)}{8\sqrt{\sec(e+fx)}} - 15\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+}\right.}{15a^3f(\sec(e+fx)+1)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-15*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sq
rt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 +
 E^((2*I)*(e + f*x))])]*Cos[(e + f*x)/2]^5 + (E^((I/2)*(e + f*x))*(67 + 80*
```

```
Cos[e + f*x] + 37*Cos[2*(e + f*x)]))/(8*Sqrt[Sec[e + f*x]]))*Sec[e + f*x]^(
7/2)*Sin[(e + f*x)/2])/(15*a^3*E^((I/2)*(e + f*x))*f*(1 + Sec[e + f*x])^3*S
qrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.54, size = 401, normalized size = 2.22

$$\left[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)}{240\left(a^3cf\cos\left(fx+e\right)^2+2a^3cf\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/240*(15*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin
(f*x + e)))*sin(f*x + e) + 4*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)
^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/120*(15*sqrt(2)*(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) -
2*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e)
 + a^3*c*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((15*i*atan(-i)-23)
/120/a^3/sqrt(-c)/sqrt(2)*sign(tan((f*x+exp(1))/2))+1/8*((-1/3*c^13*sqrt(c*
tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+1/5*c^12*sqrt(c*tan((f
```

```
*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)^2+c^14*sqrt(c*tan((f*x+exp(1
))/2)^2-c))/c^15-atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))/sqrt(c))/sqr
t(2)/a^3/sign(tan((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^2-1))
```

maple [A] time = 2.05, size = 155, normalized size = 0.86

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(3\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{5}{2}}-5\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)+15\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\right)}{60a^{3}f\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\ \sin\left(fx+e\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/60/a^3/f*(-1+cos(f*x+e))*(3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-5*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(3/2)+15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2))+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))/(c*(-1+cos(f*x+e))/cos(f*x+
e))^(1/2)/sin(f*x+e)/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^{3}\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*sqrt(-c*sec(f*x + e) + c)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^3\left(e+fx\right)+3\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)+3\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)+\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**2 + 3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

3.105 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3\sec(e+fx)+a^3\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{30af(a}$$

```
[Out] -7/32*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/c^(
3/2)/f*2^(1/2)-7/16*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(3/2)+1/5*tan(f*x+e)/
f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2)+7/30*tan(f*x+e)/a/f/(a+a*sec(f*
x+e))^2/(c-c*sec(f*x+e))^(3/2)+7/12*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*
sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.48, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3\sec(e+fx)+a^3\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{30af(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-7*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) - (7*Tan[e + f*x])/(16*a^3*f*(c - c*Sec[e + f*x])^(3
/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
 + (7*Tan[e + f*x])/(30*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/
2)) + (7*Tan[e + f*x])/(12*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*x])^
(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{7\int \frac{\text{se}}{(a+a\sec(e+fx)}}{}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{}{30af(a+a\sec}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{}{30af(a+a\sec}$$

$$= -\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\,\text{se}}$$

$$= -\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\,\text{se}}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f} - \frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{}{5}$$

Mathematica [C] time = 6.47, size = 398, normalized size = 1.88

$$\frac{\sin^3\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^5(e+fx)\left(\frac{278\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{15f}-\frac{278\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)}{15f}+\frac{2\sec^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}-\frac{56\sec^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{15f}+\frac{242\sec(}{15}\right.}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),
x]
```

```
[Out] (7*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f
*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])
]*Cos[e/2 + (f*x)/2]^6*Sec[e + f*x]^(9/2)*Sin[e/2 + (f*x)/2]^3)/(E^((I/2)*(
e + f*x))*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)) + (Cos[e/2 +
 (f*x)/2]^6*Sec[e + f*x]^5*((-278*Cos[e/2]*Cos[(f*x)/2])/(15*f) - (Cot[e/2]
*Csc[e/2 + (f*x)/2])/f + (242*Sec[e/2 + (f*x)/2])/(15*f) - (56*Sec[e/2 + (f
*x)/2]^3)/(15*f) + (2*Sec[e/2 + (f*x)/2]^5)/(5*f) + (Csc[e/2]*Csc[e/2 + (f*
x)/2]^2*Sin[(f*x)/2])/f + (278*Sin[e/2]*Sin[(f*x)/2])/(15*f))*Sin[e/2 + (f*
x)/2]^3)/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
```

fricas [A] time = 0.53, size = 483, normalized size = 2.28

$$\left[-\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2-\cos\left(fx+e\right)-1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+(3}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{960\left(a^3c^2f\cos\left(fx+e\right)^3+a^3c^2f\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/960*(105*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(139*cos(f*x + e)^4 + 21*cos(f
*x + e)^3 - 175*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^
3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)), 1/480*(105*sqrt(2)*(cos(f*
x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(c)*arctan(sqrt(2)*sqrt((
```

```
c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(
f*x + e) - 2*(139*cos(f*x + e)^4 + 21*cos(f*x + e)^3 - 175*cos(f*x + e)^2 -
 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos
(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f
)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/16*(-7/2*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))+
(-2/3*c^9*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+1/5*c
^8*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)^2+3*c^10*sqr
t(c*tan((f*x+exp(1))/2)^2-c))/c^10+1/2*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)/c/
tan((f*x+exp(1))/2)^2)/sqrt(2)/a^3/c^2/sign(tan((f*x+exp(1))/2))/sign(tan((
f*x+exp(1))/2)^2-1)
```

maple [A] time = 2.02, size = 370, normalized size = 1.75

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(15\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{9}{2}}\cos\left(fx+e\right)+15\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{9}{2}}+15\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\cos\left(fx+e\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/120/a^3/f*(-1+cos(f*x+e))^2*(15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*cos(
f*x+e)+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+15*(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(7/2)*cos(f*x+e)-15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)-21*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)+21*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5
/2)+35*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-35*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(3/2)-105*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-105
*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+105*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)+105*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))
```

```
)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^3(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(e+fx)\left(a+\frac{a}{\cos(e+fx)}\right)^3\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)-2c\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+2c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c*s
qrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 2*c*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

3.106 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{63\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}-\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{21\tan}{20f\left(a^3\sec(e+fx)+\right.}$$

```
[Out] -63/256*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/c
^(5/2)/f*2^(1/2)-21/32*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(5/2)+1/5*tan(f*x+
e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2)+3/10*tan(f*x+e)/a/f/(a+a*sec
(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+21/20*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(
c-c*sec(f*x+e))^(5/2)-63/128*tan(f*x+e)/a^3/c/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{63\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}-\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{21\tan}{20f\left(a^3\sec(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-63*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) - (21*Tan[e + f*x])/(32*a^3*f*(c - c*Sec[e + f*x])
^(5/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/
2)) + (3*Tan[e + f*x])/(10*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^
(5/2)) + (21*Tan[e + f*x])/(20*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*
x])^(5/2)) - (63*Tan[e + f*x])/(128*a^3*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{9\int \frac{\sec(}{(a+a\sec(e+fx))^2}}{10}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3}{10af(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3}{10af(a+a\sec(e}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec}$$

$$= -\frac{63\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f} - \frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{}{5}$$

Mathematica [C] time = 6.66, size = 468, normalized size = 1.90

$$\frac{\sin^5\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^6(e+fx)\left(-\frac{257\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{10f}+\frac{257\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)}{10f}-\frac{2\sec^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}+\frac{22\sec^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}-124\ldots\right.}{(a\sec(e+fx)+a)^3(c-c\,\mathrm{s}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (-63*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e +
 f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))
])]*Cos[e/2 + (f*x)/2]^6*Sec[e + f*x]^(11/2)*Sin[e/2 + (f*x)/2]^5)/(4*E^((I
/2)*(e + f*x))*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)) + (Cos[
e/2 + (f*x)/2]^6*Sec[e + f*x]^6*((257*Cos[e/2]*Cos[(f*x)/2])/(10*f) + (23*C
ot[e/2]*Csc[e/2 + (f*x)/2])/(4*f) - (Cot[e/2]*Csc[e/2 + (f*x)/2]^3)/(2*f) -
 (124*Sec[e/2 + (f*x)/2])/(5*f) + (22*Sec[e/2 + (f*x)/2]^3)/(5*f) - (2*Sec[
e/2 + (f*x)/2]^5)/(5*f) - (23*Csc[e/2]*Csc[e/2 + (f*x)/2]^2*Sin[(f*x)/2])/(
4*f) + (Csc[e/2]*Csc[e/2 + (f*x)/2]^4*Sin[(f*x)/2])/(2*f) - (257*Sin[e/2]*S
in[(f*x)/2])/(10*f))*Sin[e/2 + (f*x)/2]^5)/((a + a*Sec[e + f*x])^3*(c - c*S
ec[e + f*x])^(5/2))
```

fricas [A] time = 0.62, size = 461, normalized size = 1.87

$$\left[-\frac{315\sqrt{2}\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\ldots}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\ldots\right.}{2560\left(a^3c^3f\cos\left(f\ldots\right.}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/2560*(315*sqrt(2)*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-c)*log(
(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)
*sin(f*x + e)))*sin(f*x + e) + 4*(257*cos(f*x + e)^5 - 354*cos(f*x + e)^4 -
 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x
```

```
 + e)^2 + a^3*c^3*f)*sin(f*x + e)), 1/1280*(315*sqrt(2)*(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(257*cos(f*x
 + e)^5 - 354*cos(f*x + e)^4 - 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 31
5*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*
x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/32*(-63/8*sqrt(c)*atan(sqrt(c*tan((f*x+exp(1))/2)^2-c)/sqrt(c))
+1/8*(17*c*sqrt(c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+15*c
^2*sqrt(c*tan((f*x+exp(1))/2)^2-c))/(c*tan((f*x+exp(1))/2)^2)^2+(-c^9*sqrt(
c*tan((f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)+1/5*c^8*sqrt(c*tan((
f*x+exp(1))/2)^2-c)*(c*tan((f*x+exp(1))/2)^2-c)^2+6*c^10*sqrt(c*tan((f*x+ex
p(1))/2)^2-c))/c^10)/sqrt(2)/a^3/c^3/sign(tan((f*x+exp(1))/2))/sign(tan((f*
x+exp(1))/2)^2-1)
```

maple [B] time = 2.20, size = 631, normalized size = 2.57

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{3}\left(45\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{11}{2}}+20\cos\left(fx+e\right)\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{11}{2}}+35\left(-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/160/a^3/f*(-1+cos(f*x+e))^3*(45*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(11/2)+20*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(11/2)+35*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(9/2)*cos(f*x+e)^2-25*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(11/2)-70*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*cos(f*x+e)-45*(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)^2+35*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)
+90*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)+63*(-2*cos(f*x+e)/(1+co
```

```
s(f*x+e)))^(5/2)*cos(f*x+e)^2-45*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)-126*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)-105*cos(f*x+e)^2*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(3/2)+63*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+210*cos(f
*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+315*cos(f*x+e)^2*arctan(1/(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2))+315*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)-105*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-630*cos(f*x+e)*arctan(1
/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-630*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)+315*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+315*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(
f*x+e)^5/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^5\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^4\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^3\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^2\left(e+fx\right)+c^2 \cdots}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**5 + c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c**2*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**3 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*
*2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c))
, x)/a**3
```

3.107 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=43

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 1/3*a*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx=\frac{a(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 0.47, size = 87, normalized size = 2.02

$$\frac{c^2(-6\cos(e+fx)+3\cos(2(e+fx))+5)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] (c^2*(5 - 6*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*
x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(
12*f)
```

fricas [B] time = 0.43, size = 93, normalized size = 2.16

$$\frac{\left(3\,c^{2}\cos\left(fx+e\right)^{2}-3\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2*cos(f*x + e) + c^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-8/3*sqrt(-a*c)*(-3*c^3*(c*tan(1/2*(
f*x+exp(1)))^2-c)-c^4-3*c^2*(c*tan(1/2*(f*x+exp(1)))^2-c)^2)*abs(c)*sign(ta
n(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^
3/f
```

maple [B] time = 1.96, size = 82, normalized size = 1.91

$$-\frac{\sin\left(fx+e\right)\left(7\left(\cos^{2}\left(fx+e\right)\right)-4\cos\left(fx+e\right)+1\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}}{3f\left(-1+\cos\left(fx+e\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(7*cos(f*x+e)^2-4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*
x+e))^(5/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))^3
```

maxima [B] time = 0.83, size = 638, normalized size = 14.84

$$\frac{2\left(30\,c^2\cos\left(3\,fx+3\,e\right)\sin\left(2\,fx+2\,e\right)-9\,c^2\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-3\,c^2\sin\left(fx+e\right)-\left(3\,c^2\sin\left(5\,fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(30*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*c^2*cos(2*f*x + 2*e)*sin(
f*x + e) - 3*c^2*sin(f*x + e) - (3*c^2*sin(5*f*x + 5*e) - 6*c^2*sin(4*f*x +
 4*e) + 10*c^2*sin(3*f*x + 3*e) - 6*c^2*sin(2*f*x + 2*e) + 3*c^2*sin(f*x +
e))*cos(6*f*x + 6*e) + 9*(c^2*sin(4*f*x + 4*e) + c^2*sin(2*f*x + 2*e))*cos(
5*f*x + 5*e) - 3*(10*c^2*sin(3*f*x + 3*e) + 3*c^2*sin(f*x + e))*cos(4*f*x +
 4*e) + (3*c^2*cos(5*f*x + 5*e) - 6*c^2*cos(4*f*x + 4*e) + 10*c^2*cos(3*f*x
 + 3*e) - 6*c^2*cos(2*f*x + 2*e) + 3*c^2*cos(f*x + e))*sin(6*f*x + 6*e) - 3
*(3*c^2*cos(4*f*x + 4*e) + 3*c^2*cos(2*f*x + 2*e) + c^2)*sin(5*f*x + 5*e) +
 3*(10*c^2*cos(3*f*x + 3*e) + 3*c^2*cos(f*x + e) + 2*c^2)*sin(4*f*x + 4*e)
- 10*(3*c^2*cos(2*f*x + 2*e) + c^2)*sin(3*f*x + 3*e) + 3*(3*c^2*cos(f*x + e
) + 2*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2
+ 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*
e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(
2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 3.80, size = 136, normalized size = 3.16

$$\frac{2\,c^2\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(10\,\sin\left(e+f\,x\right)-12\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(3\,e+3\,f\,x\right)-6\,\sin\left(4\,e+\right.\right.}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (2*c^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/
cos(e + f*x))^(1/2)*(10*sin(e + f*x) - 12*sin(2*e + 2*f*x) + 13*sin(3*e + 3
```

```
*f*x) - 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x) -
2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.108 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=43

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 1/2*a*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx=\frac{a(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.31, size = 73, normalized size = 1.70

$$\frac{c(2\cos(e+fx)-1)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (c*(-1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*f)
```

fricas [B] time = 0.42, size = 78, normalized size = 1.81

$$\frac{\left(2\,c\cos\left(fx+e\right)-c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*c*cos(f*x + e) - c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2*sqrt(-a*c)*(-2*c^3*(c*tan(1/2*(f*
x+exp(1)))^2-c)-c^4)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1
))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^2/c^2/f
```

maple [A] time = 1.98, size = 72, normalized size = 1.67

$$-\frac{\sin\left(fx+e\right)\left(3\cos\left(fx+e\right)-1\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{2f\left(-1+\cos\left(fx+e\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(3*cos(f*x+e)-1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(c*(
-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^2
```

maxima [B] time = 0.78, size = 298, normalized size = 6.93

$$\frac{2\left(2\,c\cos\left(3\,fx+3\,e\right)\sin\left(2\,fx+2\,e\right)-2\,c\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-\left(c\sin\left(3\,fx+3\,e\right)-c\sin\left(2\,fx+2\,e\right)+\right.\right.}{\left(2\left(2\,\cos\left(2\,fx+2\,e\right)+1\right)\cos\left(4\,fx+4\,e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*c*cos(2*f*x + 2*e)*sin(f*x + e
) - (c*sin(3*f*x + 3*e) - c*sin(2*f*x + 2*e) + c*sin(f*x + e))*cos(4*f*x +
4*e) + (c*cos(3*f*x + 3*e) - c*cos(2*f*x + 2*e) + c*cos(f*x + e))*sin(4*f*x
 + 4*e) - (2*c*cos(2*f*x + 2*e) + c)*sin(3*f*x + 3*e) + (2*c*cos(f*x + e) +
 c)*sin(2*f*x + 2*e) - c*sin(f*x + e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin
(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^
2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 2.64, size = 78, normalized size = 1.81

$$\frac{c\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(\sin\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{f\,{\sin\left(2\,e+2\,f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] (c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*(
sin(e + f*x) - sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)*sec(e +
f*x), x)
```

3.109 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.17, size = 56, normalized size = 1.37

$$\frac{\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\sec(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Se
c[e + f*x]])/(2*f)
```

fricas [A] time = 0.45, size = 56, normalized size = 1.37

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2*sqrt(-a*c)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1
))))/f/(c*tan(1/2*(f*x+exp(1)))^2-c)
```

maple [A] time = 1.95, size = 62, normalized size = 1.51

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{f\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/
2)*sin(f*x+e)/(-1+cos(f*x+e))
```

maxima [A] time = 0.85, size = 55, normalized size = 1.34

$$\frac{2\,\sqrt{-a}\,\sqrt{c}}{f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-a)*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1))
```

mupad [B] time = 1.94, size = 47, normalized size = 1.15

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}}{f\,\sin\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2))/(f
*sin(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x
), x)
```

3.110 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A] time = 0.13, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3952}

$$\frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.84, size = 99, normalized size = 1.94

$$-\frac{i\left(-1+e^{i(e+fx)}\right)\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{a(\sec(e+fx)+1)}}{f\left(1+e^{i(e+fx)}\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] ((-I)*(-1 + E^(I*(e + f*x)))*(2*Log[1 - E^(I*(e + f*x))] - Log[1 + E^((2*I)
*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])])/((1 + E^(I*(e + f*x)))*f*Sqrt[c -
 c*Sec[e + f*x]])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}\,\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/(
c*sec(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
```

(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assumi
ng -2*a+a is positive. Hint: run assume to make assumptions on a variableWa
rning, assuming -2*a+a is positive. Hint: run assume to make assumptions on
 a variableWarning, assuming -2*a+a is positive. Hint: run assume to make a
ssumptions on a variableWarning, assuming -2*a+a is positive. Hint: run ass
ume to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to

```
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check 
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of a
bs or sign assumes constant sign by intervals (correct if the argument is r
eal):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs 
or sign assumes constant sign by intervals (correct if the argument is real
):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t
_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to 
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+
exp(1)) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to 
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
```

```
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
```

```
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
```

Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2

```
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
```

```
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
```

```
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (

```
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

```
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
```

```
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
```

x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%%{-a,0%%%}+%%%{1,2%%%})Evaluation time: 3.23Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 2.02, size = 141, normalized size = 2.76

$$-\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\right)\cos\left(fx+e\right)}{f\sin\left(fx+e\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(2*ln(-(-1+cos(f*x+e))/sin(f*x+e))
-ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-sin(f*x+e)-1+cos(f*x+e))/
sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c
```

maxima [A] time = 0.90, size = 92, normalized size = 1.80

$$-\frac{\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{c}}+\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}-\frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(c) + sqrt(-a)*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - 2*sqrt(-a)*log(sin(f*x + e)/
(cos(f*x + e) + 1))/sqrt(c))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1)
), x)
```

3.111 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -1/2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.25, size = 62, normalized size = 1.48

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}}{cf(\sec(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Sec[e
 + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.46, size = 79, normalized size = 1.88

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)a^2*(1/2-1/2/tan(1/2*(f
*x+exp(1)))^2)*sign(cos(f*x+exp(1)))/sqrt(-a*c)/c/f/abs(a)/sign(tan(1/2*(f*
x+exp(1)))^2-1)/sign(tan(1/2*(f*x+exp(1))))
```

maple [A] time = 2.05, size = 60, normalized size = 1.43

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{2f\cos\left(fx+e\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(f*x+e)/(c*(-1+cos
(f*x+e))/cos(f*x+e))^(3/2)
```

maxima [B] time = 1.33, size = 514, normalized size = 12.24

$$-\frac{}{\left(c^2\cos\left(4\,fx+4\,e\right)^2+4\,c^2\cos\left(3\,fx+3\,e\right)^2+4\,c^2\cos\left(2\,fx+2\,e\right)^2+4\,c^2\cos\left(fx+e\right)^2+c^2\sin\left(4\,fx+4\,e\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*((sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) - (cos(3*f*x + 3*e)
+ cos(f*x + e))*sin(4*f*x + 4*e) + (2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e
) - 2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*cos(f*x + e)*sin(2*f*x + 2*e) +
 2*cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(
4*f*x + 4*e)^2 + 4*c^2*cos(3*f*x + 3*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^
2*cos(f*x + e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(3*f*x + 3*e)^2 + 4*c^
2*sin(2*f*x + 2*e)^2 - 8*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(3*f*x + 3*e) - 2*c^2*cos(2
*f*x + 2*e) + 2*c^2*cos(f*x + e) - c^2)*cos(4*f*x + 4*e) - 4*(2*c^2*cos(2*f
*x + 2*e) - 2*c^2*cos(f*x + e) + c^2)*cos(3*f*x + 3*e) - 4*(2*c^2*cos(f*x +
 e) - c^2)*cos(2*f*x + 2*e) - 4*(c^2*sin(3*f*x + 3*e) - c^2*sin(2*f*x + 2*e
) + c^2*sin(f*x + e))*sin(4*f*x + 4*e) - 8*(c^2*sin(2*f*x + 2*e) - c^2*sin(
f*x + e))*sin(3*f*x + 3*e))*f)
```

mupad [B] time = 3.00, size = 118, normalized size = 2.81

$$-\frac{2\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{c^2\,f\,\left(4\,\cos\left(e+f\,x\right)+4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] -(2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/cos
(e + f*x))^(1/2)*(sin(e + f*x) - 2*sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(c
^2*f*(4*cos(e + f*x) + 4*cos(2*e + 2*f*x) - 4*cos(3*e + 3*f*x) + cos(4*e +
4*f*x) - 5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
3/2), x)
```

3.112 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(a*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.39, size = 69, normalized size = 1.60

$$-\frac{(2\cos(e+fx)-1)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] -1/2*((-1 + 2*Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c
^2*f*(-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.44, size = 106, normalized size = 2.47

$$\frac{\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)1/2*a^2*(1/4*(2*a*(-a*t
an(1/2*(f*x+exp(1)))^2+a)-a^2)/(-a*tan(1/2*(f*x+exp(1)))^2)^2+1/4)*sign(cos
(f*x+exp(1)))/c^2/sqrt(-a*c)/f/abs(a)/sign(tan(1/2*(f*x+exp(1)))^2-1)/sign(
tan(1/2*(f*x+exp(1))))
```

maple [A] time = 2.12, size = 70, normalized size = 1.63

$$-\frac{\left(3\cos\left(fx+e\right)-1\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{8f\cos\left(fx+e\right)^{2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(3*cos(f*x+e)-1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(
f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

maxima [B] time = 1.22, size = 758, normalized size = 17.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*((sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) + cos(4*f*x + 4*e)*sin(2*f*x + 2*e) - (cos(4*f*x + 4*e) + 2*cos(2*f*x + 2
*e) + 1)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 3
6*c^3*cos(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + c^3*sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36
*c^3*sin(2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 12*c^3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*
f*x + 4*e) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*
x + 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c
^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [B] time = 6.61, size = 203, normalized size = 4.72

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^3\,f}+\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^3\,f}-\frac{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^3\,f}\right)}{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,10{}\mathrm{i}-{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,8{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/2)
*4i)/(c^3*f) + (exp(e*3i + f*x*3i)*cos(2*e + 2*f*x)*(a + a/cos(e + f*x))^(1
/2)*4i)/(c^3*f) - (cos(e + f*x)*exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/
2)*4i)/(c^3*f)))/(exp(e*3i + f*x*3i)*sin(e + f*x)*10i - exp(e*3i + f*x*3i)*
sin(2*e + 2*f*x)*8i + exp(e*3i + f*x*3i)*sin(3*e + 3*f*x)*2i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
5/2), x)
```

3.113 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}{5f}$$

```
[Out] 1/10*a^2*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/5*a*(
c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.28, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*Sec[e + f*x]
]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(
5*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\tan(e+f}{5f}$$

$$= \frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(}{}$$

Mathematica [A] time = 1.06, size = 108, normalized size = 1.21

$$\frac{ac^3(-10\cos(e+fx)+20\cos(2(e+fx))-10\cos(3(e+fx))+5\cos(4(e+fx))+7)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+f\right.}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a*c^3*(7 - 10*Cos[e + f*x] + 20*Cos[2*(e + f*x)] - 10*Cos[3*(e + f*x)] + 5
*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(80*f)
```

fricas [A] time = 0.47, size = 112, normalized size = 1.26

$$\frac{\left(10\,ac^3\cos\left(fx+e\right)^4-10\,ac^3\cos\left(fx+e\right)^3+5\,ac^3\cos\left(fx+e\right)-2\,ac^3\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{10\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/10*(10*a*c^3*cos(f*x + e)^4 - 10*a*c^3*cos(f*x + e)^3 + 5*a*c^3*cos(f*x +
 e) - 2*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)8/5*a*c*sqrt(-a*c)*(-15*c^5*(c*tan(1
/2*(f*x+exp(1)))^2-c)-4*c^6-10*c^3*(c*tan(1/2*(f*x+exp(1)))^2-c)^3-20*c^4*(
c*tan(1/2*(f*x+exp(1)))^2-c)^2)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2
*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^5/f
```

maple [A] time = 1.96, size = 103, normalized size = 1.16

$$\frac{\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(13\left(\cos^{3}\left(fx+e\right)\right)-16\left(\cos^{2}\left(fx+e\right)\right)+9\cos\left(fx+e\right)-2\right)\left(\sin^{3}\left(fx+e\right)\right)}{10f\left(-1+\cos\left(fx+e\right)\right)^{5}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 1/10/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(
1/2)*(13*cos(f*x+e)^3-16*cos(f*x+e)^2+9*cos(f*x+e)-2)*sin(f*x+e)^3/(-1+cos(
f*x+e))^5/cos(f*x+e)*a
```

maxima [B] time = 0.91, size = 1680, normalized size = 18.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/5*(100*a*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 25*a*c^3*cos(2*f*x + 2*e
)*sin(f*x + e) - 5*a*c^3*sin(f*x + e) - (5*a*c^3*sin(9*f*x + 9*e) - 10*a*c^
3*sin(8*f*x + 8*e) + 20*a*c^3*sin(7*f*x + 7*e) - 10*a*c^3*sin(6*f*x + 6*e)
+ 14*a*c^3*sin(5*f*x + 5*e) - 10*a*c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*
x + 3*e) - 10*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e))*cos(10*f*x + 1
0*e) + 25*(a*c^3*sin(8*f*x + 8*e) + 2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(
4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 5*(20*a*c^3*sin(7
*f*x + 7*e) + 10*a*c^3*sin(6*f*x + 6*e) + 14*a*c^3*sin(5*f*x + 5*e) + 10*a*
c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*x + 3*e) + 5*a*c^3*sin(f*x + e))*co
s(8*f*x + 8*e) + 100*(2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(4*f*x + 4*e) +
 a*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(14*a*c^3*sin(5*f*x + 5*e) +
 20*a*c^3*sin(3*f*x + 3*e) - 5*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e
))*cos(6*f*x + 6*e) + 70*(2*a*c^3*sin(4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e)
```

```
)*cos(5*f*x + 5*e) - 50*(4*a*c^3*sin(3*f*x + 3*e) - a*c^3*sin(2*f*x + 2*e)
+ a*c^3*sin(f*x + e))*cos(4*f*x + 4*e) + (5*a*c^3*cos(9*f*x + 9*e) - 10*a*c
^3*cos(8*f*x + 8*e) + 20*a*c^3*cos(7*f*x + 7*e) - 10*a*c^3*cos(6*f*x + 6*e)
 + 14*a*c^3*cos(5*f*x + 5*e) - 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f
*x + 3*e) - 10*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e))*sin(10*f*x +
10*e) - 5*(5*a*c^3*cos(8*f*x + 8*e) + 10*a*c^3*cos(6*f*x + 6*e) + 10*a*c^3*
cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(9*f*x + 9*e) + 5*(
20*a*c^3*cos(7*f*x + 7*e) + 10*a*c^3*cos(6*f*x + 6*e) + 14*a*c^3*cos(5*f*x
+ 5*e) + 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f*x + 3*e) + 5*a*c^3*co
s(f*x + e) + 2*a*c^3)*sin(8*f*x + 8*e) - 20*(10*a*c^3*cos(6*f*x + 6*e) + 10
*a*c^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(7*f*x + 7*e
) + 10*(14*a*c^3*cos(5*f*x + 5*e) + 20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos
(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e) + a*c^3)*sin(6*f*x + 6*e) - 14*(10*a*c
^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(5*f*x + 5*e) +
10*(20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x
+ e) + a*c^3)*sin(4*f*x + 4*e) - 20*(5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(
3*f*x + 3*e) + 5*(5*a*c^3*cos(f*x + e) + 2*a*c^3)*sin(2*f*x + 2*e))*sqrt(a)
*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e
) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 10*
(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*e) + 5*cos(2*f*x
+ 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20*(5*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*cos(2*f*x + 2*e)^2
+ 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(2*sin(6*f*x + 6*e
) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 25*sin(8*f*x
+ 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 1
00*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 6.15, size = 294, normalized size = 3.30

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,28\mathrm{i}}{5\,f}-\frac{a\,c^3\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,8\mathrm{i}}{f}+\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\right.}}}{f}\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,6\mathrm{i}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*c^3*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))
^(1/2)*28i)/(5*f) - (a*c^3*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f
*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos(e
+ f*x))^(1/2)*16i)/f - (a*c^3*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a + a/co
```

```
s(e + f*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)*(a + a
/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i + exp(e*5i
 + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.114 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}{4f}$$

```
[Out] 1/6*a^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/4*a*(c
-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.28, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[a + a*Sec[e + f*x]]
) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(4
*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\tan(e+}{4f}$$

$$= \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{a+a\,\mathrm{se}}}{}$$

Mathematica [A] time = 0.59, size = 97, normalized size = 1.09

$$\frac{ac^2(5\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a*c^2*(5*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Csc[(e +
f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])/(24*f)
```

fricas [A] time = 0.45, size = 112, normalized size = 1.26

$$\frac{\left(12\,ac^2\cos\left(fx+e\right)^3-6\,ac^2\cos\left(fx+e\right)^2-4\,ac^2\cos\left(fx+e\right)+3\,ac^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a*c^2*cos(f*x + e)^3 - 6*a*c^2*cos(f*x + e)^2 - 4*a*c^2*cos(f*x +
e) + 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)4/3*a*sqrt(-a*c)*(-8*c^4*(c*tan(1/2*
(f*x+exp(1)))^2-c)-3*c^5-6*c^3*(c*tan(1/2*(f*x+exp(1)))^2-c)^2)*abs(c)*sign
(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-
c)^4/f
```

maple [A] time = 2.44, size = 93, normalized size = 1.04

$$\frac{\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(11\left(\cos^{2}\left(fx+e\right)\right)-10\cos\left(fx+e\right)+3\right)\left(\sin^{3}\left(fx+e\right)\right)a}{12f\left(-1+\cos\left(fx+e\right)\right)^{4}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(
1/2)*(11*cos(f*x+e)^2-10*cos(f*x+e)+3)*sin(f*x+e)^3/(-1+cos(f*x+e))^4/cos(f
*x+e)*a
```

maxima [B] time = 0.86, size = 1105, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a*c^2*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a*c^2*sin(f*x + e) - (3*a*c^2*sin(7*f*x + 7*e) - 3*a*c^2*
sin(6*f*x + 6*e) + 5*a*c^2*sin(5*f*x + 5*e) + 5*a*c^2*sin(3*f*x + 3*e) - 3*
a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a*c^
2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a*c^2*sin(5*f*x + 5*e) + 9*a*c^2*sin(4*f*x + 4*e) +
 10*a*c^2*sin(3*f*x + 3*e) + 6*a*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a*c^2*sin(3*f*x + 3*e) - 3*a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a*c^2*cos(7*f*x + 7*e) - 3*a*c^2*cos(6*f*x + 6*e) + 5
*a*c^2*cos(5*f*x + 5*e) + 5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*
e) + 3*a*c^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a*c^2*cos(6*f*x + 6*e) +
 6*a*c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(7*f*x + 7
```

```
*e) + (20*a*c^2*cos(5*f*x + 5*e) + 18*a*c^2*cos(4*f*x + 4*e) + 20*a*c^2*cos
(3*f*x + 3*e) + 12*a*c^2*cos(f*x + e) + 3*a*c^2)*sin(6*f*x + 6*e) - 5*(6*a*
c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5*f*x + 5*e) +
 6*(5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*e) + 3*a*c^2*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3*f*x + 3*
e) + 3*(4*a*c^2*cos(f*x + e) + a*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 5.50, size = 195, normalized size = 2.19

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,c^2\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,20\mathrm{i}}{3\,f}-\frac{a\,c^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4\mathrm{i}}{f}+\frac{a\,c^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos\left(3\,e+3\right.}{f}}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*c^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*20i)/(3*f) - (a*c^2*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a*c^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)
*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.115 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

```
[Out] -1/3*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/3*c*(
a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.27, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]
]) - (c*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(
3*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,\tan(e$$

$$= -\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{c(a+a\,\mathrm{se}}{}$$

Mathematica [A] time = 0.43, size = 78, normalized size = 0.88

$$\frac{ac(3\cos(2(e+fx))+1)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (a*c*(1 + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x
]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(12*f)
```

fricas [A] time = 0.43, size = 82, normalized size = 0.92

$$\frac{\left(3\,ac\cos\left(fx+e\right)^2 - ac\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a*c*cos(f*x + e)^2 - a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)4/3*a*sqrt(-a*c)*(-3*c^4*(c*tan(1/2*
(f*x+exp(1)))^2-c)-2*c^5)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+
exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^3/c^2/f
```

maple [A] time = 2.17, size = 83, normalized size = 0.93

$$\frac{\left(\sin^{3}\left(fx+e\right)\right)\left(2\cos\left(fx+e\right)-1\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}} a}{3f\left(-1+\cos\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*sin(f*x+e)^3*(2*cos(f*x+e)-1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(c*
(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^3/cos(f*x+e)*a
```

maxima [B] time = 0.99, size = 550, normalized size = 6.18

$$\frac{2\left(6\,ac\cos\left(3\,fx+3\,e\right)\sin\left(2\,fx+2\,e\right)+9\,ac\cos\left(fx+e\right)\sin\left(2\,fx+2\,e\right)-9\,ac\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(6*a*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 9*a*c*cos(f*x + e)*sin(2*f*x
 + 2*e) - 9*a*c*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a*c*sin(f*x + e) - (3*a*c
*sin(5*f*x + 5*e) + 2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(a*c*sin(4*f*x + 4*e) + a*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) -
 3*(2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(4*f*x + 4*e) + (3*a*c*
cos(5*f*x + 5*e) + 2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(6*f*x +
 6*e) - 3*(3*a*c*cos(4*f*x + 4*e) + 3*a*c*cos(2*f*x + 2*e) + a*c)*sin(5*f*x
 + 5*e) + 3*(2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(4*f*x + 4*e)
- 2*(3*a*c*cos(2*f*x + 2*e) + a*c)*sin(3*f*x + 3*e))*sqrt(a)*sqrt(c)/((2*(3
*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x +
6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2
 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 3.14, size = 108, normalized size = 1.21

$$\frac{2\,a\,c\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)+5\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] (2*a*c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/
2)*(2*sin(e + f*x) + 5*sin(3*e + 3*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*
e + 2*f*x) - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.116 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -(c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.28, size = 73, normalized size = 1.70

$$\frac{a(2\cos(e+fx)+1)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (a*(1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*f)
```

fricas [B] time = 0.43, size = 76, normalized size = 1.77

$$\frac{\left(2\,a\cos\left(fx+e\right)+a\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*a*cos(f*x + e) + a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2*a*c*sqrt(-a*c)*abs(c)*sign(tan(1/
2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^2/f
```

maple [A] time = 2.07, size = 73, normalized size = 1.70

$$\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sin^{3}\left(fx+e\right)\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a}{2f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^3*(c*(-1+cos(f*x+e))/c
os(f*x+e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))^2*a
```

maxima [A] time = 0.77, size = 56, normalized size = 1.30

$$-\frac{2\sqrt{-a}\,a\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*sqrt(-a)*a*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)^2)
```

mupad [B] time = 2.59, size = 76, normalized size = 1.77

$$\frac{a\sqrt{c-\frac{c}{\cos(e+f\,x)}}\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\left(\sin\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{f\sin\left(2\,e+2\,f\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (a*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*(
sin(e + f*x) + sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))*sec(e +
f*x), x)
```

3.117 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+}}{\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.42, size = 174, normalized size = 1.83

$$\frac{\sqrt{2}\,a\sin\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)\sqrt{a(\sec(e+fx)+1)}\,\left(1+\left(4\log\left(1-e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)\right)\cos(e+fx}{f\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (Sqrt[2]*a*(1 + Cos[e + f*x]*(4*Log[1 - E^(I*(e + f*x))] - 2*Log[1 + E^((2*
I)*(e + f*x))]))*Sec[e + f*x]^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Cos[(e + f*
x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/((1 + E^(I*(e + f*x)))*Sqrt[E
^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[c - c*Sec[e + f*x]])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a\sec\left(fx+e\right)^{2}+a\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assumi
ng -2*a+a is positive. Hint: run assume to make assumptions on a variableWa
rning, assuming -2*a+a is positive. Hint: run assume to make assumptions on
 a variableWarning, assuming -2*a+a is positive. Hint: run assume to make a
ssumptions on a variableWarning, assuming -2*a+a is positive. Hint: run ass
ume to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
```

4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p

```
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, integration of abs or sign assumes constant sign by interval
s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Warning, integration of abs or sign assumes constant sign by intervals (
correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinu
ities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuiti
es at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
```

```
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
```

```
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
```

ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab

```
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
```

```
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
```

```
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

```
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
```

x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi

```
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
```

k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U

```
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(t_nostep)]Sign error (%%%{-a,0%%%}+%%%{1
,2%%%})Evaluation time: 3.45Limit: Max order reached or unable to make seri
es expansion Error: Bad Argument Value
```

maple [A] time = 2.06, size = 161, normalized size = 1.69

$$-\frac{\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\cos\left(fx+e\right)-2\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\cos\left(fx+e\right)\ln\left(-\frac{-\sin\left(fx+e\right)-1}{\sin\left(fx\right.}\right.\right.}{f\sin\left(fx+e\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(4*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-2*cos(f*x+e)*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)*ln(-(-sin(f*x+e)-1+cos(f*x+e))
/sin(f*x+e))+cos(f*x+e)+1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(c*(-1+cos(f
*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c*a
```

maxima [B] time = 1.03, size = 275, normalized size = 2.89

$$-\frac{2\left(a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)+\left(a\cos\left(2fx+2e\right)^{2}+a\sin\left(2fx+2e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 + (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*(a*cos(2*f*x + 2*e)^2
+ a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 1) - (a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(2*f*x + 2*e)^2 + c*sin(2
*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/sqrt(-c*(sec(e + f*x) -
 1)), x)
```

3.118 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-a^2*ln(1-sec(
f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
 - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$
$$= -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a^2\log(1-\sec(e+fx))\tan(e}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec}}$$

Mathematica [C] time = 0.70, size = 134, normalized size = 1.35

$$-\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\left(-2\log\left(1-e^{i(e+fx)}\right)+\log\left(1+e^{2i(e+fx)}\right)+\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1\right.\right.\right.}{cf(\cos(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((a*(2 - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(2*Log[1 - E^(I*(e + f*
x))] - Log[1 + E^((2*I)*(e + f*x))]) + Log[1 + E^((2*I)*(e + f*x))])*Sqrt[a
*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*
Sec[e + f*x]]))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)^2+a\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{c^2\sec\left(fx+e\right)^2-2\,c^2\sec\left(fx+e\right)+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt(
-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.89, size = 246, normalized size = 2.48

$$\frac{\left(-1+\cos\left(fx+e\right)\right)\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\cos\left(fx+e\right)-\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\cos\left(fx+e\right)\ln}{f\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(-1+cos(f*x+e))*(2*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-cos(f*x+e
)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(-(-sin(f*x+e)-1+
cos(f*x+e))/sin(f*x+e))+cos(f*x+e)-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+ln(-(-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x
+e))+1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/c
os(f*x+e))^(3/2)/sin(f*x+e)*a
```

maxima [A] time = 0.74, size = 122, normalized size = 1.23

$$\frac{\frac{\sqrt{-a}\,a\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}\,a\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{2\sqrt{-a}\,a\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}\,a\left(\cos\left(fx+e\right)+1\right)^{2}}{c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(3/2) + sqrt(-a)*a*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 2*sqrt(-a)*a*log(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^(3/2) + sqrt(-a)*a*(cos(f*x + e) + 1)^2/(c^(3/2)
*sin(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1))
**(3/2), x)
```

3.119 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/4*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(c - c*Sec[e + f*x])^(5/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.48, size = 63, normalized size = 1.50

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{c^3f(\sec(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (a*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]*Tan[(e
+ f*x)/2])/(c^3*f*(-1 + Sec[e + f*x])^3)
```

fricas [B] time = 0.44, size = 95, normalized size = 2.26

$$\frac{a\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f
)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)a^2*(1/4*a^3/(-a*tan(1/2*(f*x+exp(1)
))^2)^2-1/4*a)/c^2/sqrt(-a*c)/f/abs(a)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [A] time = 1.94, size = 73, normalized size = 1.74

$$\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sin^{3}\left(fx+e\right)\right)a}{4f\left(-1+\cos\left(fx+e\right)\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^3/(-1+cos(f*x+e))/(c*(
-1+cos(f*x+e))/cos(f*x+e))^(5/2)/cos(f*x+e)^2*a
```

maxima [B] time = 0.98, size = 533, normalized size = 12.69

$$\overline{\left(c^3\cos\left(4\,fx+4\,e\right)^2+16\,c^3\cos\left(3\,fx+3\,e\right)^2+36\,c^3\cos\left(2\,fx+2\,e\right)^2+16\,c^3\cos\left(fx+e\right)^2+c^3\sin\left(4\,fx+4\,e\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*(6*a*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 6*a*cos(f*x + e)*sin(2*f*x + 2*e
) - 6*a*cos(2*f*x + 2*e)*sin(f*x + e) - (a*sin(3*f*x + 3*e) + a*sin(f*x + e
))*cos(4*f*x + 4*e) + (a*cos(3*f*x + 3*e) + a*cos(f*x + e))*sin(4*f*x + 4*e
) - (6*a*cos(2*f*x + 2*e) + a)*sin(3*f*x + 3*e) - a*sin(f*x + e))*sqrt(a)*s
qrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*
f*x + 2*e)^2 + 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(
3*f*x + 3*e)^2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*
x + e) + 16*c^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*
f*x + 3*e) - 6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x +
 4*e) - 8*(6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3
*e) - 12*(4*c^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x +
 3*e) - 3*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*
(3*c^3*sin(2*f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

mupad [B] time = 4.83, size = 165, normalized size = 3.93

$$-\frac{2\,a\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(6\,\sin\left(e+f\,x\right)-8\,\sin\left(2\,e+2\,f\,x\right)+7\,\sin\left(3\,e+3\,f\,x\right)-4\,\sin\left(4\,e+4\,f\right.\right.}{c^3\,f\,\left(48\,\cos\left(e+f\,x\right)+15\,\cos\left(2\,e+2\,f\,x\right)-40\,\cos\left(3\,e+3\,f\,x\right)+26\,\cos\left(4\,e+4\,f\,x\right)-8\,\cos\left(5\,e+5\,f\,x\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*a*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/c
os(e + f*x))^(1/2)*(6*sin(e + f*x) - 8*sin(2*e + 2*f*x) + 7*sin(3*e + 3*f*x
) - 4*sin(4*e + 4*f*x) + sin(5*e + 5*f*x)))/(c^3*f*(48*cos(e + f*x) + 15*co
s(2*e + 2*f*x) - 40*cos(3*e + 3*f*x) + 26*cos(4*e + 4*f*x) - 8*cos(5*e + 5*
f*x) + cos(6*e + 6*f*x) - 42))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1))
**(5/2), x)
```

3.120 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/6*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)-1/24*(a+a*s
ec(f*x+e))^(3/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.30, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(6*f*(c - c*Sec[e + f*x])^(7/2))
 - ((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(24*c*f*(c - c*Sec[e + f*x])^(
5/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx}{6c}$$
$$= -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}} - \frac{(a+a\sec(e+fx))^{3/2}\tan(e+f}{24cf(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.56, size = 80, normalized size = 0.91

$$\frac{a(3\cos(e+fx)-3\cos(2(e+fx))-4)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{6c^3f(\cos(e+fx)-1)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a*(-4 + 3*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(6*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.42, size = 133, normalized size = 1.51

$$\frac{\left(6\,a\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f
*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*si
n(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)1/2*a^2*(1/12*(3*a^3*(-a*tan(1/2*(f*
x+exp(1)))^2+a)-a^4)/(-a*tan(1/2*(f*x+exp(1)))^2)^3-1/12*a)/c^3/sqrt(-a*c)/
f/abs(a)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [A] time = 2.18, size = 83, normalized size = 0.94

$$\frac{\left(5\cos\left(fx+e\right)-1\right)\left(\sin^{3}\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a}{24f\left(-1+\cos\left(fx+e\right)\right)\cos\left(fx+e\right)^{3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 1/24/f*(5*cos(f*x+e)-1)*sin(f*x+e)^3*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(-
1+cos(f*x+e))/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*a
```

maxima [B] time = 2.07, size = 1559, normalized size = 17.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 3*(a*si
n(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) - 4*a*sin(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(2*a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
 15*a*sin(2*f*x + 2*e) + 3*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(a*sin(6*f
*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a*cos(4*f*x + 4*e) + a*cos(2*f*x +
2*e))*sin(6*f*x + 6*e) + 3*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x + 2*e) - 3*(a
*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) - 4*a*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 4*(2*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) + 3*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 2*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
```

```
 3*(a*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + a)*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*
cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^
2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225
*c^4*sin(4*f*x + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c
^4*sin(2*f*x + 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 400*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 36*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*c
os(2*f*x + 2*e) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e
) + c^4)*cos(6*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x +
4*e) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*
x + 2*e) - 20*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*
c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c
^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*
c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x
 + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [B] time = 7.03, size = 273, normalized size = 3.10

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^4\,f}-\frac{a\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,44{}\mathrm{i}}{3\,c^4\,f}+\frac{a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^4\,f}\right.}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,28{}\mathrm{i}-{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,28{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,12{}\mathrm{i}-{\mathrm{e}}^{e\,4{}\mathrm{i}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1/
2)*4i)/(c^4*f) - (a*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1
/2)*44i)/(3*c^4*f) + (a*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)*(a + a/cos(e +
f*x))^(1/2)*4i)/(c^4*f) - (a*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x)*(a + a/cos
```

```
(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)*28i - exp(e
*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e + 3*f*x)*12
i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
[Out] Timed out
```

3.121 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

```
[Out] 1/12*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a
*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)
```

Rubi [A] time = 0.28, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3953}

$$\frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(4*f*(c - c*Sec[e + f*x])^(9/2))
 + (a^2*Tan[e + f*x])/(12*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
^(7/2))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} - \frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx}{4c}$$
$$= -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} + \frac{a^2\tan(e+fx)}{12cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec}$$

Mathematica [A] time = 0.83, size = 90, normalized size = 0.98

$$-\frac{a(17\cos(e+fx)-6\cos(2(e+fx))+3\cos(3(e+fx))-8)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{12c^4f(\cos(e+fx)-1)^4\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -1/12*(a*(-8 + 17*Cos[e + f*x] - 6*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*S
qrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c^4*f*(-1 + Cos[e + f*x])^4*Sq
rt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.45, size = 158, normalized size = 1.72

$$\frac{\left(6\,a\cos\left(fx+e\right)^4-6\,a\cos\left(fx+e\right)^3+4\,a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^4 - 6*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - a*cos(f
*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos
(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)1/4*a^2*(1/24*(-4*a^4*(-a*tan(1/2*(f
*x+exp(1)))^2+a)+a^5+6*a^3*(-a*tan(1/2*(f*x+exp(1)))^2+a)^2)/(-a*tan(1/2*(f
*x+exp(1)))^2)^4-1/24*a)/c^4/sqrt(-a*c)/f/abs(a)/sign(tan(1/2*(f*x+exp(1)))
^2-1)
```

maple [A] time = 2.13, size = 93, normalized size = 1.01

$$\frac{\left(17\left(\cos^{2}\left(fx+e\right)\right)-6\cos\left(fx+e\right)+1\right)\left(\sin^{3}\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a}{96f\left(-1+\cos\left(fx+e\right)\right)\cos\left(fx+e\right)^{4}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x)
```

```
[Out] 1/96/f*(17*cos(f*x+e)^2-6*cos(f*x+e)+1)*sin(f*x+e)^3*(a*(1+cos(f*x+e))/cos(
f*x+e))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(
9/2)*a
```

maxima [B] time = 7.89, size = 2608, normalized size = 28.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(28*a*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 28*a*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e) + 2*(3*a*sin(6*f*x + 6*e) + 8*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x
+ 2*e))*cos(8*f*x + 8*e) + (3*a*sin(8*f*x + 8*e) + 36*a*sin(6*f*x + 6*e) +
82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e) - 32*a*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17
*a*sin(8*f*x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140
*a*sin(2*f*x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a*sin(8*f*
x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140*a*sin(2*f*
x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a*sin(8*f*x + 8*e) + 3
```

```
6*a*sin(6*f*x + 6*e) + 82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(3*a*cos(6*f*x + 6*e) +
 8*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - 2*(14*a*co
s(4*f*x + 4*e) - 3*a)*sin(6*f*x + 6*e) + 4*(7*a*cos(2*f*x + 2*e) + 4*a)*sin
(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f
*x + 6*e) + 82*a*cos(4*f*x + 4*e) + 36*a*cos(2*f*x + 2*e) - 32*a*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 3*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(
4*f*x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(4*f*
x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 17*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f*x + 6*e) + 82*a*cos(4*f*x + 4*e
) + 36*a*cos(2*f*x + 2*e) + 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x +
6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2 + 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*
cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*
e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 + 3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 64*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 3136*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c
^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x +
 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*
f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*
cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^
5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56
*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*
c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + c^5)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f
*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + c^5)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*
f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e)
```

```
 + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) +
 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c
^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 5
6*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*s
in(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*
(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) +
 28*c^5*sin(2*f*x + 2*e) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*
sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c
^5*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
f)
```

mupad [B] time = 7.62, size = 340, normalized size = 3.70

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,68{}\mathrm{i}}{3\,c^5\,f}-\frac{a\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,88{}\mathrm{i}}{3\,c^5\,f}+\frac{a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,8}{3\,c^5\,f}\right.}{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,84{}\mathrm{i}-{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,96{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,54{}\mathrm{i}-{\mathrm{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(1/
2)*68i)/(3*c^5*f) - (a*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))
^(1/2)*88i)/(3*c^5*f) + (a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos(e
 + f*x))^(1/2)*80i)/(3*c^5*f) - (a*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a +
 a/cos(e + f*x))^(1/2)*8i)/(c^5*f) + (a*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin(e + f*x)*
84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i)*sin(3*e
+ 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i + f*x*5i)
*sin(5*e + 5*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{9/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

```
[Out] 1/20*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)/(a+a*sec(f*x+e))^(1/2)-1/5*a
*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)
```

Rubi [A] time = 0.29, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3953}

$$\frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{9/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^(11/2)
) + (a^2*Tan[e + f*x])/(20*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]
)^(9/2))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}} - \frac{a\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{9/2}}\,dx}{5c}$$
$$= -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}} + \frac{a^2\tan(e+fx)}{20cf\sqrt{a+a\sec(e+fx)}\,(c-c\,\mathrm{se}}$$

Mathematica [A] time = 1.22, size = 100, normalized size = 1.09

$$\frac{a(75\cos(e+fx)-50\cos(2(e+fx))+15\cos(3(e+fx))-5\cos(4(e+fx))-51)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+f}}{40c^5f(\cos(e+fx)-1)^5\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a*(-51 + 75*Cos[e + f*x] - 50*Cos[2*(e + f*x)] + 15*Cos[3*(e + f*x)] - 5*C
os[4*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(40*c^5*f*(-1
 + Cos[e + f*x])^5*Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.43, size = 184, normalized size = 2.00

$$\frac{\left(20\,a\cos\left(fx+e\right)^5-30\,a\cos\left(fx+e\right)^4+30\,a\cos\left(fx+e\right)^3-15\,a\cos\left(fx+e\right)^2+3\,a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e}{\cos(fx+e}}}{20\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/20*(20*a*cos(f*x + e)^5 - 30*a*cos(f*x + e)^4 + 30*a*cos(f*x + e)^3 - 15*
a*cos(f*x + e)^2 + 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*
cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*
f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)1/8*a^2*(1/40*(5*a^5*(-a*tan(1/2*(f*
x+exp(1)))^2+a)-a^6+10*a^3*(-a*tan(1/2*(f*x+exp(1)))^2+a)^3-10*a^4*(-a*tan(
1/2*(f*x+exp(1)))^2+a)^2)/(-a*tan(1/2*(f*x+exp(1)))^2)^5-1/40*a)/c^5/sqrt(-
a*c)/f/abs(a)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [A] time = 2.17, size = 103, normalized size = 1.12

$$\frac{\left(49\left(\cos^{3}\left(fx+e\right)\right)-23\left(\cos^{2}\left(fx+e\right)\right)+7\cos\left(fx+e\right)-1\right)\left(\sin^{3}\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a}{320f\left(-1+\cos\left(fx+e\right)\right)\cos\left(fx+e\right)^{5}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x)
```

```
[Out] 1/320/f*(49*cos(f*x+e)^3-23*cos(f*x+e)^2+7*cos(f*x+e)-1)*sin(f*x+e)^3*(a*(1
+cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^5/(c*(-1+cos(f*x+
e))/cos(f*x+e))^(11/2)*a
```

maxima [B] time = 38.32, size = 3906, normalized size = 42.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
```

```
[Out] -2/5*(225*a*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 225*a*cos(4*f*x + 4*e)*sin(
2*f*x + 2*e) - 15*(a*sin(8*f*x + 8*e) + 5*a*sin(6*f*x + 6*e) + 5*a*sin(4*f*
x + 4*e) + a*sin(2*f*x + 2*e))*cos(10*f*x + 10*e) - 225*(a*sin(6*f*x + 6*e)
 + a*sin(4*f*x + 4*e))*cos(8*f*x + 8*e) - 5*(a*sin(10*f*x + 10*e) + 15*a*si
n(8*f*x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2
*f*x + 2*e) - 20*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4
8*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*a*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 10*(5*a*sin(10*f*x + 10*e) + 45*a*sin(8*f*x + 8*e) +
 150*a*sin(6*f*x + 6*e) + 150*a*sin(4*f*x + 4*e) + 45*a*sin(2*f*x + 2*e) -
```

36*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*a*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 6*(17*a*sin(10*f*x + 10*e) + 135*a*sin(8*f*x + 8*e)
 + 420*a*sin(6*f*x + 6*e) + 420*a*sin(4*f*x + 4*e) + 135*a*sin(2*f*x + 2*e)
 + 60*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 40*a*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 50*(a*sin(10*f*x + 10*e) + 9*a*sin(8*f*x + 8*e)
+ 30*a*sin(6*f*x + 6*e) + 30*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) + 2*
a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*(a*sin(10*f*x + 10*e) + 15*a*sin(8*f*
x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x +
 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(a*cos(8*f
*x + 8*e) + 5*a*cos(6*f*x + 6*e) + 5*a*cos(4*f*x + 4*e) + a*cos(2*f*x + 2*e
))*sin(10*f*x + 10*e) + 15*(15*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) -
 a)*sin(8*f*x + 8*e) - 75*(3*a*cos(2*f*x + 2*e) + a)*sin(6*f*x + 6*e) - 75*
(3*a*cos(2*f*x + 2*e) + a)*sin(4*f*x + 4*e) - 15*a*sin(2*f*x + 2*e) + 5*(a*
cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x + 6*e) + 60*a*c
os(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) - 20*a*cos(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 48*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*
sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(5*a*cos(10*f*x +
 10*e) + 45*a*cos(8*f*x + 8*e) + 150*a*cos(6*f*x + 6*e) + 150*a*cos(4*f*x +
 4*e) + 45*a*cos(2*f*x + 2*e) - 36*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 10*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 5*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(17*a*cos(1
0*f*x + 10*e) + 135*a*cos(8*f*x + 8*e) + 420*a*cos(6*f*x + 6*e) + 420*a*cos
(4*f*x + 4*e) + 135*a*cos(2*f*x + 2*e) + 60*a*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 40*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 50*
(a*cos(10*f*x + 10*e) + 9*a*cos(8*f*x + 8*e) + 30*a*cos(6*f*x + 6*e) + 30*a
*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 5*(a*cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x
+ 6*e) + 60*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 1
0*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 + 44100*c^6*cos(6*f*x + 6*e)^2 + 44100
*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 + 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*
c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*sin(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(7/2*arctan2(si

n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*sin(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) + c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 2
10*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e
) + c^6)*cos(10*f*x + 10*e) + 90*(210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*
f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(8*f*x + 8*e) + 420*(210*c^6
*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(
45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e
) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x
+ 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 252*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
0*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(9/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) +
45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4
*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*
cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*cos(10*f*x
+ 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(
4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + c^6)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x
+ 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 10*c^6*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8
*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*
cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x
+ 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + 1350*(14*c^6*sin(6*f
*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(8*f*x + 8
e) + 6300(14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(6*f*x + 6
e) - 20(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*
f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 120*c^6*s
in(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*sin(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*
(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e
) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 252*c^6*sin(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*sin(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5

```
04*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x +
6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c
^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) -
10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*sin(10*f*x + 10*e) + 45*c^
6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) +
45*c^6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*f)
```

mupad [B] time = 7.68, size = 407, normalized size = 4.42

$$\frac{\sqrt{c-\frac{c}{\cos(e+fx)}}\left(\frac{a\,e^{e\,6i+f\,x\,6i}\sqrt{a+\frac{a}{\cos(e+fx)}}\,60i}{c^6\,f}-\frac{a\,\cos(e+fx)\,e^{e\,6i+f\,x\,6i}\sqrt{a+\frac{a}{\cos(e+fx)}}\,608i}{5\,c^6\,f}+\frac{a\,e^{e\,6i+f\,x\,6i}\cos(2\,e+2\,f\,x)\sqrt{a+\frac{a}{\cos(e+fx)}}}{c^6\,f}\cdots\right.}{e^{e\,6i+f\,x\,6i}\sin(e+f\,x)\,264i-e^{e\,6i+f\,x\,6i}\sin(2\,e+2\,f\,x)\,330i+e^{e\,6i+f\,x\,6i}\sin(3\,e+3\,f\,x)\,220\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),x
)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(1/
2)*60i)/(c^6*f) - (a*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(
1/2)*608i)/(5*c^6*f) + (a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a/cos(e
+ f*x))^(1/2)*72i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(a + a/
cos(e + f*x))^(1/2)*44i)/(c^6*f) + (a*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(
a + a/cos(e + f*x))^(1/2)*12i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(5*e + 5*
f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*6i + f*x*6i)*sin(e + f
*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*si
n(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i + exp(e*6i +
f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.123 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=134

$$\frac{a^3\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}{15f}+\frac{a\tan(e+fx)(a\,\mathrm{se}}{}$$

```
[Out] 1/6*a*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f+1/15*a^3*(
c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(c-c*sec
(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.42, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}{15f}+\frac{a^3\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{15f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)(a\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]
]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x
])/(15*f) + (a*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2)*Tan[e
+ f*x])/(6*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e}{6f}$$
$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\tan(e}{15f}$$
$$= \frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sqrt{a+a}}{}$$

Mathematica [A] time = 1.30, size = 113, normalized size = 0.84

$$\frac{a^2c^3(78\cos(e+fx)+5(7\cos(3(e+fx))-3\cos(4(e+fx))+3\cos(5(e+fx))-5))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+\right.}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*c^3*(78*Cos[e + f*x] + 5*(-5 + 7*Cos[3*(e + f*x)] - 3*Cos[4*(e + f*x)]
 + 3*Cos[5*(e + f*x)]))*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^5*Sq
rt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(480*f)
```

fricas [A] time = 0.48, size = 152, normalized size = 1.13

$$\frac{\left(30\,a^2c^3\cos\left(fx+e\right)^5-15\,a^2c^3\cos\left(fx+e\right)^4-20\,a^2c^3\cos\left(fx+e\right)^3+15\,a^2c^3\cos\left(fx+e\right)^2+6\,a^2c^3\cos\left(fx+\right.\right.}{30\,f\cos\left(fx+e\right)^5\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/30*(30*a^2*c^3*cos(f*x + e)^5 - 15*a^2*c^3*cos(f*x + e)^4 - 20*a^2*c^3*co
s(f*x + e)^3 + 15*a^2*c^3*cos(f*x + e)^2 + 6*a^2*c^3*cos(f*x + e) - 5*a^2*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-16/15*a^2*c*sqrt(-a*c)*(-36*c^6*(c*
tan(1/2*(f*x+exp(1)))^2-c)-10*c^7-20*c^4*(c*tan(1/2*(f*x+exp(1)))^2-c)^3-45
*c^5*(c*tan(1/2*(f*x+exp(1)))^2-c)^2)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+t
an(1/2*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^6/f
```

maple [A] time = 2.43, size = 105, normalized size = 0.78

$$-\frac{\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(21\left(\cos^{3}\left(fx+e\right)\right)-33\left(\cos^{2}\left(fx+e\right)\right)+21\cos\left(fx+e\right)-5\right)\left(\sin^{5}\left(fx+e\right.\right.}{30f\left(-1+\cos\left(fx+e\right)\right)^{6}\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/30/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^
(1/2)*(21*cos(f*x+e)^3-33*cos(f*x+e)^2+21*cos(f*x+e)-5)*sin(f*x+e)^5/(-1+co
s(f*x+e))^6/cos(f*x+e)^2*a^2
```

maxima [B] time = 0.61, size = 2454, normalized size = 18.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(210*a^2*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 90*a^2*c^3*cos(2*f*x
+ 2*e)*sin(f*x + e) - 15*a^2*c^3*sin(f*x + e) - (15*a^2*c^3*sin(11*f*x + 11
*e) - 15*a^2*c^3*sin(10*f*x + 10*e) + 35*a^2*c^3*sin(9*f*x + 9*e) + 78*a^2*
c^3*sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x +
 5*e) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*
c^3*sin(f*x + e))*cos(12*f*x + 12*e) + 15*(6*a^2*c^3*sin(10*f*x + 10*e) + 1
5*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4
```

```
*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(11*f*x + 11*e) - 3*(70*a^2*c^
3*sin(9*f*x + 9*e) + 75*a^2*c^3*sin(8*f*x + 8*e) + 156*a^2*c^3*sin(7*f*x +
7*e) + 156*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*
c^3*sin(3*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(10*f*x + 10*e) + 35*(15
*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4*
f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 15*(78*a^2*c^3*
sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x + 5*e
) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*c^3*
sin(f*x + e))*cos(8*f*x + 8*e) + 78*(20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c
^3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(15
6*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*c^3*sin(3
*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(6*f*x + 6*e) + 234*(5*a^2*c^3*si
n(4*f*x + 4*e) + 2*a^2*c^3*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 75*(7*a^2*c
^3*sin(3*f*x + 3*e) - 3*a^2*c^3*sin(2*f*x + 2*e) + 3*a^2*c^3*sin(f*x + e))*
cos(4*f*x + 4*e) + (15*a^2*c^3*cos(11*f*x + 11*e) - 15*a^2*c^3*cos(10*f*x +
 10*e) + 35*a^2*c^3*cos(9*f*x + 9*e) + 78*a^2*c^3*cos(7*f*x + 7*e) - 50*a^2
*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x + 5*e) + 35*a^2*c^3*cos(3*f*x
+ 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c^3*cos(f*x + e))*sin(12*f*x
+ 12*e) - 15*(6*a^2*c^3*cos(10*f*x + 10*e) + 15*a^2*c^3*cos(8*f*x + 8*e) +
20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2
*f*x + 2*e) + a^2*c^3)*sin(11*f*x + 11*e) + 3*(70*a^2*c^3*cos(9*f*x + 9*e)
+ 75*a^2*c^3*cos(8*f*x + 8*e) + 156*a^2*c^3*cos(7*f*x + 7*e) + 156*a^2*c^3*
cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*a^2*c^3*cos(3*f*x + 3*e
) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(10*f*x + 10*e) - 35*(15*a^2*c^
3*cos(8*f*x + 8*e) + 20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4
*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(9*f*x + 9*e) + 15*(78*a^2*c
^3*cos(7*f*x + 7*e) - 50*a^2*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x +
5*e) + 35*a^2*c^3*cos(3*f*x + 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c
^3*cos(f*x + e))*sin(8*f*x + 8*e) - 78*(20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^
2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(7*f*x +
7*e) + 10*(156*a^2*c^3*cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*
a^2*c^3*cos(3*f*x + 3*e) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(6*f*x +
 6*e) - 78*(15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*
c^3)*sin(5*f*x + 5*e) + 75*(7*a^2*c^3*cos(3*f*x + 3*e) - 3*a^2*c^3*cos(2*f*
x + 2*e) + 3*a^2*c^3*cos(f*x + e))*sin(4*f*x + 4*e) - 35*(6*a^2*c^3*cos(2*f
*x + 2*e) + a^2*c^3)*sin(3*f*x + 3*e) + 15*(6*a^2*c^3*cos(f*x + e) + a^2*c^
3)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(6*cos(10*f*x + 10*e) + 15*cos(8*f
*x + 8*e) + 20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e)
+ 1)*cos(12*f*x + 12*e) + cos(12*f*x + 12*e)^2 + 12*(15*cos(8*f*x + 8*e) +
20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(10*
f*x + 10*e) + 36*cos(10*f*x + 10*e)^2 + 30*(20*cos(6*f*x + 6*e) + 15*cos(4*
f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + 225*cos(8*f*x + 8*e
)^2 + 40*(15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) +
400*cos(6*f*x + 6*e)^2 + 30*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 225
*cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 2*(6*sin(10*f*x + 10*e) + 15*
```

```
sin(8*f*x + 8*e) + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x
+ 2*e))*sin(12*f*x + 12*e) + sin(12*f*x + 12*e)^2 + 12*(15*sin(8*f*x + 8*e)
 + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(10*f
*x + 10*e) + 36*sin(10*f*x + 10*e)^2 + 30*(20*sin(6*f*x + 6*e) + 15*sin(4*f
*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 225*sin(8*f*x + 8*e)^2 +
 120*(5*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 400*sin(6
*f*x + 6*e)^2 + 225*sin(4*f*x + 4*e)^2 + 180*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 36*sin(2*f*x + 2*e)^2 + 12*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 6.23, size = 307, normalized size = 2.29

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,20{}\mathrm{i}}{3\,f}+\frac{a^2\,c^3\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,104{}\mathrm{i}}{5\,f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\cdots}}{3\,f}\cdots\right.}{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,10{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c^3*cos(e + f*x)*exp(e*6i + f*x*6i)*(a +
a/cos(e + f*x))^(1/2)*104i)/(5*f) - (a^2*c^3*exp(e*6i + f*x*6i)*(a + a/cos(
e + f*x))^(1/2)*20i)/(3*f) + (a^2*c^3*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(
a + a/cos(e + f*x))^(1/2)*28i)/(3*f) - (a^2*c^3*exp(e*6i + f*x*6i)*cos(4*e
+ 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c^3*exp(e*6i + f*x*6i)*cos
(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*6i + f*x*6i)*sin(2*
e + 2*f*x)*10i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*8i + exp(e*6i + f*x*6i
)*sin(6*e + 6*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.124 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2c^3\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{15f\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{5f}-\frac{c\tan(e+fx)(a$$

```
[Out] -1/5*c*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/15*c^3*
(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/5*c^2*(a+a*sec
(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.42, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$-\frac{2c^3\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{15f\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{5f}-\frac{c\tan(e+fx)(a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f
*x]]) - (c^2*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*
x])/(5*f) - (c*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(5*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e}{5f}$$

$$= -\frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,\tan(e}{5f}$$

$$= -\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{c^2(a+a\,\mathrm{s}}{}$$

Mathematica [A] time = 0.78, size = 92, normalized size = 0.69

$$\frac{a^2c^2(20\cos(2(e+fx))+15\cos(4(e+fx))+29)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(\sec(e+fx)+1)}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a^2*c^2*(29 + 20*Cos[2*(e + f*x)] + 15*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]*
Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])/(240*f)
```

fricas [A] time = 0.46, size = 106, normalized size = 0.79

$$\frac{\left(15\,a^2c^2\cos\left(fx+e\right)^4-10\,a^2c^2\cos\left(fx+e\right)^2+3\,a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*a^2*c^2*cos(f*x + e)^4 - 10*a^2*c^2*cos(f*x + e)^2 + 3*a^2*c^2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))/(f*cos(f*x + e)^4*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-16/15*a^2*sqrt(-a*c)*(-15*c^5*(c*ta
n(1/2*(f*x+exp(1)))^2-c)-6*c^6-10*c^4*(c*tan(1/2*(f*x+exp(1)))^2-c)^2)*abs(
c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(
1)))^2-c)^5/f
```

maple [A] time = 2.32, size = 95, normalized size = 0.71

$$-\frac{\left(\sin^{5}\left(fx+e\right)\right)\left(8\left(\cos^{2}\left(fx+e\right)\right)-9\cos\left(fx+e\right)+3\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}a^{2}}{15f\left(-1+\cos\left(fx+e\right)\right)^{5}\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/15/f*sin(f*x+e)^5*(8*cos(f*x+e)^2-9*cos(f*x+e)+3)*(a*(1+cos(f*x+e))/cos(
f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^5/cos(f*
x+e)^2*a^2
```

maxima [B] time = 0.59, size = 1526, normalized size = 11.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(100*a^2*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 75*a^2*c^2*cos(f*x +
e)*sin(2*f*x + 2*e) - 75*a^2*c^2*cos(2*f*x + 2*e)*sin(f*x + e) - 15*a^2*c^2
*sin(f*x + e) - (15*a^2*c^2*sin(9*f*x + 9*e) + 20*a^2*c^2*sin(7*f*x + 7*e)
+ 58*a^2*c^2*sin(5*f*x + 5*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*si
n(f*x + e))*cos(10*f*x + 10*e) + 75*(a^2*c^2*sin(8*f*x + 8*e) + 2*a^2*c^2*s
in(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*co
s(9*f*x + 9*e) - 5*(20*a^2*c^2*sin(7*f*x + 7*e) + 58*a^2*c^2*sin(5*f*x + 5*
```

```
e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(8*f*x + 8*e
) + 100*(2*a^2*c^2*sin(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*
sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(58*a^2*c^2*sin(5*f*x + 5*e) + 20*a
^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(
2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 5
0*(4*a^2*c^2*sin(3*f*x + 3*e) + 3*a^2*c^2*sin(f*x + e))*cos(4*f*x + 4*e) +
(15*a^2*c^2*cos(9*f*x + 9*e) + 20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos
(5*f*x + 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(
10*f*x + 10*e) - 15*(5*a^2*c^2*cos(8*f*x + 8*e) + 10*a^2*c^2*cos(6*f*x + 6*
e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*si
n(9*f*x + 9*e) + 5*(20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos(5*f*x + 5*
e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(8*f*x + 8*e
) - 20*(10*a^2*c^2*cos(6*f*x + 6*e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(7*f*x + 7*e) + 10*(58*a^2*c^2*cos(5*f*x
+ 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(6*f*x +
 6*e) - 58*(10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*
c^2)*sin(5*f*x + 5*e) + 50*(4*a^2*c^2*cos(3*f*x + 3*e) + 3*a^2*c^2*cos(f*x
+ e))*sin(4*f*x + 4*e) - 20*(5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(3*f*
x + 3*e))*sqrt(a)*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 1
0*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*
x + 10*e)^2 + 10*(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x +
 2*e) + 1)*cos(8*f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*
e) + 5*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20
*(5*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*co
s(2*f*x + 2*e)^2 + 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x
+ 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(
2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + 25*sin(8*f*x + 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin
(6*f*x + 6*e) + 100*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e)
+ 1)*f)
```

mupad [B] time = 5.61, size = 215, normalized size = 1.60

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,116\mathrm{i}}{15\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,16\mathrm{i}}{3\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a}}{f}\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,6\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x
))^(1/2)*116i)/(15*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
```

```
/cos(e + f*x))^(1/2)*16i)/(3*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(4*e + 4*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i
+ exp(e*5i + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f
*x)*2i)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.125 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

```
[Out] -1/6*c^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/4*c*(
a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.27, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[c - c*Sec[e + f*x]
]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(
4*f)
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\ \tan(e$$

$$= -\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}} - \frac{c(a+a\,\mathrm{se}}{}$$

Mathematica [A] time = 0.59, size = 96, normalized size = 1.08

$$\frac{a^2c(5\cos(e+fx)+3(\cos(2(e+fx))+\cos(3(e+fx))))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (a^2*c*(5*Cos[e + f*x] + 3*(Cos[2*(e + f*x)] + Cos[3*(e + f*x)]))*Csc[(e +
f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])/(24*f)
```

fricas [A] time = 0.44, size = 112, normalized size = 1.26

$$\frac{\left(12\,a^2c\cos\left(fx+e\right)^3+6\,a^2c\cos\left(fx+e\right)^2-4\,a^2c\cos\left(fx+e\right)-3\,a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e)^2 - 4*a^2*c*cos(f*x +
e) - 3*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-4/3*a^2*sqrt(-a*c)*(-4*c^5*(c*tan(1
/2*(f*x+exp(1)))^2-c)-3*c^6)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f
*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)^4/c^2/f
```

maple [A] time = 2.18, size = 85, normalized size = 0.96

$$-\frac{\left(\sin^5\left(fx+e\right)\right)\left(5\cos\left(fx+e\right)-3\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}a^2}{12f\left(-1+\cos\left(fx+e\right)\right)^4\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*sin(f*x+e)^5*(5*cos(f*x+e)-3)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^4/cos(f*x+e)^2*a^2
```

maxima [B] time = 0.56, size = 1106, normalized size = 12.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a^2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a^2*c*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a^2*c*sin(f*x + e) - (3*a^2*c*sin(7*f*x + 7*e) + 3*a^2*c*
sin(6*f*x + 6*e) + 5*a^2*c*sin(5*f*x + 5*e) + 5*a^2*c*sin(3*f*x + 3*e) + 3*
a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a^2*
c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a^2*c*sin(5*f*x + 5*e) - 9*a^2*c*sin(4*f*x + 4*e) +
 10*a^2*c*sin(3*f*x + 3*e) + 6*a^2*c*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a^2*c*sin(3*f*x + 3*e) + 3*a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a^2*c*cos(7*f*x + 7*e) + 3*a^2*c*cos(6*f*x + 6*e) + 5
*a^2*c*cos(5*f*x + 5*e) + 5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*
e) + 3*a^2*c*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a^2*c*cos(6*f*x + 6*e) +
 6*a^2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(7*f*x + 7
*e) + (20*a^2*c*cos(5*f*x + 5*e) - 18*a^2*c*cos(4*f*x + 4*e) + 20*a^2*c*cos
(3*f*x + 3*e) + 12*a^2*c*cos(f*x + e) - 3*a^2*c)*sin(6*f*x + 6*e) - 5*(6*a^
```

```
2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5*f*x + 5*e) +
 6*(5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*e) + 3*a^2*c*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3*f*x + 3*
e) + 3*(4*a^2*c*cos(f*x + e) - a^2*c)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

mupad [B] time = 5.44, size = 195, normalized size = 2.19

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,c\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,20{}\mathrm{i}}{3\,f}+\frac{a^2\,c\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{f}+\frac{a^2\,c\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos\left(3\,e+3\right.}{f}}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*20i)/(3*f) + (a^2*c*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)
*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.126 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/3*c*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -(c*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [B] time = 0.47, size = 88, normalized size = 2.05

$$\frac{a^2\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(4\cos(e+fx)+\cos^2(e+fx)\sec^2\left(\frac{1}{2}(e+\right.\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (a^2*Cot[(e + f*x)/2]*(2 + 4*Cos[e + f*x] + Cos[e + f*x]^2*Sec[(e + f*x)/2]
^2)*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(6*
f)
```

fricas [B] time = 0.44, size = 93, normalized size = 2.16

$$\frac{\left(3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)8/3*a^2*c^2*sqrt(-a*c)*abs(c)*sign(t
an(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/(c*tan(1/2*(f*x+exp(1)))^2-c)
^3/f
```

maple [A] time = 2.30, size = 75, normalized size = 1.74

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sin^5\left(fx+e\right)\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a^2}{3f\cos\left(fx+e\right)^2\left(-1+\cos\left(fx+e\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^5*(c*(-1+cos(f*x+e))/
cos(f*x+e))^(1/2)/cos(f*x+e)^2/(-1+cos(f*x+e))^3*a^2
```

maxima [A] time = 0.46, size = 58, normalized size = 1.35

$$\frac{8\sqrt{-a}\,a^2\sqrt{c}}{3\,f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^3\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 8/3*sqrt(-a)*a^2*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^3*(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)^3)
```

mupad [B] time = 3.61, size = 136, normalized size = 3.16

$$\frac{2\,a^2\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(10\,\sin\left(e+f\,x\right)+12\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(3\,e+3\,f\,x\right)+6\,\sin\left(4\,e+\right.\right.}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/
cos(e + f*x))^(1/2)*(10*sin(e + f*x) + 12*sin(2*e + 2*f*x) + 13*sin(3*e + 3
*f*x) + 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x) -
2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.127 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^3}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4*a^3*ln(1
-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+2*a
^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^3}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[c - c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+}{\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+f}{2f\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+fx)}\,\tan(}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 6.60, size = 328, normalized size = 2.33

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e+fx)(a(\sec(e+fx)+1))^{5/2}\sqrt{(\cos(e+fx)+1)\sec(e+fx)}\left(\frac{5\sec\left(\frac{e}{2}+\frac{fx}{2}\right)}{2f}+\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e+fx)}{f}\right)}{(\sec(e+fx)+1)^{5/2}\sqrt{c-c\sec(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (4*Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(
e + f*x)))]*(2*Log[1 - E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))])*Sqr
t[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2])/((1 + E^(I
*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*(1 + Sec[e +
 f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (Sec[e + f*x]*Sqrt[(1 + Cos[e + f*
x])*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*((5*Sec[e/2 + (f*x)/2])/(2*f
) + (Cos[e/2 + (f*x)/2]*Sec[e + f*x])/f)*Sin[e/2 + (f*x)/2])/((1 + Sec[e +
f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^3+2\,a^2\sec\left(fx+e\right)^2+a^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assumi
ng -2*a+a is positive. Hint: run assume to make assumptions on a variableWa
rning, assuming -2*a+a is positive. Hint: run assume to make assumptions on
 a variableWarning, assuming -2*a+a is positive. Hint: run assume to make a
ssumptions on a variableWarning, assuming -2*a+a is positive. Hint: run ass
ume to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
```

```
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
```

```
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integ
ration of abs or sign assumes constant sign by intervals (correct if the ar
gument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t
_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(
f*t_nostep+exp(1)) were not checkedUnable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
```

able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>

(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign

```
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
```

```
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
```

```
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
```

```
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
```

```
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
```

)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/

```
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
```

ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab

```
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Warning, integration
 of abs or sign assumes constant sign by intervals (correct if the argument
 is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of ab
s or sign assumes constant sign by intervals (correct if the argument is re
al):Check [abs(t_nostep)]Sign error (%%%{-a,0%%%}+%%%{1,2%%%})Evaluation ti
me: 4.25Limit: Max order reached or unable to make series expansion Error:
Bad Argument Value
```

maple [A] time = 2.47, size = 189, normalized size = 1.34

$$\frac{\left(8\ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\left(\cos^2\left(fx+e\right)\right)+8\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2\left(fx+e\right)\right)-16\left(\cos^2\left(fx+e\right.\right.\right.}{2f\sin\left(fx+e\right)\cos\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(8*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+8*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-16*cos(f*x+e)^2*ln(-(-1+cos
(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^2-6*cos(f*x+e)-1)*(a*(1+cos(f*x+e))/cos(f
*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)/c*a
^2
```

maxima [B] time = 0.59, size = 737, normalized size = 5.23

$$-\frac{2\left(a^2\cos\left(2fx+2e\right)\sin\left(4fx+4e\right)-a^2\cos\left(4fx+4e\right)\sin\left(2fx+2e\right)-a^2\sin\left(2fx+2e\right)+2\left(a^2\cos\left(4\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - a^2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e) - a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
 + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(
2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin
(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x
 + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)
*cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 3*(a^2*si
n(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 3*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a^2*cos(4*f*x + 4*e)
+ 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 3*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(4*f*
x + 4*e)^2 + 4*c*cos(2*f*x + 2*e)^2 + c*sin(4*f*x + 4*e)^2 + 4*c*sin(4*f*x
```

```
+ 4*e)*sin(2*f*x + 2*e) + 4*c*sin(2*f*x + 2*e)^2 + 2*(2*c*cos(2*f*x + 2*e)
+ c)*cos(4*f*x + 4*e) + 4*c*cos(2*f*x + 2*e) + c)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.128 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-4*a^3*ln(1-se
c(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*a^
2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3954, 3955, 3952}

$$-\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)) - (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]) - (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{2a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+}{cf\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{4a^3\log(1-\sec(e+fx))\tan(e}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec}}$$

Mathematica [C] time = 1.50, size = 188, normalized size = 1.30

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\,\left(-4\log\left(1-e^{i(e+fx)}\right)+2\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1-e^{i(e+f}\right.\right.\right.}{cf(\cos(e+fx)-1)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] (a^2*(1 - 4*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-5 + 8*Log[1 - E^(I*(e
 + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + 2*Log[1 + E^((2*I)*(e + f*x))
] + Cos[2*(e + f*x)]*(-4*Log[1 - E^(I*(e + f*x))] + 2*Log[1 + E^((2*I)*(e +
 f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-
1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\sec\left(fx+e\right)^3+2\,a^2\sec\left(fx+e\right)^2+a^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{c^2\sec\left(fx+e\right)^2-2\,c^2\sec\left(fx+e\right)+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2
*sec(f*x + e) + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.20, size = 289, normalized size = 1.99

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(4\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^{2}\left(fx+e\right)\right)+4\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^{2}\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(4*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+
e)^2+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*cos(f*x+e)
^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3*cos(f*x+e)^2-4*cos(f*x+e)*ln(-(-sin(f*
x+e)-1+cos(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))+8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-2*cos(f*x+e)+1)*(a
*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e
))^(3/2)/sin(f*x+e)*a^2
```

maxima [B] time = 0.74, size = 2035, normalized size = 14.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2*(8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a
^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*a^2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 2*a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
+ 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a
^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 4*(a^2*cos(4*f*x + 4*e) + 2*a
^2*cos(2*f*x + 2*e) - 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*
cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*
e) - 2*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^
2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*a
rctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + (16*a^2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 5*a^2*sin(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x +
 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (5*a^2*s
in(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x + 4*e) + 2*a^2*
cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*a^2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*a^2*cos(4*f*x + 4*e) - 6*a^2*cos(2*f*x + 2*e) - 5*a^2 - 8*(
a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 (5*a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 5*a^2 + 8*(a^2*sin(4*f*
x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt
(c)/((c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/2*arctan2(
```

```
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*
f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*
f*x + 2*e) - 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c
^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x
 + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e) - 2*c^
2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f
*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{a^3\tan(e+fx)\log(1-\sec(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+a^2*(a+a*
sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-sec(f*x+e)
)*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$\frac{a^3\tan(e+fx)\log(1-\sec(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2
)) + (a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(c*f*(c - c*Sec[e + f*x])^
(3/2)) + (a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+}{cf(c-c\sec(e+fx))^{3/2}}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+}{cf(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] time = 1.34, size = 182, normalized size = 1.26

$$-\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\left(-6\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1-e^{i(e+fx)}\right)-4\log\right.\right.}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\,s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -1/2*(a^2*(4 - 6*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(8*Log[1 - E^(I*(e
 + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + 3*Log[1 + E^((2*I)*(e + f*x))
] + Cos[2*(e + f*x)]*(-2*Log[1 - E^(I*(e + f*x))] + Log[1 + E^((2*I)*(e + f
*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c^2*f*(-1 + Cos[e + f
*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^3+2\,a^2\sec\left(fx+e\right)^2+a^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^
3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make a
ssumptions on a variableWarning, assuming -2*a+a is positive. Hint: run ass
ume to make assumptions on a variableWarning, assuming -2*a+a is positive.
Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a
is positive. Hint: run assume to make assumptions on a variableUnable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
```

pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)

```
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Warning, integration of abs or sign assumes constant sign by in
tervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1))
)]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Disc
ontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(
1)) were not checkedUnable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
```

)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si

```
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
```

e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
```

```
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x

```
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
```

x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check

```
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Warni
ng, integration of abs or sign assumes constant sign by intervals (correct
if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, in
tegration of abs or sign assumes constant sign by intervals (correct if the
 argument is real):Check [abs(t_nostep)]Sign error (%%%{-a,0%%%}+%%%{1,2%%%
})Evaluation time: 6.5Limit: Max order reached or unable to make series exp
ansion Error: Bad Argument Value
```

maple [B] time = 2.29, size = 364, normalized size = 2.51

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(4\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^{2}\left(fx+e\right)\right)-2\ln\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(-(-sin(f*x+e)-1+
cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(
f*x+e)+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)
*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+cos(f*x+e)^2+4*ln(-(-1+cos(f*x+
e))/sin(f*x+e))-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*ln(-(-sin(f*
x+e)-1+cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)-3)*(a*(1+cos(f*x+e))/cos(f*x+e)
)^(1/2)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)*a^2
```

maxima [A] time = 0.45, size = 169, normalized size = 1.17

$$-\frac{\frac{2\sqrt{-a}\,a^{2}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{c^{\frac{5}{2}}}+\frac{2\sqrt{-a}\,a^{2}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{5}{2}}}-\frac{4\sqrt{-a}\,a^{2}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^{\frac{5}{2}}}+\frac{\left(\sqrt{-a}\,a^{2}\sqrt{c}+\frac{2\sqrt{-a}\,a^{2}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\cdots\right.}{c^{3}\sin\left(fx+e\right)^{4}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(5/2) + 2*s
qrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 4*sqrt(-a)*a
^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) + (sqrt(-a)*a^2*sqrt(c) + 2
*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) +
1)^4/(c^3*sin(f*x + e)^4))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/6*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*(c - c*Sec[e + f*x])^(7/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A] time = 0.56, size = 76, normalized size = 1.81

$$\frac{a^2(3\cos(2(e+fx))+5)\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{48c^3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a^2*(5 + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]^5*Sec[(e + f*x)/2]*Sqrt[a*(1
 + Sec[e + f*x])])/(48*c^3*f*Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.44, size = 126, normalized size = 3.00

$$\frac{\left(3\,a^2\cos\left(fx+e\right)^3+a^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^3 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f*cos(f*x + e)^3 -
 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)a^2*(1/6*a^2+1/6*a^5/(-a*tan(1/2*(f*
x+exp(1)))^2)^3)/c^3/sqrt(-a*c)/f/abs(a)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [B] time = 1.96, size = 75, normalized size = 1.79

$$-\frac{\left(\sin^5\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\;a^2}{6f\left(-1+\cos\left(fx+e\right)\right)^2\cos\left(fx+e\right)^3\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/6/f*sin(f*x+e)^5*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))^2/c
os(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*a^2
```

maxima [B] time = 0.58, size = 1815, normalized size = 43.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(208*a^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 48*a^2*cos(f*x + e)*sin(2*
f*x + 2*e) - 48*a^2*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a^2*sin(f*x + e) - (3
*a^2*sin(7*f*x + 7*e) + 13*a^2*sin(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) +
 3*a^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(8*a^2*sin(6*f*x + 6*e) + 15*a^2*
sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(13*a^2*si
n(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(6*f*x +
6*e) + 26*(15*a^2*sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*
e) - 30*(13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(4*f*x + 4*e) + (
3*a^2*cos(7*f*x + 7*e) + 13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e)
+ 3*a^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(16*a^2*cos(6*f*x + 6*e) + 30*a^
2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2)*sin(7*f*x + 7*e) + 16*(
13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin
(6*f*x + 6*e) - 13*(30*a^2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2
)*sin(5*f*x + 5*e) + 30*(13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin(
4*f*x + 4*e) - 13*(16*a^2*cos(2*f*x + 2*e) + a^2)*sin(3*f*x + 3*e))*sqrt(a)
*sqrt(c)/((c^4*cos(8*f*x + 8*e)^2 + 36*c^4*cos(7*f*x + 7*e)^2 + 256*c^4*cos
(6*f*x + 6*e)^2 + 676*c^4*cos(5*f*x + 5*e)^2 + 900*c^4*cos(4*f*x + 4*e)^2 +
 676*c^4*cos(3*f*x + 3*e)^2 + 256*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x +
 e)^2 + c^4*sin(8*f*x + 8*e)^2 + 36*c^4*sin(7*f*x + 7*e)^2 + 256*c^4*sin(6*
f*x + 6*e)^2 + 676*c^4*sin(5*f*x + 5*e)^2 + 900*c^4*sin(4*f*x + 4*e)^2 + 67
6*c^4*sin(3*f*x + 3*e)^2 + 256*c^4*sin(2*f*x + 2*e)^2 - 192*c^4*sin(2*f*x +
 2*e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*
(6*c^4*cos(7*f*x + 7*e) - 16*c^4*cos(6*f*x + 6*e) + 26*c^4*cos(5*f*x + 5*e)
 - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2
*e) + 6*c^4*cos(f*x + e) - c^4)*cos(8*f*x + 8*e) - 12*(16*c^4*cos(6*f*x + 6
*e) - 26*c^4*cos(5*f*x + 5*e) + 30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x
+ 3*e) + 16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(7*f*x + 7*
e) - 32*(26*c^4*cos(5*f*x + 5*e) - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f
*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x +
 6*e) - 52*(30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x + 3*e) + 16*c^4*cos(
2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 60*(26*c^4*cos(
3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*
x + 4*e) - 52*(16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*
x + 3*e) - 32*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 4*(3*c^4*sin(7*
```

```
f*x + 7*e) - 8*c^4*sin(6*f*x + 6*e) + 13*c^4*sin(5*f*x + 5*e) - 15*c^4*sin(
4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3*c^4*sin
(f*x + e))*sin(8*f*x + 8*e) - 24*(8*c^4*sin(6*f*x + 6*e) - 13*c^4*sin(5*f*x
 + 5*e) + 15*c^4*sin(4*f*x + 4*e) - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f
*x + 2*e) - 3*c^4*sin(f*x + e))*sin(7*f*x + 7*e) - 64*(13*c^4*sin(5*f*x + 5
*e) - 15*c^4*sin(4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x +
 2*e) + 3*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 104*(15*c^4*sin(4*f*x + 4*e)
 - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f*x + 2*e) - 3*c^4*sin(f*x + e))*s
in(5*f*x + 5*e) - 120*(13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3
*c^4*sin(f*x + e))*sin(4*f*x + 4*e) - 104*(8*c^4*sin(2*f*x + 2*e) - 3*c^4*s
in(f*x + e))*sin(3*f*x + 3*e))*f)
```

mupad [B] time = 6.04, size = 199, normalized size = 4.74

$$-\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,52{}\mathrm{i}}{3\,c^4\,f}+\frac{a^2\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^4\,f}\right)}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,28{}\mathrm{i}-{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,28{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,12{}\mathrm{i}-{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] -((c - c/cos(e + f*x))^(1/2)*((a^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/c
os(e + f*x))^(1/2)*52i)/(3*c^4*f) + (a^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x
)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)
*28i - exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e
 + 3*f*x)*12i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.131 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

```
[Out] -1/8*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)-1/48*(a+a*s
ec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.30, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(8*f*(c - c*Sec[e + f*x])^(9/2))
 - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Sec[e + f*x])^(
7/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}dx}{8c}$$
$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+f}{48cf(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A] time = 0.83, size = 92, normalized size = 1.05

$$-\frac{a^2(17\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx))-5)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{12c^4f(\cos(e+fx)-1)^4\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -1/12*(a^2*(-5 + 17*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])
*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c^4*f*(-1 + Cos[e + f*x])^4*
Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.45, size = 166, normalized size = 1.89

$$\frac{\left(6\,a^2\cos\left(fx+e\right)^4 - 3\,a^2\cos\left(fx+e\right)^3 + 4\,a^2\cos\left(fx+e\right)^2 - a^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^5f\cos\left(fx+e\right)^4 - 4\,c^5f\cos\left(fx+e\right)^3 + 6\,c^5f\cos\left(fx+e\right)^2 - 4\,c^5f\cos\left(fx+e\right) + c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x + e)^2 - a
^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)1/2*a^2*(1/24*(4*a^5*(-a*tan(1/2*(f*
x+exp(1)))^2+a)-a^6)/(-a*tan(1/2*(f*x+exp(1)))^2)^4+1/24*a^2)/c^4/sqrt(-a*c
)/f/abs(a)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [A] time = 1.99, size = 85, normalized size = 0.97

$$-\frac{\left(7\cos\left(fx+e\right)-1\right)\left(\sin^{5}\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a^{2}}{48f\left(-1+\cos\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)^{4}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/48/f*(7*cos(f*x+e)-1)*sin(f*x+e)^5*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(
-1+cos(f*x+e))^2/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(9/2)*a^2
```

maxima [B] time = 4.43, size = 2719, normalized size = 30.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(70*a^2*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 70*a^2*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 3*a^2*sin(2*f*x + 2*e) + (3*a^2*sin(6*f*x + 6*e) + 10*a^2*s
in(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) + (3*a^2*sin(8*f
*x + 8*e) + 60*a^2*sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin
(2*f*x + 2*e) - 32*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 32*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*
a^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e)
+ 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*a
^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e) +
 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a^2*sin(8*f*x + 8*e) + 60*a^2*
sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin(2*f*x + 2*e))*cos(
```

```
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(6*f*x + 6*e)
+ 10*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - (70*
a^2*cos(4*f*x + 4*e) - 3*a^2)*sin(6*f*x + 6*e) + 10*(7*a^2*cos(2*f*x + 2*e)
 + a^2)*sin(4*f*x + 4*e) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*x + 6*e
) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) - 32*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 3*a^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*e) + 630
*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*
e) + 630*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*
x + 6*e) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) + 3*a^2)*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos
(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x + 6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2
+ 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 3136*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8
*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 +
3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 6
4*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5
*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5
)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4
*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x
 + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)
*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x
+ 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f
*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8
*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28
*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) -
 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 2
8*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(2*c^5*sin
(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x
```

```
+ 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x +
6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*
x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e
) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*
f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 8*c^5*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x +
 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e))*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [B] time = 6.79, size = 350, normalized size = 3.98

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,68{}\mathrm{i}}{3\,c^5\,f}-\frac{a^2\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,52{}\mathrm{i}}{3\,c^5\,f}+\frac{a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{3\,c^5\,f}\right.}{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,84{}\mathrm{i}-{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,96{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,54{}\mathrm{i}-\mathrm{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(
1/2)*68i)/(3*c^5*f) - (a^2*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f
*x))^(1/2)*52i)/(3*c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
/cos(e + f*x))^(1/2)*80i)/(3*c^5*f) - (a^2*exp(e*5i + f*x*5i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(4*
e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin
(e + f*x)*84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i
)*sin(3*e + 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i
 + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{240c^2f(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{40cf(c-c\sec(e+fx))^{9/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

```
[Out] -1/10*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)-1/40*(a+a
*sec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)-1/240*(a+a*sec(f*x
+e))^(5/2)*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.46, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{240c^2f(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{40cf(c-c\sec(e+fx))^{9/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(10*f*(c - c*Sec[e + f*x])^(11/2
)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(40*c*f*(c - c*Sec[e + f*x])
^(9/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(240*c^2*f*(c - c*Sec[e
 + f*x])^(7/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

```
&& !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx}{5c}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}}$$

Mathematica [A] time = 1.24, size = 102, normalized size = 0.77

$$\frac{a^2(170\cos(e+fx)-140\cos(2(e+fx))+30\cos(3(e+fx))-15\cos(4(e+fx))-141)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e}{120c^5f(\cos(e+fx)-1)^5\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a^2*(-141 + 170*Cos[e + f*x] - 140*Cos[2*(e + f*x)] + 30*Cos[3*(e + f*x)]
- 15*Cos[4*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(120*c^
5*f*(-1 + Cos[e + f*x])^5*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.44, size = 194, normalized size = 1.46

$$\frac{\left(15\,a^2\cos\left(fx+e\right)^5-15\,a^2\cos\left(fx+e\right)^4+20\,a^2\cos\left(fx+e\right)^3-10\,a^2\cos\left(fx+e\right)^2+2\,a^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(f}{\cos(}}}{15\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/15*(15*a^2*cos(f*x + e)^5 - 15*a^2*cos(f*x + e)^4 + 20*a^2*cos(f*x + e)^3
 - 10*a^2*cos(f*x + e)^2 + 2*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
```

```
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5
- 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^
2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)1/4*a^2*(1/60*(-5*a^6*(-a*tan(1/2*(f
*x+exp(1)))^2+a)+a^7+10*a^5*(-a*tan(1/2*(f*x+exp(1)))^2+a)^2)/(-a*tan(1/2*(
f*x+exp(1)))^2)^5+1/60*a^2)/c^5/sqrt(-a*c)/f/abs(a)/sign(tan(1/2*(f*x+exp(1
)))^2-1)
```

maple [A] time = 2.03, size = 95, normalized size = 0.71

$$-\frac{\left(31\left(\cos^{2}\left(fx+e\right)\right)-8\cos\left(fx+e\right)+1\right)\left(\sin^{5}\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,a^{2}}{240f\left(-1+\cos\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)^{5}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x)
```

```
[Out] -1/240/f*(31*cos(f*x+e)^2-8*cos(f*x+e)+1)*sin(f*x+e)^5*(a*(1+cos(f*x+e))/co
s(f*x+e))^(1/2)/(-1+cos(f*x+e))^2/cos(f*x+e)^5/(c*(-1+cos(f*x+e))/cos(f*x+e
))^(11/2)*a^2
```

maxima [B] time = 23.29, size = 4108, normalized size = 30.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
```

```
[Out] -2/15*(1350*a^2*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 1350*a^2*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e) - 30*a^2*sin(2*f*x + 2*e) - 10*(3*a^2*sin(8*f*x + 8*e)
+ 17*a^2*sin(6*f*x + 6*e) + 17*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e
```

```
))*cos(10*f*x + 10*e) - 1350*(a^2*sin(6*f*x + 6*e) + a^2*sin(4*f*x + 4*e))*
cos(8*f*x + 8*e) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) +
290*a^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*
e) - 80*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*
sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e
) + 450*a^2*sin(6*f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x
 + 2*e) - 72*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*
a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(47*a^2*sin(10*f*x + 10*e) + 855*a^
2*sin(8*f*x + 8*e) + 2730*a^2*sin(6*f*x + 6*e) + 2730*a^2*sin(4*f*x + 4*e)
+ 855*a^2*sin(2*f*x + 2*e) + 240*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 160*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10
*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e) + 450*a^2*sin(6*f*x + 6*e) + 450*a^
2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x + 2*e) + 20*a^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) + 290*a
^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(3*a^2*cos(8*f*x +
 8*e) + 17*a^2*cos(6*f*x + 6*e) + 17*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x
 + 2*e))*sin(10*f*x + 10*e) + 30*(45*a^2*cos(6*f*x + 6*e) + 45*a^2*cos(4*f*
x + 4*e) - a^2)*sin(8*f*x + 8*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*s
in(6*f*x + 6*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*sin(4*f*x + 4*e) +
 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x + 8*e) + 290*a^2*cos(6*f*x
+ 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(2*f*x + 2*e) - 80*a^2*cos(7/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*cos(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 3*a^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 20*(7*a^2*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^
2*cos(6*f*x + 6*e) + 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) -
72*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*a^2*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(47*a^2*cos(10*f*x + 10*e) + 855*a^2*
cos(8*f*x + 8*e) + 2730*a^2*cos(6*f*x + 6*e) + 2730*a^2*cos(4*f*x + 4*e) +
855*a^2*cos(2*f*x + 2*e) + 240*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 160*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 47*a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*(7*a^2
*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^2*cos(6*f*x + 6*e) +
 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) + 20*a^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x
 + 8*e) + 290*a^2*cos(6*f*x + 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(
2*f*x + 2*e) + 3*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))
```

```
*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 10*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 +
 44100*c^6*cos(6*f*x + 6*e)^2 + 44100*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos
(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 14400*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
63504*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^
6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 +
 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin
(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*si
n(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 14400*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 63504*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c
^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) +
 c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(
4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(10*f*x + 10*e) + 90*(210*
c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) +
 c^6)*cos(8*f*x + 8*e) + 420*(210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x +
 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f
*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*
cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120
*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + c^6)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos
(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^
6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 504*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 21
0*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e)
 - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*
cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45
*c^6*cos(2*f*x + 2*e) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + c^6)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(
c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e)
 + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^
6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(
10*f*x + 10*e) + 1350*(14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) +
3*c^6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*c^6*sin(4*f*x + 4*e) +
3*c^6*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 20*(c^6*sin(10*f*x + 10*e) + 45*
```

```
c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e)
+ 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8
*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*
sin(2*f*x + 2*e) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^
6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*sin(10*f*x + 10*e) + 45*c^6*si
n(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c
^6*sin(2*f*x + 2*e) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10
*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*
x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x
 + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [B] time = 7.17, size = 419, normalized size = 3.15

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,136{}\mathrm{i}}{3\,c^6\,f}-\frac{a^2\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,1688{}\mathrm{i}}{15\,c^6\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\right.}}}{3\,c^6\,f}\right.}{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),x
)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(
1/2)*136i)/(3*c^6*f) - (a^2*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e +
f*x))^(1/2)*1688i)/(15*c^6*f) + (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a
 + a/cos(e + f*x))^(1/2)*160i)/(3*c^6*f) - (a^2*exp(e*6i + f*x*6i)*cos(3*e
+ 3*f*x)*(a + a/cos(e + f*x))^(1/2)*124i)/(3*c^6*f) + (a^2*exp(e*6i + f*x*6
i)*cos(4*e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*8i)/(c^6*f) - (a^2*exp(e*6i
+ f*x*6i)*cos(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*
6i + f*x*6i)*sin(e + f*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i +
 exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*
f*x)*88i + exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin
(6*e + 6*f*x)*2i)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
[Out] Timed out
```

3.133 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-4*c^3*ln(
1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*
c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$-\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[a + a*Sec[e + f*x]]) - (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f
*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
```

```
!(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}+(2c)\int \frac{\sec(e+fx)(c-c\sec(e+}{\sqrt{a+a\sec(e+fx}}$$
$$= -\frac{2c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+}{2f\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\sqrt{c-c\sec(e+fx)}\,\tan}{f\sqrt{a+a\sec(e+f}}$$

Mathematica [C] time = 1.48, size = 141, normalized size = 1.01

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)-6\cos(e+fx)+\left(8\right.\right.}{2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(1 - 6*Cos[e + f*x] + 8*Log[1 + E^(I*(e + f*x))] + Co
s[2*(e + f*x)]*(8*Log[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]
) - 4*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]]
)/(2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)8*c^2*(1/4*(4*c^2*sqrt(-a*c)*(c*tan(
1/2*(f*x+exp(1)))^2-c)+c^3*sqrt(-a*c)+3*c*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)
))^2-c)^2)/(c*tan(1/2*(f*x+exp(1)))^2-c)^2/a/abs(c)-1/2*c*sqrt(-a*c)*ln(c*t
an(1/2*(f*x+exp(1)))^2-c)/a/abs(c))*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f
*x+exp(1))))/f
```

maple [A] time = 2.06, size = 165, normalized size = 1.19

$$-\frac{\left(8\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^{2}\left(fx+e\right)\right)+8\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^{2}\left(fx+e\right)\right)+7\left(\cos^{2}\left(fx+e\right)\right.}{2f\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+8*ln(-(-s
in(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+7*cos(f*x+e)^2+6*cos(f*x+e
)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(a*(1+cos(f*x+e))/cos(
f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))^2/a
```

maxima [B] time = 0.60, size = 737, normalized size = 5.30

$$\frac{2\left(c^{2}\cos\left(2fx+2e\right)\sin\left(4fx+4e\right)-c^{2}\cos\left(4fx+4e\right)\sin\left(2fx+2e\right)-c^{2}\sin\left(2fx+2e\right)+2\left(c^{2}\cos\left(4fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c^2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e) - c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
+ 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
```

```
 + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2
*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(
4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x
+ 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*
cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 3*(c^2*sin
(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 3*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(c^2*cos(4*f*x + 4*e) +
 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 3*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(4*f*x
 + 4*e)^2 + 4*a*cos(2*f*x + 2*e)^2 + a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*f*x + 2*e)^2 + 2*(2*a*cos(2*f*x + 2*e) +
 a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.134 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=94

$$-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)-c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[
a + a*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + (2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+}{\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{c\sqrt{c-c\sec(e+fx)}\,\tan(e}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 1.83, size = 173, normalized size = 1.84

$$\frac{ce^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2\cos\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}\,\left(-1+\left(4\log\left(1+e^{i(e+f}\right.\right.\right.}{2f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c*(1 + E^((2*I)*(e + f*x)))^2*Cos[(e + f*x)/2]*Cot[(e + f*x)/2]*(-1 + Cos[
e + f*x]*(4*Log[1 + E^(I*(e + f*x))] - 2*Log[1 + E^((2*I)*(e + f*x))]))*Sec
[e + f*x]^3*Sqrt[c - c*Sec[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2]
))/(2*E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])
])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)^2 - c\sec\left(fx+e\right)\right)\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(-c*sec(f*x + e) + c)/sqr
t(a*sec(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)4*c*(1/2*(c^2*sqrt(-a*c)*(c*tan(1/2*
(f*x+exp(1)))^2-c)+c^3*sqrt(-a*c))/a/(c*tan(1/2*(f*x+exp(1)))^2-c)/abs(c)-1
/2*c^2*sqrt(-a*c)*ln(c*tan(1/2*(f*x+exp(1)))^2-c)/a/abs(c))*sign(tan(1/2*(f
*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/c/f
```

maple [A] time = 1.89, size = 149, normalized size = 1.59

$$-\frac{\left(2\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+2\cos\left(fx+e\right)\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\cos\left(fx+e\right)+1\right)\cos}{f\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(2*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*
ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+cos(f*x+e)+1)*cos(f*x+e)*(c*(-1+
cos(f*x+e))/cos(f*x+e))^(3/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e
)/(-1+cos(f*x+e))/a
```

maxima [B] time = 0.58, size = 276, normalized size = 2.94

$$-\frac{2\left(c\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)-\left(c\cos\left(2fx+2e\right)^{2}+c\sin\left(2fx+2e\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(c*cos(2*f*x + 2*e)^2
+ c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 1) - (c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/sqrt(a*(sec(e + f*x) +
 1)), x)
```

3.135 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=50

$$-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Rubi [A] time = 0.12, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3952}

$$-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.43, size = 140, normalized size = 2.80

$$\frac{i\left(1+e^{i(e+fx)}\right)\sqrt{\frac{c\left(-1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)}{f\left(-1+e^{i(e+fx)}\right)\sqrt{\frac{a\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (I*(1 + E^(I*(e + f*x)))*Sqrt[(c*(-1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e
 + f*x)))]*(2*Log[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]))/((-
1 + E^(I*(e + f*x)))*Sqrt[(a*(1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*
x)))]*f)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c\sec\left(fx+e\right)+c}\,\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)c^2*sign(t
an(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*sign(cos(f*x+exp(1)))*ln(abs(
c*tan(1/2*(f*x+exp(1)))^2-c))/sqrt(-a*c)/f/abs(c)/sign(tan(1/2*(f*x+exp(1))
)^2-1)
```

maple [B] time = 1.98, size = 116, normalized size = 2.32

$$-\frac{\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\cos\left(fx+e\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}}{f\sin\left(fx+e\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-sin(f*x+e)-1+cos(f*x
+e))/sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos
(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/a
```

maxima [A] time = 0.45, size = 64, normalized size = 1.28

$$-\frac{\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{-a}}+\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{-a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(-a) + sqrt(c)*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(-a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)
), x)
```

3.136 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -arctanh(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3959, 3770}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -((ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\int \csc(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.79, size = 94, normalized size = 2.00

$$\frac{4i\left(-1+e^{i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)}{f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),
x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^(I*(e + f*x))]*Cos[(e + f*x)/2]^2*S
ec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c
*Sec[e + f*x]])
```

fricas [A] time = 0.57, size = 204, normalized size = 4.34

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)}{2\,acf}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\ldots}{ac\sin(fx\ldots}\right)}{acf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x +
e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), sqr
t(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))/(a*c*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableUnab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
```

```
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
```

```
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discon
tinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integra
tion of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*p
```

```
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Disc
ontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
```

```
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
```

: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n

```
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
```

```
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
```

ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable

```
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
```

```
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/
```

```
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not che
ckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or
 sign assumes constant sign by intervals (correct if the argument is real):
Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_n
ostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) w
ere not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant s
ign by intervals (correct if the argument is real):Check [abs(t_nostep),abs
(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by
 intervals (correct if the argument is real):Check [abs(t_nostep)]Sign erro
r %%%{c,2%%%}Evaluation time: 2.24Limit: Max order reached or unable to mak
e series expansion Error: Bad Argument Value
```

maple [A] time = 1.90, size = 85, normalized size = 1.81

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{f\sin\left(fx+e\right)ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*ln(-(-1+cos(f*x+e))/sin(f*x+e))*(c
*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)/c/a
```

maxima [A] time = 0.55, size = 44, normalized size = 0.94

$$-\frac{\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)-\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(arctan2(sin(f*x + e), cos(f*x + e) + 1) - arctan2(sin(f*x + e), cos(f*x +
 e) - 1))/(sqrt(a)*sqrt(c)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1
))), x)
```

3.137 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)-1/2*arctanh
(cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] -Tan[e + f*x]/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
```

```
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{\int \frac{\sec(\ldots}{\sqrt{a+a\sec(e+fx)}\ldots}}{2\ldots}$$

$$= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+\ldots}{2c\sqrt{a+a\sec(\ldots}}$$

$$= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} - \frac{\tanh^{-1}(\mathrm{co}\ldots}{2cf\sqrt{a+a\sec\ldots}}$$

Mathematica [C] time = 0.90, size = 79, normalized size = 0.83

$$-\frac{\tan(e+fx)\left(1+2(\cos(e+fx)-1)\tanh^{-1}\left(e^{i(e+fx)}\right)\right)}{2cf(\cos(e+fx)-1)\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)
),x]
```

```
[Out] -1/2*((1 + 2*ArcTanh[E^(I*(e + f*x))]*(-1 + Cos[e + f*x]))*Tan[e + f*x])/(c
*f*(-1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

fricas [B] time = 0.55, size = 382, normalized size = 4.02

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2+\left(ac\cos\left(fx+e\right)^2+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^2-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+\ldots\right.}{4\left(ac^2f\cos\left(fx+e\right)-ac^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e) - a*c^2*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) - 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e)
 - a*c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
```

```
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
```

```
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
```

```
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checked
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)War
ning, integration of abs or sign assumes constant sign by intervals (correc
t if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t
_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
```

```
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
```

nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch

```
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
```

```
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
```

ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p

```
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
```

```
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
```

```
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign
 assumes constant sign by intervals (correct if the argument is real):Check
 [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep
 were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t
_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nos
tep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_n
ostep^2-1)]Warning, integration of abs or sign assumes constant sign by int
```

```
ervals (correct if the argument is real):Check [abs(t_nostep)]Sign error %%
%{c,2%%%}Evaluation time: 2.28Limit: Max order reached or unable to make se
ries expansion Error: Bad Argument Value
```

maple [A] time = 1.98, size = 131, normalized size = 1.38

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\cos\left(fx+e\right)-\cos\left(fx+e\right)-2\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-1\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}}{4f\cos\left(fx+e\right)\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sin\left(fx+e\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*(-1+cos(f*x+e))*(2*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-cos(f*
x+e)-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/
2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)/a
```

maxima [B] time = 0.59, size = 406, normalized size = 4.27

$$\frac{\left(\left(2\left(2\cos\left(fx+e\right)-1\right)\cos\left(2fx+2e\right)-\cos\left(2fx+2e\right)^{2}-4\cos\left(fx+e\right)^{2}-\sin\left(2fx+2e\right)^{2}+4\sin\left(2fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(
f*x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f
*x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (
2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x +
e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e
)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*cos(f
*x + e)*sin(2*f*x + 2*e) - 2*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a*c^2*cos(2*f*x + 2*e)^2 + 4*a*c^2*cos(f*x + e)^2 + a*c
^2*sin(2*f*x + 2*e)^2 - 4*a*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a*c^2*sin
(f*x + e)^2 - 4*a*c^2*cos(f*x + e) + a*c^2 - 2*(2*a*c^2*cos(f*x + e) - a*c^
2)*cos(2*f*x + 2*e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(3/2)), x)
```

3.138 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e}{4f\sqrt{a\sec(e+fx)+}}$$

```
[Out] -1/4*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-1/4*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e
))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e}{4f\sqrt{a\sec(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] -Tan[e + f*x]/(4*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)) - T
an[e + f*x]/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[
c - c*Sec[e + f*x]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{\int \frac{\sec}{\sqrt{a+a\sec(e+fx)}}}{}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{}{4cf\sqrt{a+a\sec}}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{}{4cf\sqrt{a+a\sec}}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{}{4cf\sqrt{a+a\sec}}$$

Mathematica [C] time = 0.83, size = 91, normalized size = 0.65

$$-\frac{\tan(e+fx)\left(3\cos(e+fx)+8\sin^4\left(\frac{1}{2}(e+fx)\right)\tanh^{-1}\left(e^{i(e+fx)}\right)-2\right)}{4c^2f(\cos(e+fx)-1)^2\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
),x]
```

```
[Out] -1/4*((-2 + 3*Cos[e + f*x] + 8*ArcTanh[E^(I*(e + f*x))]*Sin[(e + f*x)/2]^4)
*Tan[e + f*x])/(c^2*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt
[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.55, size = 456, normalized size = 3.26

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\,\sqrt{-ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(f\right.\right.}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right.}{8\left(ac^{3}f\cos\left(fx+e\right)^{2}-2\,ac^{3}f\cos\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 - 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 - 2*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*
sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
```

```
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
gn by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep
+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
```

p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi

/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) wer
e not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Warning, integration of abs or sign assumes constant sign by inte
rvals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (

```
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were
 not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
```

```
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
```

```
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
```

t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)

```
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
```

ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una

```
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
```

k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration o
f abs or sign assumes constant sign by intervals (correct if the argument i
s real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nost
ep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warn
ing, integration of abs or sign assumes constant sign by intervals (correct
 if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities a
t zeroes of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes o

```
f cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by inte
rvals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2
-1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]Sign error %%%{c,2%%
%}Evaluation time: 2.62Limit: Max order reached or unable to make series ex
pansion Error: Bad Argument Value
```

maple [A] time = 2.28, size = 170, normalized size = 1.21

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(4\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-8\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\cos\left(fx+e\right)-5\left(\cos^{2}\left(fx+e\right)\right)}{16f\cos\left(fx+e\right)^{2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sin\left(fx+e\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/16/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-8*l
n(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-5*cos(f*x+e)^2+4*ln(-(-1+cos(f*x+
e))/sin(f*x+e))-2*cos(f*x+e)+3)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x
+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)/a
```

maxima [B] time = 0.66, size = 1201, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e)
+ 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos
(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
+ 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x
+ 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*s
in(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e
) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x +
 e) + 1) - (2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1
)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x
 + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) -
1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(
3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(
4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e)
- 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f
*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos
(f*x + e) - 1) - 2*(3*sin(3*f*x + 3*e) - 4*sin(2*f*x + 2*e) + 3*sin(f*x + e
))*cos(4*f*x + 4*e) + 2*(3*cos(3*f*x + 3*e) - 4*cos(2*f*x + 2*e) + 3*cos(f*
x + e))*sin(4*f*x + 4*e) - 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) + 4*
(cos(f*x + e) + 2)*sin(2*f*x + 2*e) + 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) -
 4*cos(2*f*x + 2*e)*sin(f*x + e) - 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a*c^3*
cos(4*f*x + 4*e)^2 + 16*a*c^3*cos(3*f*x + 3*e)^2 + 36*a*c^3*cos(2*f*x + 2*e
)^2 + 16*a*c^3*cos(f*x + e)^2 + a*c^3*sin(4*f*x + 4*e)^2 + 16*a*c^3*sin(3*f
*x + 3*e)^2 + 36*a*c^3*sin(2*f*x + 2*e)^2 - 48*a*c^3*sin(2*f*x + 2*e)*sin(f
*x + e) + 16*a*c^3*sin(f*x + e)^2 - 8*a*c^3*cos(f*x + e) + a*c^3 - 2*(4*a*c
^3*cos(3*f*x + 3*e) - 6*a*c^3*cos(2*f*x + 2*e) + 4*a*c^3*cos(f*x + e) - a*c
^3)*cos(4*f*x + 4*e) - 8*(6*a*c^3*cos(2*f*x + 2*e) - 4*a*c^3*cos(f*x + e) +
 a*c^3)*cos(3*f*x + 3*e) - 12*(4*a*c^3*cos(f*x + e) - a*c^3)*cos(2*f*x + 2*
e) - 4*(2*a*c^3*sin(3*f*x + 3*e) - 3*a*c^3*sin(2*f*x + 2*e) + 2*a*c^3*sin(f
*x + e))*sin(4*f*x + 4*e) - 16*(3*a*c^3*sin(2*f*x + 2*e) - 2*a*c^3*sin(f*x
+ e))*sin(3*f*x + 3*e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(5/2)), x)
```

3.139 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a\sec(e+fx)+a}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)+4*c^3*ln(1+sec
(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+2*c^2
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3954, 3955, 3952}

$$\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a\sec(e+fx)+a}}+\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sq
rt[c - c*Sec[e + f*x]]) + (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a*
f*Sqrt[a + a*Sec[e + f*x]]) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$

$$= \frac{2c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

$$= \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 1.05, size = 183, normalized size = 1.29

$$-\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}\,\left(4\log\left(1+e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1+e^{i(e+fx)}\right)\right.\right.}{af(\cos(e+fx)+1)\sqrt{a(s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c^2*Cot[(e + f*x)/2]*(-1 + 4*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(-5
 + 8*Log[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e
+ f*x)]*(4*Log[1 + E^(I*(e + f*x))] - 2*Log[1 + E^((2*I)*(e + f*x))]) - 2*L
og[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1
 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2
*sec(f*x + e) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-4*c^2*(1/2*(2*c*sqrt(-a*c)*(c*tan(1
/2*(f*x+exp(1)))^2-c)+c^2*sqrt(-a*c))/a^2/(c*tan(1/2*(f*x+exp(1)))^2-c)/abs
(c)-1/2*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)/a^2/abs(c)-c*sqrt(-a*c)*ln
(c*tan(1/2*(f*x+exp(1)))^2-c)/a^2/abs(c))*sign(tan(1/2*(f*x+exp(1)))^3+tan(
1/2*(f*x+exp(1))))/f
```

maple [A] time = 1.87, size = 235, normalized size = 1.65

$$-\frac{\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^2\left(fx+e\right)\right)+4\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+4*ln(-(-sin
(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-cos(f*x+e)^2+4*cos(f*x+e)*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-sin(f*x+e)-1+co
s(f*x+e))/sin(f*x+e))+4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*
cos(f*x+e)^2*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e
))/a^2
```

maxima [B] time = 0.75, size = 2035, normalized size = 14.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*(8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*c^2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e) + 2*c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
 + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(
2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*
c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) + 4*(c^2*cos(4*f*x + 4*e) + 2*
c^2*cos(2*f*x + 2*e) + 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2
*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2
*e) + 2*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c
^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (16*c^2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 5*c^2*sin(4*f*x + 4*e) - 6*c^2*sin(2*f*x + 2*e) + 8*(c^2*cos(4*f*x
+ 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (5*c^2*
sin(4*f*x + 4*e) + 6*c^2*sin(2*f*x + 2*e) - 8*(c^2*cos(4*f*x + 4*e) + 2*c^2
*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*c^2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*
(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ (5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*(c^2*sin(4*f
*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
```

```
*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqr
t(c)/((a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2
*f*x + 2*e) + 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e) + 2*a
^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*
f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)

[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)

[Out] Timed out
```

3.140 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] c^2*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a
*Sec[e + f*x])^(3/2))
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$
$$= \frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{c\sqrt{c-c\sec(e+fx)}\,\tan(e}{f(a+a\sec(e+fx))^{3}}$$

Mathematica [C] time = 1.06, size = 132, normalized size = 1.39

$$-\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\,\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)+\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e\right.\right.\right.}{af(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c*Cot[(e + f*x)/2]*(-2 + 2*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(2*Lo
g[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]) - Log[1 + E^((2*I)*(
e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + S
ec[e + f*x])]))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)^{2}-c\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2*c^3*(1/2*c*ln(c*tan(1/2*(f*x+exp(
1)))^2-c)+1/2*(c*tan(1/2*(f*x+exp(1)))^2-c))*sign(tan(1/2*(f*x+exp(1)))^3+t
an(1/2*(f*x+exp(1))))/sqrt(-a*c)/a/c/f/abs(c)
```

maple [B] time = 1.93, size = 193, normalized size = 2.03

$$-\frac{\left(\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\cos\left(fx+e\right)\ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)-\cos\left(fx+e\right)+\ln\left(-\frac{-1+\mathrm{co}\cdots}{\cdots}\right.\right.}{f\sin\left(fx+e\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(-
(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)+ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+1)*(c*(-1+cos(
f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/
sin(f*x+e)^3/a^2
```

maxima [A] time = 0.47, size = 99, normalized size = 1.04

$$\frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}\,a}+\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}\,a}+\frac{c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\sqrt{-a}\,a\left(\cos\left(fx+e\right)+1\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a) + c^(3/2)*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a) + c^(3/2)*sin(f*x + e)^
2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}{\cos\left(e+fx\right)\left(a+\frac{a}{\cos(e+fx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1))
**(3/2), x)
```

3.141 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] 1/2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$\frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.19, size = 42, normalized size = 1.00

$$\frac{\csc(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (Csc[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [B] time = 0.45, size = 78, normalized size = 1.86

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\left(a^2f\cos\left(fx+e\right)+a^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-1/2*c^2*(c*tan(1/2*(f*
x+exp(1)))^2-c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*sign(co
s(f*x+exp(1)))/sqrt(-a*c)/a/c/f/abs(c)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [A] time = 2.07, size = 73, normalized size = 1.74

$$\frac{\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^2}{2f\sin\left(fx+e\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1
/2)*cos(f*x+e)*(-1+cos(f*x+e))^2/sin(f*x+e)^3/a^2
```

maxima [A] time = 0.47, size = 54, normalized size = 1.29

$$\frac{\sqrt{c}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{2\sqrt{-a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/(sqrt(-a)*a*f)
```

mupad [B] time = 2.55, size = 50, normalized size = 1.19

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{a\,f\,\sin\left(e+f\,x\right)\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] (c - c/cos(e + f*x))^(1/2)/(a*f*sin(e + f*x)*((a*(cos(e + f*x) + 1))/cos(e
+ f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
3/2), x)
```

3.142 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arctanh(
cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
```

```
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+f}{\sqrt{a+a\sec(e+fx)}\sqrt{c}}}{2a}$$

$$= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx}{2a\sqrt{a+a\sec(e+}}$$

$$= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\tanh^{-1}(\cos(}{2af\sqrt{a+a\sec(e}}$$

Mathematica [C] time = 1.40, size = 157, normalized size = 1.65

$$-\frac{\sin\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(1+2(\cos(e+fx)+1)\tanh^{-1}\left(e^{i(e+fx)}\right)\right)}{\sqrt{2}\,af\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -(((1 + 2*ArcTanh[E^(I*(e + f*x))]*(1 + Cos[e + f*x]))*Sec[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/(Sqrt[2]*a*(1 + E^
(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

fricas [B] time = 0.55, size = 380, normalized size = 4.00

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)}{4\left(a^{2}cf\cos\left(fx+e\right)+a^{2}cf\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e) + a^2*c*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) + 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e)
 + a^2*c*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
```

```
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
```

```
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
```

```
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checked
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)War
ning, integration of abs or sign assumes constant sign by intervals (correc
t if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t
_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
```

```
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
```

```
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
```

```
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
```

```
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
```

```
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
```

```
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
```

(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos

```
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign
 assumes constant sign by intervals (correct if the argument is real):Check
 [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep
 were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t
_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nos
tep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assum
es constant sign by intervals (correct if the argument is real):Check [abs(
```

```
t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes con
stant sign by intervals (correct if the argument is real):Check [abs(t_nost
ep)]Sign error %%%{c,2%%%}Evaluation time: 2.59Limit: Max order reached or
unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 2.10, size = 123, normalized size = 1.29

$$\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(-1+\cos\left(fx+e\right)\right)^{2}\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\cos\left(fx+e\right)+\cos\left(fx+e\right)+2\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-}{4f\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(2*ln(-(-1+cos(
f*x+e))/sin(f*x+e))*cos(f*x+e)+cos(f*x+e)+2*ln(-(-1+cos(f*x+e))/sin(f*x+e))
-1)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/a^2
```

maxima [B] time = 0.58, size = 397, normalized size = 4.18

$$-\frac{\left(\left(2\left(2\cos\left(fx+e\right)+1\right)\cos\left(2fx+2e\right)+\cos\left(2fx+2e\right)^{2}+4\cos\left(fx+e\right)^{2}+\sin\left(2fx+2e\right)^{2}+4\sin\left(2f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos
(f*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(
f*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) -
(2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x +
 e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x +
e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 2*cos(
f*x + e)*sin(2*f*x + 2*e) + 2*cos(2*f*x + 2*e)*sin(f*x + e) + 2*sin(f*x + e
))*sqrt(a)*sqrt(c)/((a^2*c*cos(2*f*x + 2*e)^2 + 4*a^2*c*cos(f*x + e)^2 + a^
2*c*sin(2*f*x + 2*e)^2 + 4*a^2*c*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*c*si
n(f*x + e)^2 + 4*a^2*c*cos(f*x + e) + a^2*c + 2*(2*a^2*c*cos(f*x + e) + a^2
*c)*cos(2*f*x + 2*e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

3.143 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\csc(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2acf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*csc(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arct
anh(cos(f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Rubi [A] time = 0.18, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3959, 2611, 3770}

$$\frac{\csc(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2acf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Csc[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) -
(ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
```

```
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)\int \cot^2(e+fx)\csc(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+}{2ac\sqrt{a+a\sec(}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tanh^{-1}(\mathrm{co}}{2acf\sqrt{a+a\sec}}$$

Mathematica [C] time = 0.80, size = 69, normalized size = 0.66

$$\frac{\csc(e+fx)-2\tan(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)}{2acf\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] (Csc[e + f*x] - 2*ArcTanh[E^(I*(e + f*x))]*Tan[e + f*x])/(2*a*c*f*Sqrt[a*(1
 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.60, size = 402, normalized size = 3.87

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)\sin\left(fx+e\right.}{4\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f
*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^2*f*cos(f*x + e)^2
- a^2*c^2*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e)^2 - 1)*arctan(sqrt
(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^
2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
```

```
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
```

```
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
```

```
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checked
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)War
ning, integration of abs or sign assumes constant sign by intervals (correc
t if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant
sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t
_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
```

```
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
```

tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl

```
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
```

```
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
```

```
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
```

```
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
```

```
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
```

able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign
 assumes constant sign by intervals (correct if the argument is real):Check
 [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep
 were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t
_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nos
tep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_n
ostep^2-1)]Warning, integration of abs or sign assumes constant sign by int
ervals (correct if the argument is real):Check [abs(t_nostep)]Sign error %%
%{c,2%%%}Evaluation time: 2.45Limit: Max order reached or unable to make se
ries expansion Error: Bad Argument Value

maple [A] time = 2.04, size = 133, normalized size = 1.28

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\cos\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}}{2f\sin\left(fx+e\right)^{3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\cos\left(fx+e\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))^2*(cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-ln(-(
-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e))*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/
sin(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)/a^2
```

maxima [B] time = 0.58, size = 567, normalized size = 5.45

$$\frac{\left(\left(2\left(2\cos\left(2fx+2e\right)-1\right)\cos\left(4fx+4e\right)-\cos\left(4fx+4e\right)^{2}-4\cos\left(2fx+2e\right)^{2}-\sin\left(4fx+4e\right)^{2}+4\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*
cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), c
os(f*x + e) + 1) - (2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x
 + 4*e)^2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(s
in(f*x + e), cos(f*x + e) - 1) - 2*(sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*
f*x + 4*e) + 2*(cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*e) + 2*(2*co
s(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
- 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin
(f*x + e))*sqrt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f
*x + 2*e)^2 + a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a
^2*c^2 - 2*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+fx\right)\left(a+\frac{a}{\cos\left(e+fx\right)}\right)^{3/2}\left(c-\frac{c}{\cos\left(e+fx\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1)
)**(3/2)), x)
```

3.144 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\csc(e+fx)}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan}{4f(a\sec(e+fx)+}$$

```
[Out] 3/8*csc(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2)-3/8*arctanh(cos(f*
x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3960, 3959, 2611, 3770}

$$\frac{3\csc(e+fx)}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan}{4f(a\sec(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) - Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
```

```
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} + \frac{3\int \frac{}{(a+a\sec(e-}}{}$$

$$= -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} - \frac{(3\tan(e+f}{4ac^2\sqrt{a+a}}$$

$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{}{4f(a+a\sec(e}$$

$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{}{4f(a+a\sec(e}$$

Mathematica [C] time = 1.58, size = 122, normalized size = 0.84

$$-\frac{\tan(e+fx)\left(-2\cos(e+fx)+5\cos(2(e+fx))+24\sin^2\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)+1\right)}{16ac^2f(\cos(e+fx)-1)^2(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] -1/16*((1 - 2*Cos[e + f*x] + 5*Cos[2*(e + f*x)] + 24*ArcTanh[E^(I*(e + f*x)
)]*Sin[(e + f*x)/2]^2*Sin[e + f*x]^2)*Tan[e + f*x])/(a*c^2*f*(-1 + Cos[e +
f*x])^2*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*
x]])
```

fricas [A] time = 0.59, size = 544, normalized size = 3.73

$$\left[-\frac{3\left(\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)+1\right)\sqrt{-ac}\,\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\right.\right.}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right.}{16\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 -
a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 -
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3
*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
```

2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(c
os(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableWarning, assuming -2*a+a is positive. Hint: run assum
e to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi

```
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
```

)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is rea l):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+ t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4 *pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep /2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t _nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi /t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2) >(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/ 2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_ nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4* pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(- 4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/ t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep /2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t _nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4 *pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2) Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s

```
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
```

```
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
```

```
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
```

ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una

```
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
```

```
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
```

```
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning,
integration of abs or sign assumes constant sign by intervals (correct if t
he argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zer
oes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
```

```
vals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Disc
ontinuities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuitie
s at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant
 sign by intervals (correct if the argument is real):Check [abs(t_nostep),a
bs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign er
ror %%%{c,2%%%}Evaluation time: 2.97Limit: Max order reached or unable to m
ake series expansion Error: Bad Argument Value
```

maple [A] time = 2.17, size = 211, normalized size = 1.45

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(12\left(\cos^{3}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-12\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-5\left(\cos^{3}\left(f\right.\right.\right.}{32f\sin\left(fx+e\right)^{3}\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/32/f*(-1+cos(f*x+e))^2*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1
2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^3-12*ln(-(-1+co
s(f*x+e))/sin(f*x+e))*cos(f*x+e)-15*cos(f*x+e)^2+12*ln(-(-1+cos(f*x+e))/sin
```

```
(f*x+e))+9*cos(f*x+e)+3)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/c
os(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.145 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{c^3\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af(a\sec(e+fx)+a)^{3/2}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] 1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-c^3*ln(1+s
ec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-c
^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$-\frac{c^3\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af(a\sec(e+fx)+a)^{3/2}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c - c*Sec[e + f*x]])) - (c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a
*f*(a + a*Sec[e + f*x])^(3/2)) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(2*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a}$$

$$= -\frac{c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}}$$

$$= -\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{c^2\sqrt{c-c\sec(e+fx)}\,\tan}{af(a+a\sec(e+fx)}$$

Mathematica [C] time = 1.39, size = 178, normalized size = 1.23

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\,\left(6\log\left(1+e^{i(e+fx)}\right)-3\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+\right.\right.\right.}{2a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(-4 + 6*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(8*Lo
g[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e + f*x)]
*(2*Log[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]) - 3*Log[1 + E^
((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(2*a^2*f*(1 + Cos[e + f*x])^2
*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sqrt{-c\sec\left(fx+e\right)+c}}{a^3\sec\left(fx+e\right)^3+3\,a^3\sec\left(fx+e\right)^2+3\,a^3\sec\left(fx+e\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3
*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2*c^4*(1/4*(4*c^3*(c*tan(1/2*(f*x+ex
p(1)))^2-c)+c^2*(c*tan(1/2*(f*x+exp(1)))^2-c)^2)/c^4+1/2*ln(c*tan(1/2*(f*x+
exp(1)))^2-c))*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/a^2/sqrt
(-a*c)/f/abs(c)
```

maple [B] time = 2.08, size = 281, normalized size = 1.94

$$-\frac{\left(2\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^2\left(fx+e\right)\right)+2\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*ln(-(-s
in(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-cos(f*x+e)^2+4*cos(f*x+e)*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-sin(f*x+e)-1+
cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))+2*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+3)*(c*(-1+cos(f*x+e))/c
os(f*x+e))^(5/2)*cos(f*x+e)^3*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e
)^5/a^3
```

maxima [A] time = 0.47, size = 133, normalized size = 0.92

$$-\frac{\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{-a}\,a^2}+\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{-a}\,a^2}-\frac{\frac{2\,\sqrt{-a}\,c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\sqrt{-a}\,c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}{a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a^2) + 2
*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a^2) - (2*sqrt(
-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(-a)*c^(5/2)*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.146 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] 1/4*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.31, size = 68, normalized size = 1.62

$$\frac{c\cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{4a^2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (c*Cos[e + f*x]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e + f*x]
])/(4*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [B] time = 0.44, size = 95, normalized size = 2.26

$$\frac{c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f
)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)1/2*c^2*(1/2*(c*tan(1/2*(f*x+exp(1)))^2-c)^2*sign(tan(1/2*(f*x+exp(
1)))^3+tan(1/2*(f*x+exp(1))))+c*(c*tan(1/2*(f*x+exp(1)))^2-c)*sign(tan(1/2*
(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1)))))/a^2/sqrt(-a*c)/c/f/abs(c)
```

maple [B] time = 2.06, size = 75, normalized size = 1.79

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\cos^{2}\left(fx+e\right)\right)\left(-1+\cos\left(fx+e\right)\right)^{3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{4f\sin\left(fx+e\right)^{5}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(-1+cos(f*x+e))^3*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^5/a^3
```

maxima [**B**] time = 0.47, size = 98, normalized size = 2.33

$$-\frac{\sqrt{-a}\,c^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\sin\left(fx+e\right)^{4}}{4\left(a^{3}-\frac{a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)f\left(\cos\left(fx+e\right)+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*sqrt(-a)*c^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)*sin(f*x + e)^4/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2)*f*(cos(f*x + e) + 1)^4)
```

mupad [**B**] time = 3.51, size = 119, normalized size = 2.83

$$-\frac{2\,c\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(\sin\left(e+f\,x\right)+2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{a^2\,f\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*c*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(sin(e + f*x) + 2*sin(2*e
 + 2*f*x) + sin(3*e + 3*f*x)))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x))
^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4*e
+ 4*f*x) - 5))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1))
**(5/2), x)
```

3.147 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.26, size = 71, normalized size = 1.65

$$\frac{(2\cos(e+fx)+1)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{8a^2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2
),x]
```

```
[Out] ((1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e
+ f*x]])/(8*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [B] time = 0.45, size = 104, normalized size = 2.42

$$\frac{\left(2\,\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f
*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)1/8*(c*tan(1/2*(f*x+exp
(1)))^2-c)^2*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*sign(cos(f
*x+exp(1)))/a^2/sqrt(-a*c)/f/abs(c)/sign(tan(1/2*(f*x+exp(1)))^2-1)
```

maple [B] time = 2.17, size = 83, normalized size = 1.93

$$-\frac{\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)\left(3\cos\left(fx+e\right)+1\right)\left(-1+\cos\left(fx+e\right)\right)^{3}}{8f\sin\left(fx+e\right)^{5}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(
1/2)*cos(f*x+e)*(3*cos(f*x+e)+1)*(-1+cos(f*x+e))^3/sin(f*x+e)^5/a^3
```

maxima [A] time = 0.46, size = 58, normalized size = 1.35

$$-\frac{\sqrt{c}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{2}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{2}}{8\sqrt{-a}\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/8*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)^2/(sqrt(-a)*a^2*f)
```

mupad [B] time = 3.25, size = 120, normalized size = 2.79

$$-\frac{2\,\left(3\,\sin\left(e+f\,x\right)+3\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}}{a^{2}\,f\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*(3*sin(e + f*x) + 3*sin(2*e + 2*f*x) + sin(3*e + 3*f*x))*((c*(cos(e + f
*x) - 1))/cos(e + f*x))^(1/2))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x))
^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4*e
+ 4*f*x) - 5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
5/2), x)
```

3.148 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=140

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4a^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(}{4f(a\sec(e+fx)+a}$$

```
[Out] 1/4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+
e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e)
)*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4a^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(}{4f(a\sec(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + Ta
n[e + f*x]/(4*a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e\cdot}{(a+a\sec(e+fx))^{3/}}}{2}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{4af(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{4af(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{4af(a+a\sec(e}$$

Mathematica [C] time = 1.04, size = 91, normalized size = 0.65

$$-\frac{\tan(e+fx)\left(3\cos(e+fx)+8\cos^4\left(\frac{1}{2}(e+fx)\right)\tanh^{-1}\left(e^{i(e+fx)}\right)+2\right)}{4a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -1/4*((2 + 8*ArcTanh[E^(I*(e + f*x))]*Cos[(e + f*x)/2]^4 + 3*Cos[e + f*x])*
Tan[e + f*x])/(a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

fricas [A] time = 0.56, size = 456, normalized size = 3.26

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2+2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\,\sqrt{-ac}\,\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(f\cdots\right.}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\cdots\right.}{8\left(a^3cf\cos\left(fx+e\right)^2+2\,a^3cf\cos\left(fx+e\right)\cdots\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 + 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 + 2*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*
sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
```

check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
gn by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep
+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste

```
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
```

/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) wer
e not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, i
ntegration of abs or sign assumes constant sign by intervals (correct if th
e argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (

```
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by
 intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep
)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
```

```
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
```

ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/

```
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
```

```
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
```

```
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
```

```
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
```

```
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
```

```
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(t
_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were n
ot checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep
^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp
(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant s
ign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sig
n error %%%{c,2%%%}Evaluation time: 2.85Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

maple [A] time = 2.26, size = 164, normalized size = 1.17

$$-\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\,\left(-1+\cos\left(fx+e\right)\right)^{3}\left(4\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+8\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\cos\left(fx+e\right)+}{16f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sin\left(fx+e\right)^{5}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/16/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^3*(4*cos(f*x+e)
^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*
x+e)+5*cos(f*x+e)^2+4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)-3)/(c*(-
1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3
```

maxima [B] time = 0.64, size = 1191, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos
(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e)
 + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*co
s(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
 + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x
 + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*
sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x +
e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x
+ e) + 1) - (2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) +
1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*
x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin
(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin
(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e)
 + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) + 2*(3*sin(3*f*x + 3*e) + 4*sin(2*f*x + 2*e) + 3*sin(f*x +
e))*cos(4*f*x + 4*e) - 2*(3*cos(3*f*x + 3*e) + 4*cos(2*f*x + 2*e) + 3*cos(f
*x + e))*sin(4*f*x + 4*e) + 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) - 4
*(cos(f*x + e) - 2)*sin(2*f*x + 2*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
+ 4*cos(2*f*x + 2*e)*sin(f*x + e) + 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*c
*cos(4*f*x + 4*e)^2 + 16*a^3*c*cos(3*f*x + 3*e)^2 + 36*a^3*c*cos(2*f*x + 2*
e)^2 + 16*a^3*c*cos(f*x + e)^2 + a^3*c*sin(4*f*x + 4*e)^2 + 16*a^3*c*sin(3*
f*x + 3*e)^2 + 36*a^3*c*sin(2*f*x + 2*e)^2 + 48*a^3*c*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*a^3*c*sin(f*x + e)^2 + 8*a^3*c*cos(f*x + e) + a^3*c + 2*(4*a^
3*c*cos(3*f*x + 3*e) + 6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e) + a^
3*c)*cos(4*f*x + 4*e) + 8*(6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e)
+ a^3*c)*cos(3*f*x + 3*e) + 12*(4*a^3*c*cos(f*x + e) + a^3*c)*cos(2*f*x + 2
*e) + 4*(2*a^3*c*sin(3*f*x + 3*e) + 3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(
f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(f*x
 + e))*sin(3*f*x + 3*e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

3.149 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\csc(e+fx)}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan}{4f(a\sec(e+fx)+}$$

```
[Out] 3/8*csc(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+1/4*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2)-3/8*arctanh(cos(f*
x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3960, 3959, 2611, 3770}

$$\frac{3\csc(e+fx)}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan}{4f(a\sec(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) + Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
```

```
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{}{(a+a\sec(e+f}}{}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} - \frac{(3\tan(e+fx)}{4a^2c\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{4f(a+a\sec(e}$$

$$= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{4f(a+a\sec(e}$$

Mathematica [C] time = 1.39, size = 130, normalized size = 0.89

$$-\frac{\tan(e+fx)\left(2\cos(e+fx)+5\cos(2(e+fx))-3(\cos(e+fx)-2\cos(2(e+fx))-\cos(3(e+fx))+2)\tanh^{-1}\left(\right.\right.}{16a^2cf(\cos(e+fx)-1)(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] -1/16*((1 + 2*Cos[e + f*x] + 5*Cos[2*(e + f*x)] - 3*ArcTanh[E^(I*(e + f*x))
]*(2 + Cos[e + f*x] - 2*Cos[2*(e + f*x)] - Cos[3*(e + f*x)]))*Tan[e + f*x])
/(a^2*c*f*(-1 + Cos[e + f*x])*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x]
)]*Sqrt[c - c*Sec[e + f*x]])
```

fricas [A] time = 0.58, size = 536, normalized size = 3.67

$$\left[-\frac{3\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^{2}+\left(ac\right.\right.}{\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)}\right.}{16\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}+a^{3}c^{2}f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
e)^3 + cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 +
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2
*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
```

```
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(c
os(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableWarning, assuming -2*a+a is positive. Hint: run assum
e to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
```

```
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
```

```
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_noste
p+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Warning, integration of abs or sign assumes constant sign by intervals (
correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
```

```
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes con
stant sign by intervals (correct if the argument is real):Check [abs(t_nost
ep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not ch
eckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
```

ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/

```
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
```

```
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
```

ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una

ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec

```
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
```

```
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
```

```
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs o
r sign assumes constant sign by intervals (correct if the argument is real)
:Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_
nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, in
tegration of abs or sign assumes constant sign by intervals (correct if the
 argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroe
s of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f
*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant
 sign by intervals (correct if the argument is real):Check [abs(t_nostep),a
bs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign er
ror %%%{c,2%%%}Evaluation time: 2.89Limit: Max order reached or unable to m
ake series expansion Error: Bad Argument Value
```

maple [A] time = 2.21, size = 204, normalized size = 1.40

$$-\frac{\left(12\left(\cos^3\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+5\left(\cos^3\left(fx+e\right)\right)+12\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-15\left(\cos^2\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/32/f*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))+5*cos(f*x+e)^3+12*
cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-15*cos(f*x+e)^2-12*ln(-(-1+cos
(f*x+e))/sin(f*x+e))*cos(f*x+e)-9*cos(f*x+e)-12*ln(-(-1+cos(f*x+e))/sin(f*x
+e))+3)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))
/cos(f*x+e))^(3/2)/c^3/sin(f*x+e)^5/a^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.150 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+f}{8a^2c^2f\sqrt{a\sec(e+}}$$

```
[Out] 3/8*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*
cot(f*x+e)^2*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-3/8*arctanh(cos(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3959, 2611, 3770}

$$\frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+f}{8a^2c^2f\sqrt{a\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f
*x]]) - (Cot[e + f*x]^2*Csc[e + f*x])/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]]) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*
c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
```

```
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^4(e+fx)\csc(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(3\tan(e+f}{4a^2c^2\sqrt{a+}}$$
$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\mathrm{cot}}{4a^2c^2f\sqrt{a+a}}$$
$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\mathrm{cot}}{4a^2c^2f\sqrt{a+a}}$$

Mathematica [C] time = 1.38, size = 84, normalized size = 0.52

$$\frac{(1-5\cos(2(e+fx)))\csc^3(e+fx)-12\tan(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)}{16a^2c^2f\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] ((1 - 5*Cos[2*(e + f*x)])*Csc[e + f*x]^3 - 12*ArcTanh[E^(I*(e + f*x))]*Tan[
e + f*x])/(16*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]
)
```

fricas [A] time = 0.57, size = 482, normalized size = 3.01

$$\left[-\frac{3\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2+\left(ac\cos\left(fx+e\right)^2+ac\right)\mathrm{si}\right.}{\left(\cos\left(fx+e\right)^2-1\right)\sin\left(fx+e\right)}\right.}{16\left(a^3c^3f\cos\left(fx+e\right)^4-2a^3c^3f\cos\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-4*(2*sqrt
(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos
(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e)^4 - 3*cos(
f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 +
 a^3*c^3*f)*sin(f*x + e)), 1/8*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*s
qrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*
x + e)^4 - 3*cos(f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f
*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
gn by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep
+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
```

```
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
```

```
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
```

```
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) wer
e not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, i
ntegration of abs or sign assumes constant sign by intervals (correct if th
e argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by
 intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep
)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable t
```

```
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
```

n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_

```
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
```

```
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
```

```
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
```

```
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
```

```
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
```

```
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(t
_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were n
ot checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep
^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp
(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
```

```
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, i
ntegration of abs or sign assumes constant sign by intervals (correct if th
e argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integra
tion of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(t_nostep)]Sign error %%%{c,2%%%}Evaluation time: 2
.97Limit: Max order reached or unable to make series expansion Error: Bad A
rgument Value
```

maple [A] time = 2.57, size = 173, normalized size = 1.08

$$-\frac{(-1+\cos(fx+e))^3 \left(3(\cos^4(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) - 6(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) - 5(\cos^3(fx\right.}{8f\sin(fx+e)^5\cos(fx+e)^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))^3*(3*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-6*
cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^3+3*ln(-(-1+cos(f
*x+e))/sin(f*x+e))+3*cos(f*x+e))*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*
x+e)^5/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/a^3
```

maxima [B] time = 0.82, size = 1659, normalized size = 10.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/8*(3*(2*(4*cos(6*f*x + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1
)*cos(8*f*x + 8*e) - cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f
*x + 2*e) + 1)*cos(6*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x +
 2*e) - 1)*cos(4*f*x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2
 + 4*(2*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f
*x + 8*e) - sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 3*(2*(4*cos(6*f*x + 6*e) -
 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x
+ 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
 - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 3
6*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*si
n(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2
+ 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*
f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) - 2*(5*sin(7*f*x + 7*e) + 3*sin(5*f*x + 5*e) + 3*sin(3*f*x
+ 3*e) + 5*sin(f*x + e))*cos(8*f*x + 8*e) - 20*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 8*(3*sin(5*f*x + 5*e)
 + 3*sin(3*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 12*(3*sin(4*f*x
+ 4*e) - 2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 12*(3*sin(3*f*x + 3*e) + 5*
sin(f*x + e))*cos(4*f*x + 4*e) + 2*(5*cos(7*f*x + 7*e) + 3*cos(5*f*x + 5*e)
 + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(8*f*x + 8*e) + 10*(4*cos(6*f*x
+ 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 8*
(3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e)
 - 6*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) + 12*(3
*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(4*f*x + 4*e) + 6*(4*cos(2*f*x + 2*e
) - 1)*sin(3*f*x + 3*e) - 24*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 40*cos(f*x
 + e)*sin(2*f*x + 2*e) + 40*cos(2*f*x + 2*e)*sin(f*x + e) - 10*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e
)^2 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c
^3*sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*
x + 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(
2*f*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*
f*x + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x
 + 2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x
 + 4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(
```

```
4*f*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+fx\right)\left(a+\frac{a}{\cos\left(e+fx\right)}\right)^{5/2}\left(c-\frac{c}{\cos\left(e+fx\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.151 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=101

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,{}_2F_1\left(m+\frac{1}{2},\frac{1}{2}-n;m+\frac{3}{2};\frac{1}{2}(\sec$$
$$f(2m+1)$$

```
[Out] -2^(1/2+n)*c*hypergeom([1/2+m, 1/2-n],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f*
x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(1+2*
m)
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,{}_2F_1\left(m+\frac{1}{2},\frac{1}{2}-n;m+\frac{3}{2};\frac{1}{2}(\sec$$
$$f(2m+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*c*Hypergeometric2F1[1/2 + m, 1/2 - n, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+\cdots}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+\cdots}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\cdots\right.}{f\cdots}$$

$$= -\frac{2^{\frac{1}{2}+n}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}-n;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)\cdots\right.}{\cdots}$$

Mathematica [F] time = 1.35, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m\left(-c\sec\left(fx+e\right)+c\right)^n\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec \left(fx+e\right)+a\right)^{m}\left(-c\sec \left(fx+e\right)+c\right)^{n}\sec \left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

maple [F] time = 2.96, size = 0, normalized size = 0.00

$$\int \sec \left(fx+e\right)\left(a+a\sec \left(fx+e\right)\right)^{m}\left(c-c\sec \left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec \left(fx+e\right)+a\right)^{m}\left(-c\sec \left(fx+e\right)+c\right)^{n}\sec \left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos \left(e+f\,x\right)}\right)}^m\,{\left(c-\frac{c}{\cos \left(e+f\,x\right)}\right)}^n}{\cos \left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{n}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n*sec(e + f*x),
 x)
```

3.152 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=92

$$\frac{a 2^{m+\frac{1}{2}} \tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+f\right.}{5f}$$

```
[Out] 1/5*2^(1/2+m)*a*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x
+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))^2*tan(f*x+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3961, 70, 69}

$$\frac{a 2^{m+\frac{1}{2}} \tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+f\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c-cx)^{3/2}\,

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*(a*
sec(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(fx+e\right)-c\right)^{2}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

maple [F] time = 2.34, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(fx+e\right)-c\right)^{2}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{2}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(a\sec\left(e+fx\right)+a\right)^m\sec\left(e+fx\right)dx+\int\left(-2\left(a\sec\left(e+fx\right)+a\right)^m\sec^2\left(e+fx\right)\right)dx+\int\left(a\sec\left(e+fx\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral((a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral(-2*(a*se
c(e + f*x) + a)**m*sec(e + f*x)**2, x) + Integral((a*sec(e + f*x) + a)**m*s
ec(e + f*x)**3, x))
```

3.153 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=90

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+f$$
$$3f$$

```
[Out] 1/3*2^(1/2+m)*a*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x
+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))*tan(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3961, 70, 69}

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+f$$
$$3f$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])*Tan[e + f*x])/(3*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}\sqrt{c-cx}\,dx, \right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+f x)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\right.}{f\sqrt{c}}$$

$$= \frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(}{}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.94

$$-\frac{c2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)-1)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(2^(1/2 + m)*c*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])
/2]*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x])
)^m*Tan[e + f*x])/f
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c\sec\left(fx+e\right)^2-c\sec\left(fx+e\right)\right)\left(a\sec\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(c \sec \left(f x + e\right) - c\right) \left(a \sec \left(f x + e\right) + a\right)^{m} \sec \left(f x + e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="giac
")

[Out] integrate(-(c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)

maple [F] time = 1.76, size = 0, normalized size = 0.00

$$\int \sec \left(f x + e\right) \left(a + a \sec \left(f x + e\right)\right)^{m} \left(c - c \sec \left(f x + e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)

[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(c \sec \left(f x + e\right) - c\right) \left(a \sec \left(f x + e\right) + a\right)^{m} \sec \left(f x + e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")

[Out] -integrate((c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a + \frac{a}{\cos\left(e + f\,x\right)}\right)}^{m}\,\left(c - \frac{c}{\cos\left(e + f\,x\right)}\right)}{\cos\left(e + f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x),x)

[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-\left(a\sec\left(e+fx\right)+a\right)^{m}\sec\left(e+fx\right)\right)dx+\int\left(a\sec\left(e+fx\right)+a\right)^{m}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral((a*sec(e
+ f*x) + a)**m*sec(e + f*x)**2, x))
```

3.154 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=90

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(c-c\sec(e+fx))}$$

```
[Out] -2^(1/2+m)*a*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e
))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + m)*a*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(f*(c - c*Sec[e + f*x])))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\mathrm{Sub}}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a}{f(c-c\sec(e+fx))}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]), x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a \sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)}{c \sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}}{c-c\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a \sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)}{c \sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}}{c-c\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] -int((a + a/cos(e + f*x))^m/(c - c*cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a\sec\left(e+fx\right)+a\right)^{m}\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x) - 1), x)/c
```

3.155 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -1/3*2^(1/2+m)*a*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sec(f*x+e))*(1+sec(
f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -(2^(1/2 + m)*a*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(3*f*(c - c*Sec[e + f*x])^2)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\text{Sub}}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a}{3f(c-c\sec(e+fx))^2}$$

Mathematica [F] time = 2.59, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2, x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{c^2\sec\left(fx+e\right)^2-2\,c^2\sec\left(fx+e\right)+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2*sec(f*x + e)^2 - 2*c^2*se
c(f*x + e) + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

maple [F] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}}{\left(c-c\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(a\sec\left(e+fx\right)+a\right)^m\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e +
f*x) + 1), x)/c**2
```

3.156 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=160

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}\,(a\sec(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}-\frac{2c\tan$$

```
[Out] -2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(5+2*m)-64*c^3*
(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(5+2*m)/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2
)-16*c^2*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(4*m^2+16*m
+15)
```

Rubi [A] time = 0.38, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{16c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}\,(a\sec(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(5 + 2*m)*(3 + 8*m + 4*m^2
)*Sqrt[c - c*Sec[e + f*x]]) - (16*c^2*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec
[e + f*x]]*Tan[e + f*x])/(f*(15 + 16*m + 4*m^2)) - (2*c*(a + a*Sec[e + f*x]
)^m*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(5 + 2*m))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
```

```
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e}{f(5+2m)}$$
$$= -\frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,\tan(}{f\left(15+16m+4m^2\right)}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(15+46m+36m^2+8m^3\right)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F] time = 15.65, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2), x
]
```

fricas [A] time = 0.45, size = 191, normalized size = 1.19

$$\frac{2\left(4\,c^2m^2+\left(4\,c^2m^2+24\,c^2m+43\,c^2\right)\cos\left(fx+e\right)^3+8\,c^2m-\left(4\,c^2m^2+8\,c^2m-29\,c^2\right)\cos\left(fx+e\right)^2+3\,c^2-(}{\left(8\,fm^3+36\,fm^2+46\,fm+15\,f\right)\cos\left(fx+e\right)^2\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2*(4*c^2*m^2 + (4*c^2*m^2 + 24*c^2*m + 43*c^2)*cos(f*x + e)^3 + 8*c^2*m - (
4*c^2*m^2 + 8*c^2*m - 29*c^2)*cos(f*x + e)^2 + 3*c^2 - (4*c^2*m^2 + 24*c^2*
m + 11*c^2)*cos(f*x + e))*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x
+ e)^2*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(5/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

maxima [A] time = 0.48, size = 228, normalized size = 1.42

$$-\frac{2\left(\frac{\sqrt{2}\left(2^{m+5}m+5\cdot 2^{m+4}\right)(-a)^{m}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{\sqrt{2}\left(2^{m+4}m^{2}+2^{m+6}m+15\cdot 2^{m+2}\right)(-a)^{m}c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-2^{m+\frac{11}{2}}\left(-a\right)^{m}c^{\frac{5}{2}}\right)e^{\left(-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\right)}}{\left(8\,m^{3}+36\,m^{2}+46\,m+15\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*(2^(m + 5)*m + 5*2^(m + 4))*(-a)^m*c^(5/2)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - sqrt(2)*(2^(m + 4)*m^2 + 2^(m + 6)*m + 15*2^(m + 2))*(-a)
^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2^(m + 11/2)*(-a)^m*c^(5/2
))*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.157 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}\,(a\sec(e+fx)+a)^m}{f(2m+3)}$$

```
[Out] -8*c^2*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2)
-2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(3+2*m)
```

Rubi [A] time = 0.22, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}\,(a\sec(e+fx)+a)^m}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + 8*m + 4*m^2)*Sqrt[c -
c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]]*Tan
[e + f*x])/(f*(3 + 2*m))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,\tan(e+}{f(3+2m)}$$

$$= -\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(3+8m+4m^2\right)\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a}{}$$

Mathematica [F] time = 38.44, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2), x
]
```

fricas [A] time = 0.44, size = 112, normalized size = 1.12

$$\frac{2\left((2\,cm+5\,c)\cos\left(fx+e\right)^2-2\,cm+4\,c\cos\left(fx+e\right)-c\right)\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^m\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(4\,fm^2+8\,fm+3\,f\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2*((2*c*m + 5*c)*cos(f*x + e)^2 - 2*c*m + 4*c*cos(f*x + e) - c)*((a*cos(f*x
 + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((4*f*m^
2 + 8*f*m + 3*f)*cos(f*x + e)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(3/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

maple [F] time = 1.63, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

maxima [A] time = 0.47, size = 171, normalized size = 1.71

$$-\frac{2\left(\sqrt{2}\,2^{m+2}\left(-a\right)^{m}c^{\frac{3}{2}}-\frac{\sqrt{2}\left(2^{m+2}m+3\cdot 2^{m+1}\right)\left(-a\right)^{m}c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)e^{\left(-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\right)}}{\left(4\,m^{2}+8\,m+3\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*2^(m + 2)*(-a)^m*c^(3/2) - sqrt(2)*(2^(m + 2)*m + 3*2^(m + 1))*
(-a)^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*e^(-m*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/((4*m
^2 + 8*m + 3)*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(3/2))
```

mupad [B] time = 3.59, size = 154, normalized size = 1.54

$$-\frac{2\,c\left(\frac{a\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}\right)^{m}\sqrt{\frac{c\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(5\,\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)+5\,\sin\left(3\,e+3\,f\,x\right)+2\,m\,\sin\left(e+f\right.\right.}{f\,\left(4\,m^{2}+8\,m+3\right)\,\left(3\,\cos\left(e+f\,x\right)-2\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(3\,e+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] -(2*c*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m*((c*(cos(e + f*x) - 1))/cos(e
 + f*x))^(1/2)*(5*sin(e + f*x) - 2*sin(2*e + 2*f*x) + 5*sin(3*e + 3*f*x) +
```

```
2*m*sin(e + f*x) - 4*m*sin(2*e + 2*f*x) + 2*m*sin(3*e + 3*f*x)))/(f*(8*m +
4*m^2 + 3)*(3*cos(e + f*x) - 2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.158 $\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=46

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

[Out] -2*c*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]

[Out] (-2*c*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx=-\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 18.90, size = 163, normalized size = 3.54

$$\frac{\sqrt{2}e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\,(\sec(e+fx)+1)^{-m}(a(\sec}{(2fm+f)\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
))]*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*Csc[(e + f*x)/2]*
(a*(1 + Sec[e + f*x]))^m*Sqrt[c - c*Sec[e + f*x]])/(E^((I/2)*(e + f*x))*(f
+ 2*f*m)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m)
```

fricas [A] time = 0.47, size = 70, normalized size = 1.52

$$\frac{2 \left(\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}} \left(\cos\left(fx+e\right)+1\right)}{\left(2 fm+f\right) \sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*(cos(f*x + e) + 1)/((2*f*m + f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c \sec\left(fx+e\right)+c} \left(a \sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

maple [F] time = 1.86, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right) \left(a+a \sec\left(fx+e\right)\right)^{m} \sqrt{c-c \sec\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

maxima [B] time = 0.46, size = 114, normalized size = 2.48

$$\frac{2^{m+\frac{3}{2}} \left(-a\right)^{m} \sqrt{c}\, e^{\left(-m \log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m \log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\right)}}{f(2 m+1)\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1} \sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2^(m + 3/2)*(-a)^m*sqrt(c)*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)
, x)
```

3.159 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=69

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*tan(f*
x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3961, 68}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[
e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{c-cx}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]], x
]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*
sec(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}}{\sqrt{c-c\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1))
, x)
```

3.160 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*ta
n(f*x+e)/c/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3961, 68}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[e
 + f*x])^m*Tan[e + f*x])/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{{}_2F_1\left(2, \frac{1}{2}+m; \frac{3}{2}+m; \frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
 x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{c^2\sec\left(fx+e\right)^2-2\,c^2\sec\left(fx+e\right)+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2
*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

maple [F] time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}}{\left(c-c\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(3
/2), x)
```

3.161 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*hypergeom([3, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*ta
n(f*x+e)/c^2/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3961, 68}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[e
 + f*x])^m*Tan[e + f*x])/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F] time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
 x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^
3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sig
n by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_no
step)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
```

step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab

```
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
```

sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p

```
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
```

```
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
```

```
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
```

```
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
```

eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign as
sumes constant sign by intervals (correct if the argument is real):Check [a
bs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep we
re not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration
 of abs or sign assumes constant sign by intervals (correct if the argument
 is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_no
step^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep
+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check

```
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning,
 integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Evaluation tim
e: 2.06Unable to divide, perhaps due to rounding error%%%{1,[0,4,1,0]%%%}+%
%%{2,[0,2,1,1]%%%}+%%%{1,[0,0,1,2]%%%} / %%%{1,[0,0,0,2]%%%} Error: Bad Arg
ument Value
```

maple [F] time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}}{\left(c-c\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos\left(e+f\,x\right)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c - \frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.162 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=169

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{a^2f(2m+1)\left(4m^2+16m+15\right)}+\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+f$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/f/(1+2*m)+2*(a+a*sec
(f*x+e))^(1+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)-2*(a+a*
sec(f*x+e))^(2+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a^2/f/(1+2*m)/(4*m^2+1
6*m+15)
```

Rubi [A] time = 0.37, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{a^2f(2m+1)\left(4m^2+16m+15\right)}+\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+f$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + (2*(a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-3 - m)*T
an[e + f*x])/(a*f*(3 + 8*m + 4*m^2)) - (2*(a + a*Sec[e + f*x])^(2 + m)*(c -
 c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(a^2*f*(1 + 2*m)*(15 + 16*m + 4*m^2
))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
```

```
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan}{f(1+2m)}$$

$$= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan}{f(1+2m)}$$

$$= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan}{f(1+2m)}$$

Mathematica [C] time = 9.08, size = 321, normalized size = 1.90

$$-\frac{i2^{m+3}\left(1+e^{i(e+fx)}\right)\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left(\left(4m^2+12m+7\right)e^{4i(e+fx)}+\left(8\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)
,x]
```

```
[Out] ((-I)*2^(3 + m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I
)*(e + f*x))))^m*(7 + 12*m + 4*m^2 - 4*E^(I*(e + f*x))*(3 + 2*m) - 4*E^((3*
I)*(e + f*x))*(3 + 2*m) + E^((4*I)*(e + f*x))*(7 + 12*m + 4*m^2) + E^((2*I)
*(e + f*x))*(22 + 24*m + 8*m^2))*Sec[e + f*x]^(3 + m)*(a*(1 + Sec[e + f*x])
)^m*(c - c*Sec[e + f*x])^(-3 - m))/((-1 + E^(I*(e + f*x)))^5*(((-I)*(-1 + E
^(I*(e + f*x))))/E^((I/2)*(e + f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^m*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(1 + Sec[e + f*x])^m*Sin[e/2
 + (f*x)/2]^(2*(-3 - m)))
```

fricas [A] time = 0.53, size = 120, normalized size = 0.71

$$-\frac{\left(\left(4\,m^2+12\,m+7\right)\cos\left(fx+e\right)^2-2\,(2\,m+3)\cos\left(fx+e\right)+2\right)\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^m\left(\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}\right)^{-m-3}\sin\left(fx+\right.}{\left(8\,fm^3+36\,fm^2+46\,fm+15\,f\right)\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="fricas")
```

```
[Out] -((4*m^2 + 12*m + 7)*cos(f*x + e)^2 - 2*(2*m + 3)*cos(f*x + e) + 2)*((a*cos
(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^(-m - 3)
*sin(f*x + e)/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m-3}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 3)*sec(f*x + e
), x)
```

maple [F] time = 3.88, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

maxima [A] time = 0.66, size = 156, normalized size = 0.92

$$\frac{\left(\left(4\,m^{2}+8\,m+3\right)(-a)^{m}-\frac{2\left(4\,m^{2}+12\,m+5\right)(-a)^{m}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\left(4\,m^{2}+16\,m+15\right)(-a)^{m}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)c^{-m-3}\left(\cos\left(fx+e\right)+1\right)^{5}}{4\left(8\,m^{3}+36\,m^{2}+46\,m+15\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2\,m}\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="maxima")
```

```
[Out] 1/4*((4*m^2 + 8*m + 3)*(-a)^m - 2*(4*m^2 + 12*m + 5)*(-a)^m*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*(-a)^m*sin(f*x + e)^4/(cos(f*x +
```

```
e) + 1)^4)*c^(-m - 3)*(cos(f*x + e) + 1)^5/((8*m^3 + 36*m^2 + 46*m + 15)*f
*(sin(f*x + e)/(cos(f*x + e) + 1))^(2*m)*sin(f*x + e)^5)
```

mupad [B] time = 10.65, size = 290, normalized size = 1.72

$$-\frac{\left(\cos\left(3\,e+3\,f\,x\right)-\sin\left(3\,e+3\,f\,x\right)\,1\mathrm{i}\right)\left(\frac{\sin\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\left(\cos\left(3\,e+3\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\,1\mathrm{i}\right)\,\left(4\,m^2+12\,m+15\right)\,2\mathrm{i}}{f\,\left(m^3\,8\mathrm{i}+m^2\,36\mathrm{i}+m\,46\mathrm{i}+15\mathrm{i}\right)}-\frac{\sin(2\cdots}{\cdots}\right.}{8\,\cos\left(e+f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 3)),x)
```

```
[Out] -((cos(3*e + 3*f*x) - sin(3*e + 3*f*x)*1i)*((sin(e + f*x)*(a + a/cos(e + f*
x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 15)*2i)/(f*(
m*46i + m^2*36i + m^3*8i + 15i)) - (sin(2*e + 2*f*x)*(8*m + 12)*(a + a/cos(
e + f*x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*2i)/(f*(m*46i + m^2*36
i + m^3*8i + 15i)) + (sin(3*e + 3*f*x)*(a + a/cos(e + f*x))^m*(cos(3*e + 3*
f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 7)*2i)/(f*(m*46i + m^2*36i + m^
3*8i + 15i))))/(8*cos(e + f*x)^3*(c - c/cos(e + f*x))^(m + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

3.163 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=104

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af\left(4m^2+8m+3\right)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/f/(1+2*m)+(a+a*sec(f
*x+e))^(1+m)*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)
```

Rubi [A] time = 0.22, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af\left(4m^2+8m+3\right)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + ((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-2 - m)*Tan
[e + f*x])/(a*f*(3 + 8*m + 4*m^2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan}{f(1+2m)}$$
$$= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan}{f(1+2m)}$$

Mathematica [C] time = 3.05, size = 250, normalized size = 2.40

$$\frac{i2^{m+3}\left(1+e^{i(e+fx)}\right)\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^{m}\left((m+1)e^{2i(e+fx)}-e^{i(e+fx)}+m+\right.}{f(2m+1)(2m+3)\left(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)
,x]
```

```
[Out] (I*2^(3 + m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(
e + f*x))))^m*(1 - E^(I*(e + f*x)) + m + E^((2*I)*(e + f*x))*(1 + m))*Sec[e
 + f*x]^(2 + m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-2 - m)*Sin[
(e + f*x)/2]^(2*(2 + m)))/((-1 + E^(I*(e + f*x)))^3*(((-I)*(-1 + E^(I*(e +
f*x))))/E^((I/2)*(e + f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^m*f*(1 + 2*m)*(3 + 2*m)*(1 + Sec[e + f*x])^m)
```

fricas [A] time = 0.46, size = 93, normalized size = 0.89

$$-\frac{\left(2\,(m+1)\cos\left(fx+e\right)-1\right)\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m}\left(\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}\right)^{-m-2}\sin\left(fx+e\right)}{\left(4\,fm^2+8\,fm+3\,f\right)\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="fricas")
```

```
[Out] -(2*(m + 1)*cos(f*x + e) - 1)*((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos
(f*x + e) - c)/cos(f*x + e))^(-m - 2)*sin(f*x + e)/((4*f*m^2 + 8*f*m + 3*f)
*cos(f*x + e)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sec\left(fx+e\right)+a\right)^{m} \left(-c \sec\left(fx+e\right)+c\right)^{-m-2} \sec\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 2)*sec(f*x + e
), x)
```

maple [F] time = 3.81, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right) \left(a+a \sec\left(fx+e\right)\right)^{m} \left(c-c \sec\left(fx+e\right)\right)^{-2-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

maxima [A] time = 0.83, size = 107, normalized size = 1.03

$$-\frac{\left(\left(-a\right)^{m} \left(2\,m+1\right)-\frac{\left(-a\right)^{m} \left(2\,m+3\right) \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right) c^{-m-2} \left(\cos\left(fx+e\right)+1\right)^{3}}{2 \left(4\,m^{2}+8\,m+3\right) f \left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2\,m} \sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="maxima")
```

```
[Out] -1/2*((-a)^m*(2*m + 1) - (-a)^m*(2*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*c^(-m - 2)*(cos(f*x + e) + 1)^3/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)/(cos
(f*x + e) + 1))^(2*m)*sin(f*x + e)^3)
```

mupad [B] time = 8.00, size = 145, normalized size = 1.39

$$\frac{\sin\left(e+f\,x\right) \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m} 1\mathrm{i}}{f \cos\left(e+f\,x\right)^{2} \left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{m+2} \left(m^{2}\,4\mathrm{i}+m\,8\mathrm{i}+3\mathrm{i}\right)}-\frac{\sin\left(2\,e+2\,f\,x\right) \left(2\,m+2\right) \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m} 1\mathrm{i}}{2\,f \cos\left(e+f\,x\right)^{2} \left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{m+2} \left(m^{2}\,4\mathrm{i}+m\,8\mathrm{i}+3\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 2)),x)
```

```
[Out] (sin(e + f*x)*(a + a/cos(e + f*x))^m*1i)/(f*cos(e + f*x)^2*(c - c/cos(e + f
*x))^(m + 2)*(m*8i + m^2*4i + 3i)) - (sin(2*e + 2*f*x)*(2*m + 2)*(a + a/cos
(e + f*x))^m*1i)/(2*f*cos(e + f*x)^2*(c - c/cos(e + f*x))^(m + 2)*(m*8i + m
^2*4i + 3i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

3.164 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*m)
```

Rubi [A] time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/(f*(1
 + 2*m)))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\tan(}{f(1+2m)}$$

Mathematica [C] time = 1.18, size = 208, normalized size = 4.43

$$-\frac{2^{m+1}e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m-1}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\sin^{2(m+1)}\left(\frac{1}{2}(e+fx)\right)}{2fm+f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)
,x]
```

```
[Out] -((2^(1 + m)*(((-I)*(-1 + E^(I*(e + f*x))))/E^((I/2)*(e + f*x)))^(-1 - 2*m)
*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^
m*Sec[e + f*x]^(1 + m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-1 -
m)*Sin[(e + f*x)/2]^(2*(1 + m))))/(E^((I/2)*(e + f*x))*(E^(I*(e + f*x))/(1 +
 E^((2*I)*(e + f*x))))^m*(f + 2*f*m)*(1 + Sec[e + f*x])^m))
```

fricas [A] time = 0.49, size = 72, normalized size = 1.53

$$-\frac{\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m}\left(\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}\right)^{-m-1}\sin\left(fx+e\right)}{\left(2\,fm+f\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="fricas")
```

```
[Out] -((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^
(-m - 1)*sin(f*x + e)/((2*f*m + f)*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m-1}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 1)*sec(f*x + e
), x)
```

maple [F] time = 3.82, size = 0, normalized size = 0.00

$$\int\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

maxima [A] time = 0.60, size = 62, normalized size = 1.32

$$\frac{(-a)^m c^{-m-1}\left(\cos\left(fx+e\right)+1\right)}{f(2m+1)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2m}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="maxima")
```

```
[Out] (-a)^m*c^(-m - 1)*(cos(f*x + e) + 1)/(f*(2*m + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1))^(2*m)*sin(f*x + e))
```

mupad [B] time = 2.91, size = 105, normalized size = 2.23

$$-\frac{\left(\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,\left(\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}\right)^m}{c\,f\,\left(2\,m+1\right)\,\left(\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}\right)^m\,\left(3\,\cos\left(e+f\,x\right)-2\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 1)),x)
```

```
[Out] -((sin(e + f*x) + sin(3*e + 3*f*x))*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m
)/(c*f*(2*m + 1)*((c*(cos(e + f*x) - 1))/cos(e + f*x))^m*(3*cos(e + f*x) -
2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.165 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$

Optimal. Leaf size=101

$$-\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\,{}_2F_1\left(m+\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}\right.}{f(2m+1)}$$

```
[Out] -2^(1/2-m)*c*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f*
x+e))^(1/2+m)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*
m)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3961, 70, 69}

$$-\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\,{}_2F_1\left(m+\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*c*Hypergeometric2F1[1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 + m)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 - m)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}-}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+}}$$

$$= -\frac{\left(2^{-\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-1-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}+m}\right.}{f}$$

$$= -\frac{2^{\frac{1}{2}-m}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx\right.}{}$$

Mathematica [C] time = 1.42, size = 257, normalized size = 2.54

$$\frac{2^{m-1}\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left({}_2F_1\left(1,-2m;1-2m;\frac{i\left(-1+e^{i(e+fx)}\right)}{1+e^{i(e+fx)}}\right)-{}_2F_1\left(1,-2\right.\right.}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] (2^(-1 + m)*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*(-Hyperge
ometric2F1[1, -2*m, 1 - 2*m, ((-I)*(-1 + E^(I*(e + f*x))))/(1 + E^(I*(e + f
*x)))] + Hypergeometric2F1[1, -2*m, 1 - 2*m, (I*(-1 + E^(I*(e + f*x))))/(1
+ E^(I*(e + f*x)))])*(Sec[e + f*x]/(1 + Sec[e + f*x]))^m*(a*(1 + Sec[e + f*
x]))^m*Sin[(e + f*x)/2]^(2*m))/((((-I)*(-1 + E^(I*(e + f*x))))/E^((I/2)*(e
+ f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*f*m*(c - c*Sec
[e + f*x])^m)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

maple [F] time = 2.55, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{m}}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/((c-c*sec(f*x+e))**m),x)
```

```
[Out] Timed out
```

3.166 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$

Optimal. Leaf size=99

$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(s\right.}{f(2m+1)}$$

```
[Out] -2^(3/2-m)*c*hypergeom([-1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f
*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))^m
)
```

Rubi [A] time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3961, 70, 69}

$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(s\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(3/2 - m)*c*Hypergeometric2F1[-1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x]
)/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c-cx)^{\frac{1}{2}-\ldots}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e-\ldots}}$$

$$= -\frac{\left(2^{\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{-\frac{1}{2}+m}\ldots\right.}{f\ldots}$$

$$= -\frac{2^{\frac{3}{2}-m}c\,{}_2F_1\left(-\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+\ldots\right.}{\ldots}$$

Mathematica [F] time = 2.26, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
 x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m+1}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m+1}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

maple [F] time = 3.68, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m+1}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{1-m}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1-m),x)
[Out] Timed out
```

3.167 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$

Optimal. Leaf size=101

$$-\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{3}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\right.}{f(2m+1)}$$

```
[Out] -2^(5/2-m)*c^2*hypergeom([-3/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec
(f*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))
^m)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3961, 70, 69}

$$-\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{3}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),x]
```

```
[Out] -((2^(5/2 - m)*c^2*Hypergeometric2F1[-3/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e
 + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{\frac{3}{2}-}\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e-}}$$

$$= -\frac{\left(2^{\frac{3}{2}-m}ac^2(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{-\frac{1}{2}+m}\right.}{f}$$

$$= -\frac{2^{\frac{5}{2}-m}c^2\,{}_2F_1\left(-\frac{3}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+\right.}{}$$

Mathematica [F] time = 2.71, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
 x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m\left(-c\sec\left(fx+e\right)+c\right)^{-m+2}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+2}\sec(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

maple [F] time = 3.99, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\left(a+a\sec(fx+e)\right)^m\left(c-c\sec(fx+e)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+2}\sec(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^{2-m}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(2-m),x)
[Out] Timed out
```

3.168 $\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=105

$$-\frac{a^3c\tan^5(e+fx)}{5f}-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{2f}+\frac{a^3c\tan(e+fx)}{4f}$$

```
[Out] 1/4*a^3*c*arctanh(sin(f*x+e))/f+1/4*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/2*a^3*c
*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f-1/5*a^3*c*tan(f*x+e)^5/
f
```

Rubi [A] time = 0.19, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3962, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^3c\tan^5(e+fx)}{5f}-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{2f}+\frac{a^3c\tan(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*ArcTanh[Sin[e + f*x]])/(4*f) + (a^3*c*Sec[e + f*x]*Tan[e + f*x])/(4*
f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) - (2*a^3*c*Tan[e + f*x]^3)/(
3*f) - (a^3*c*Tan[e + f*x]^5)/(5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a^2\sec^2(e+fx)\tan^2(e+fx)+2a^2\sec^3(e+\right.\right.$$

$$= -\left(\left(a^3c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^3c\right)\int \mathrm{se}$$

$$= -\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^3c\right)\int \sec^3(e+f$$

$$= \frac{a^3c\sec(e+fx)\tan(e+fx)}{4f}-\frac{a^3c\sec^3(e+fx)\tan(e-}{2f}$$

$$= \frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}+\frac{a^3c\sec(e+fx)\tan(e+fx}{4f}$$

Mathematica [A] time = 0.30, size = 68, normalized size = 0.65

$$\frac{a^3c\left(15\tanh^{-1}(\sin(e+fx))-\tan(e+fx)\left(12\tan^4(e+fx)+40\tan^2(e+fx)+30\sec^3(e+fx)-15\sec(e+fx\right.\right.}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*(15*ArcTanh[Sin[e + f*x]] - Tan[e + f*x]*(-15*Sec[e + f*x] + 30*Sec[
e + f*x]^3 + 40*Tan[e + f*x]^2 + 12*Tan[e + f*x]^4)))/(60*f)
```

fricas [A] time = 0.43, size = 131, normalized size = 1.25

$$\frac{15\,a^3c\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(28\,a^3c\cos\left(fx+\right.\right.}{120\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*a^3*c*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e
)^5*log(-sin(f*x + e) + 1) + 2*(28*a^3*c*cos(f*x + e)^4 + 15*a^3*c*cos(f*x
+ e)^3 - 16*a^3*c*cos(f*x + e)^2 - 30*a^3*c*cos(f*x + e) - 12*a^3*c)*sin(f*
x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(a^3*c/8*ln(abs(tan((f*x+exp(1))/2)-1))-a^3*c/8*ln(abs(tan((f*
x+exp(1))/2)+1))-(-15*tan((f*x+exp(1))/2)^9*a^3*c+70*tan((f*x+exp(1))/2)^7*
a^3*c-128*tan((f*x+exp(1))/2)^5*a^3*c+250*tan((f*x+exp(1))/2)^3*a^3*c+15*ta
n((f*x+exp(1))/2)*a^3*c)*1/60/(tan((f*x+exp(1))/2)^2-1)^5)
```

maple [A] time = 1.40, size = 130, normalized size = 1.24

$$\frac{a^3c\sec\left(fx+e\right)\tan\left(fx+e\right)}{4f}+\frac{a^3c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{4f}+\frac{7a^3c\tan\left(fx+e\right)}{15f}-\frac{a^3c\left(\sec^3\left(fx+e\right)\right)\tan}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/4*a^3*c*sec(f*x+e)*tan(f*x+e)/f+1/4/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+7/1
5*a^3*c*tan(f*x+e)/f-1/2*a^3*c*sec(f*x+e)^3*tan(f*x+e)/f-1/5/f*a^3*c*tan(f*
x+e)*sec(f*x+e)^4-4/15/f*a^3*c*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.46, size = 172, normalized size = 1.64

$$-\frac{8\left(3\tan\left(fx+e\right)^{5}+10\tan\left(fx+e\right)^{3}+15\tan\left(fx+e\right)\right)a^{3}c-15a^{3}c\left(\frac{2\left(3\sin\left(fx+e\right)^{3}-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\sin\left(fx+e\right)^{2}+1}-3\log\left(\sin\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c -
15*a^3*c*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
 + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 60*a^3*
c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 120*a^3*c*tan(f*x + e))/f
```

mupad [B] time = 6.32, size = 175, normalized size = 1.67

$$\frac{-\frac{c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2}+\frac{7\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}-\frac{64\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{25\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+\frac{c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}+\frac{a^3\,c\,\mathrm{atanh}\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] ((a^3*c*tan(e/2 + (f*x)/2))/2 + (25*a^3*c*tan(e/2 + (f*x)/2)^3)/3 - (64*a^3
*c*tan(e/2 + (f*x)/2)^5)/15 + (7*a^3*c*tan(e/2 + (f*x)/2)^7)/3 - (a^3*c*tan
(e/2 + (f*x)/2)^9)/2)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4
+ 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10
- 1)) + (a^3*c*atanh(tan(e/2 + (f*x)/2)))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^{3}c\left(\int\left(-\sec^{2}\left(e+fx\right)\right)dx+\int\left(-2\sec^{3}\left(e+fx\right)\right)dx+\int 2\sec^{5}\left(e+fx\right)dx+\int\sec^{6}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) +
Integral(2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

3.169 $\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=86

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2c\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{a^2c\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] 1/8*a^2*c*arctanh(sin(f*x+e))/f+1/8*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^2*c
*sec(f*x+e)^3*tan(f*x+e)/f-1/3*a^2*c*tan(f*x+e)^3/f
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3962, 2607, 30, 2611, 3768, 3770}

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2c\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{a^2c\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(8*f) + (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(8*
f) - (a^2*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (a^2*c*Tan[e + f*x]^3)/(3*
f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a\sec^2(e+fx)\tan^2(e+fx)+a\sec^3(e+\right.\right.$$
$$= -\left(\left(a^2c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int$$
$$= -\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{4}\left(a^2c\right)\int \sec^3(e+$$
$$= \frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c\sec^3(e+fx)\tan(e}{4f}$$
$$= \frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c\sec(e+fx)\tan(e+f}{8f}$$

Mathematica [A] time = 0.18, size = 57, normalized size = 0.66

$$\frac{a^2c\left(3\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(-8\tan^2(e+fx)-6\sec^3(e+fx)+3\sec(e+fx)\right)\right)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*Sec[e + f*x] - 6*Sec[e +
f*x]^3 - 8*Tan[e + f*x]^2)))/(24*f)
```

fricas [A] time = 0.46, size = 117, normalized size = 1.36

$$\frac{3\,a^2c\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left(8\,a^2c\cos\left(fx+e\right)^3+\right.}{48\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*a^2*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^4
*log(-sin(f*x + e) + 1) + 2*(8*a^2*c*cos(f*x + e)^3 + 3*a^2*c*cos(f*x + e)^
2 - 8*a^2*c*cos(f*x + e) - 6*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(a^2*c/16*ln(abs(tan((f*x+exp(1))/2)-1))-a^2*c/16*ln(abs(tan((
f*x+exp(1))/2)+1))+(3*tan((f*x+exp(1))/2)^7*a^2*c-11*tan((f*x+exp(1))/2)^5*
a^2*c+53*tan((f*x+exp(1))/2)^3*a^2*c+3*tan((f*x+exp(1))/2)*a^2*c)*1/24/(tan
((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.37, size = 107, normalized size = 1.24

$$\frac{a^2c\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{a^2c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}+\frac{a^2c\tan\left(fx+e\right)}{3f}-\frac{a^2c\tan\left(fx+e\right)\left(\sec^2\left(fx\right.\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/8*a^2*c*sec(f*x+e)*tan(f*x+e)/f+1/8/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))+1/3
*a^2*c*tan(f*x+e)/f-1/3/f*a^2*c*tan(f*x+e)*sec(f*x+e)^2-1/4*a^2*c*sec(f*x+e
)^3*tan(f*x+e)/f
```

maxima [B] time = 0.48, size = 160, normalized size = 1.86

$$-\frac{16\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{2}c-3\,a^{2}c\left(\frac{2\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\right.\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 12*a^2*c*(2*sin(f*x + e)/(sin(f
*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*
tan(f*x + e))/f
```

mupad [B] time = 4.92, size = 146, normalized size = 1.70

$$\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{11\,c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{53\,c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] (a^2*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((a^2*c*tan(e/2 + (f*x)/2))/4 + (
53*a^2*c*tan(e/2 + (f*x)/2)^3)/12 - (11*a^2*c*tan(e/2 + (f*x)/2)^5)/12 + (a
^2*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x
)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}c\left(\int\left(-\sec^{2}\left(e+fx\right)\right)dx+\int\left(-\sec^{3}\left(e+fx\right)\right)dx+\int\sec^{4}\left(e+fx\right)dx+\int\sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sec(e + f*x)**2, x) + Integral(-sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

3.170 $\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=17

$$-\frac{ac\tan^3(e+fx)}{3f}$$

```
[Out] -1/3*a*c*tan(f*x+e)^3/f
```

Rubi [A] time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3962, 2607, 30}

$$-\frac{ac\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)$$
$$= -\frac{(ac)\,\text{Subst}\left(\int x^2\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{ac\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$-\frac{ac\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a*c*Tan[e + f*x]^3)/f
```

fricas [B] time = 0.43, size = 35, normalized size = 2.06

$$\frac{\left(ac\cos\left(fx+e\right)^2 - ac\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/3*(a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

giac [A] time = 1.39, size = 16, normalized size = 0.94

$$-\frac{ac\tan\left(fx+e\right)^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/3*a*c*tan(f*x + e)^3/f
```

maple [B] time = 0.94, size = 36, normalized size = 2.12

$$\frac{ca\tan\left(fx+e\right)+ca\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/f*(c*a*tan(f*x+e)+c*a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

maxima [B] time = 0.56, size = 36, normalized size = 2.12

$$-\frac{\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)ac-3\,ac\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a*c - 3*a*c*tan(f*x + e))/f
```

mupad [B] time = 1.70, size = 15, normalized size = 0.88

$$-\frac{a\,c\tan\left(e+f\,x\right)^{3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] -(a*c*tan(e + f*x)^3)/(3*f)
```

sympy [A] time = 2.30, size = 51, normalized size = 3.00

$$\begin{cases} \frac{-ac\left(\frac{\tan^{3}\left(e+fx\right)}{3}+\tan\left(e+fx\right)\right)+ac\tan\left(e+fx\right)}{f} & \text{for}\: f \neq 0 \\ x\left(a\sec\left(e\right)+a\right)\left(-c\sec\left(e\right)+c\right)\sec^{2}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] Piecewise(((-a*c*(tan(e + f*x)**3/3 + tan(e + f*x)) + a*c*tan(e + f*x))/f,
Ne(f, 0)), (x*(a*sec(e) + a)*(-c*sec(e) + c)*sec(e)**2, True))
```

3.171 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{c\tan(e+fx)}{af}+\frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

```
[Out] 2*c*arctanh(sin(f*x+e))/a/f-c*tan(f*x+e)/a/f-2*c*tan(f*x+e)/f/(a+a*sec(f*x+
e))
```

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4008, 3787, 3770, 3767, 8}

$$-\frac{c\tan(e+fx)}{af}+\frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c*ArcTanh[Sin[e + f*x]])/(a*f) - (c*Tan[e + f*x])/(a*f) - (2*c*Tan[e + f
*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx &= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{\int \sec(e+fx)(-2ac+ac\sec(e+fx))\,dx}{a^2}\\ &= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{c\int \sec^2(e+fx)\,dx}{a}+\frac{(2c)\int \sec(e+fx)\,dx}{a}\\ &= \frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{c\,\mathrm{Subst}(\int 1\,dx,x,-\tan}{af}\\ &= \frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{c\tan(e+fx)}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.62, size = 154, normalized size = 2.75

$$-\frac{c\left(\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f}+\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\log\left(\sin\left(\right.\right.}{}\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*((2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f - (2*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]])/f + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])) + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) +
 (2*Tan[(e + f*x)/2])/f))/a)
```

fricas [A] time = 0.44, size = 105, normalized size = 1.88

$$\frac{\left(c\cos\left(fx+e\right)^2+c\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(c\cos\left(fx+e\right)^2+c\cos\left(fx+e\right)\right)\log\left(-\sin\left(fx+e\right)+\right.}{af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] ((c*cos(f*x + e)^2 + c*cos(f*x + e))*log(sin(f*x + e) + 1) - (c*cos(f*x + e
)^2 + c*cos(f*x + e))*log(-sin(f*x + e) + 1) - (3*c*cos(f*x + e) + c)*sin(f
*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(tan((f*x+exp(1))/2)*c/a-tan((f*x+exp(1))/2)*c/a/(tan((f*x+exp
(1))/2)^2-1)+c/a*ln(abs(tan((f*x+exp(1))/2)-1))-c/a*ln(abs(tan((f*x+exp(1))
/2)+1)))
```

maple [A] time = 0.58, size = 104, normalized size = 1.86

$$-\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{fa}+\frac{c}{fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}-\frac{2c\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fa}+\frac{2c\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{fa}+\frac{c}{fa\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/f/a*c*tan(1/2*e+1/2*f*x)+1/f/a*c/(tan(1/2*e+1/2*f*x)-1)-2/f/a*c*ln(tan(1
/2*e+1/2*f*x)-1)+2/f/a*c*ln(tan(1/2*e+1/2*f*x)+1)+1/f/a*c/(tan(1/2*e+1/2*f*
x)+1)
```

maxima [B] time = 0.41, size = 194, normalized size = 3.46

$$\frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(}{\cos(}\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] (c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*(log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) -
 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))))/f
```

mupad [B] time = 1.74, size = 71, normalized size = 1.27

$$\frac{4\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}-\frac{2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\right)}-\frac{2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))),x)
```

```
[Out] (4*c*atanh(tan(e/2 + (f*x)/2)))/(a*f) - (2*c*tan(e/2 + (f*x)/2))/(f*(a - a*
tan(e/2 + (f*x)/2)^2)) - (2*c*tan(e/2 + (f*x)/2))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**3/(sec(e + f*x) + 1), x))/a
```

3.172 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] -c*arctanh(sin(f*x+e))/a^2/f+7/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-2/3*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.16, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4008, 3998, 3770, 3794}

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a^2*f)) + (7*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec
[e + f*x])) - (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-4ac+3ac\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}$$

$$= -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{c\int \sec(e+fx)\,dx}{a^2} + \frac{(7c)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= -\frac{c\tanh^{-1}(\sin(e+fx))}{a^2 f} - \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{7c\tan(e+fx)}{3f\left(a^2+a^2\sec(e+f\right.}$$

Mathematica [B] time = 0.46, size = 335, normalized size = 4.79

$$\frac{c\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(-6\sin\left(e+\frac{fx}{2}\right)+10\sin\left(e+\frac{3fx}{2}\right)+3\cos\left(e+\frac{3fx}{2}\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*Cos[(e + f*x)/2]*Sec[e/2]*Sec[e + f*x]^2*(3*Cos[e + (3*f*x)/2]*Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] + 3*Cos[2*e + (3*f*x)/2]*Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] + 9*Cos[(f*x)/2]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Cos[e + (f*x)/2]*(Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]]) - 3*Cos[e + (3*f*x)/2]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 3
*Cos[2*e + (3*f*x)/2]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 24*Sin[(f*
x)/2] - 6*Sin[e + (f*x)/2] + 10*Sin[e + (3*f*x)/2]))/(6*a^2*f*(1 + Sec[e +
f*x])^2)
```

fricas [A] time = 0.47, size = 123, normalized size = 1.76

$$-\frac{3\left(c\cos\left(fx+e\right)^2+2\,c\cos\left(fx+e\right)+c\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c\cos\left(fx+e\right)^2+2\,c\cos\left(fx+e\right)+c\right)\log\left(\right.}{6\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - 3
*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(-sin(f*x + e) + 1) - 2*(5*c*
cos(f*x + e) + 7*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x +
 e) + a^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)-2/f*(-c*1/2/a^2*ln(abs(tan((f*x+exp(1))/2)-1))+c*1/2/a^2*ln(abs(ta
n((f*x+exp(1))/2)+1))+(-4/3*tan((f*x+exp(1))/2)^3*c*a^4-8*tan((f*x+exp(1))/
2)*c*a^4)*1/8/a^6)
```

maple [A] time = 0.71, size = 81, normalized size = 1.16

$$\frac{c\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f\,a^{2}}+\frac{2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^{2}}+\frac{c\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{f\,a^{2}}-\frac{c\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{f\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/3/f*c/a^2*tan(1/2*e+1/2*f*x)^3+2/f*c/a^2*tan(1/2*e+1/2*f*x)+1/f*c/a^2*ln(
tan(1/2*e+1/2*f*x)-1)-1/f*c/a^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.61, size = 144, normalized size = 2.06

$$\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a^{2}}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{2}}\right)+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^2) + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.69, size = 44, normalized size = 0.63

$$\frac{c\left(6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-6\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\right)}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (c*(6*tan(e/2 + (f*x)/2) - 6*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*x)/2)
^3))/(3*a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.173 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$-\frac{4c\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{11c\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] -2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+11/15*c*tan(f*x+e)/a/f/(a+a*sec(f*x+
e))^2-4/15*c*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4008, 4000, 3794}

$$-\frac{4c\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{11c\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (11*c*Tan[e + f*x])/(15*
a*f*(a + a*Sec[e + f*x])^2) - (4*c*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f
*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-6ac+5ac\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{(4c)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}$$

$$= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{4c\tan(e+f}{15f\left(a^3+a^3\sec(e\right.}$$

Mathematica [A] time = 0.17, size = 43, normalized size = 0.50

$$-\frac{c(\cos(e+fx)+4)\tan^3\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{30a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -1/30*(c*(4 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(a^3*f)
```

fricas [A] time = 0.40, size = 78, normalized size = 0.91

$$\frac{\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - 4*c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

giac [A] time = 0.82, size = 39, normalized size = 0.45

$$-\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}{30\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 + 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

maple [A] time = 0.76, size = 37, normalized size = 0.43

$$\frac{c\left(-\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3}\right)}{2f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f*c/a^3*(-1/5*tan(1/2*e+1/2*f*x)^5-1/3*tan(1/2*e+1/2*f*x)^3)
```

maxima [A] time = 0.51, size = 115, normalized size = 1.34

$$-\frac{\frac{c\left(\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}-\frac{3\,c\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x +
e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.70, size = 35, normalized size = 0.41

$$-\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\left(3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+5\right)}{30\,a^{3}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^3),x)
```

```
[Out] -(c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 + 5))/(30*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f
*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.174 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+}{2}}}{}$$

```
[Out] -1/3*a^2*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(
f*x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f-1/3*a^2*c*(cos(f*x+e)^2)^(2+1/2*p
)*hypergeom([3/2, 2+1/2*p],[5/2],sin(f*x+e)^2)*(g*sec(f*x+e))^(1+p)*tan(f*x
+e)^3/f/g
```

Rubi [A] time = 0.20, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3962, 2617, 16}

$$-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a^2*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/2,
 Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f) - (a^2*c*(Cos[e +
 f*x]^2)^((4 + p)/2)*Hypergeometric2F1[3/2, (4 + p)/2, 5/2, Sin[e + f*x]^2]
*(g*Sec[e + f*x])^(1 + p)*Tan[e + f*x]^3)/(3*f*g)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a(g\sec(e+fx))^p\tan^2(e+fx)+a\sec(e\right.\right.$$
$$= -\left(\left(a^2c\right)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)-\left(a^2c\right.$$
$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(}{3f}$$
$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(}{3f}$$

Mathematica [C] time = 30.61, size = 7087, normalized size = 50.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2c\sec\left(fx+e\right)^3+a^2c\sec\left(fx+e\right)^2-a^2c\sec\left(fx+e\right)-a^2c\right)\left(g\sec\left(fx+e\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(a^2*c*sec(f*x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e)
 - a^2*c)*(g*sec(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a\sec\left(fx+e\right)+a\right)^{2}\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

maple [F] time = 3.00, size = 0, normalized size = 0.00

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a+a\sec\left(fx+e\right)\right)^{2}\left(c-c\sec\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+fx\right)}\right)^{2}\left(c-\frac{c}{\cos\left(e+fx\right)}\right)\left(\frac{g}{\cos\left(e+fx\right)}\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
```

```
[Out] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2c\left(\int\left(-\left(g\sec\left(e+fx\right)\right)^p\right)dx+\int\left(-\left(g\sec\left(e+fx\right)\right)^p\sec\left(e+fx\right)\right)dx+\int\left(g\sec\left(e+fx\right)\right)^p\sec^2\left(e+fx\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral(-(g*sec(e + f*x))**p*
sec(e + f*x), x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)**2, x) + Integ
ral((g*sec(e + f*x))**p*sec(e + f*x)**3, x))
```

3.175 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=65

$$-\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

```
[Out] -1/3*a*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(f*
x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3962, 2617}

$$-\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/2, S
in[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f)
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)$$
$$= -\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(g\,s}{3f}$$

Mathematica [A] time = 0.33, size = 72, normalized size = 1.11

$$-\frac{ac\tan(e+fx)(g\sec(e+fx))^p\left(\frac{{}_2F_1\left(\frac{1}{2},\frac{p}{2};\frac{p+2}{2};\sec^2(e+fx)\right)}{\sqrt{-\tan^2(e+fx)}}+p\right)}{fp(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*c*(g*Sec[e + f*x])^p*Tan[e + f*x]*(p + Hypergeometric2F1[1/2, p/2, (2
+ p)/2, Sec[e + f*x]^2]/Sqrt[-Tan[e + f*x]^2]))/(f*p*(1 + p)))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(ac\sec\left(fx+e\right)^2-ac\right)\left(g\sec\left(fx+e\right)\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(a*c*sec(f*x + e)^2 - a*c)*(g*sec(f*x + e))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a\sec\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

maple [F] time = 2.62, size = 0, normalized size = 0.00

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a+a\sec\left(fx+e\right)\right)\left(c-c\sec\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a\sec\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right){\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
```

```
[Out] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac\left(\int\left(-\left(g\sec\left(e+fx\right)\right)^{p}\right)dx+\int\left(g\sec\left(e+fx\right)\right)^{p}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] -a*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral((g*sec(e + f*x))**p*sec(
e + f*x)**2, x))
```

3.176 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=180

$$-\frac{cg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{af(1-p)\sqrt{\sin^2(e+fx)}}+\frac{2c\sin(e+fx)(g\sec(e+fx))^p\,{}_2F_1\left(\frac{1}{2},-\right.}{af\sqrt{\sin^2(e+fx)}}$$

```
[Out] -c*g*(1-2*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*sec(f*
x+e))^(-1+p)*sin(f*x+e)/a/f/(1-p)/(sin(f*x+e)^2)^(1/2)+2*c*hypergeom([1/2,
-1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a/f/(sin(f*x+e)
^2)^(1/2)-2*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] time = 0.23, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4020, 3787, 3772, 2643}

$$-\frac{cg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{af(1-p)\sqrt{\sin^2(e+fx)}}+\frac{2c\sin(e+fx)(g\sec(e+fx))^p\,{}_2F_1\left(\frac{1}{2},-\right.}{af\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*g*(1 - 2*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^
2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a*f*(1 - p)*Sqrt[Sin[e + f*x]^2
])) + (2*c*Hypergeometric2F1[1/2, -p/2, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec[e
 + f*x])^p*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) - (2*c*(g*Sec[e + f*x])
^p*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{\int(g\sec(e+fx))^p(ac(1-2p)+2a}{a^2}$$

$$= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{(c(1-2p))\int(g\sec(e+fx))^p\,dx}{a}$$

$$= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{\left(c(1-2p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+\right.}{a}$$

$$= -\frac{c(1-2p)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+fx))}{af(1-p)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 16.81, size = 3396, normalized size = 18.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-6*c*Sec[e + f*x]^p*(g*Sec[e + f*x])^p*Tan[(e + f*x)/2]^3*(-((AppellF1[1/2
, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^
2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
```

```
 + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(a*f*(3*Sec[(e + f*x)/2]^2*S
ec[e + f*x]^p*(-((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1
[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2,
 p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2,
p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 +
 p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
 + 6*p*Sec[e + f*x]^(1 + p)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-((AppellF1[1/2,
 p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2
)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 6*Sec[e + f*x]^p*Tan[(e + f*
x)/2]*((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Cos[(e + f*x)/2]*Sin[(e + f*x)/2])/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (Cos
[(e + f*x)/2]^2*(-1/3*((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (p*AppellF
1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + ((p*Appel
lF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)/(3*Ap
pellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(App
ellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appell
F1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
f*x)/2]^2) - (AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
```

```
2]^2]*(2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((p*AppellF1[3/2, p, 1 - p, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*p*Tan[(e + f*x)/2]^2*
((-3*(1 - p)*AppellF1[5/2, p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (6*p*AppellF1[5/2, 1 + p,
1 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5/2, 2 + p, -p, 7/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/(3*Appe
llF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(Appel
lF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1
[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2 + (AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[(e + f*x)/2]^2*(2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
 + 3*(-1/3*((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (p*AppellF1[3/2, 1 +
p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + p)*((-3*(2 - p)*AppellF1
[5/2, p, 3 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5 + (3*p*AppellF1[5/2, 1 + p, 2 - p, 7/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + p
*((-3*(1 - p)*AppellF1[5/2, 1 + p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5
/2, 2 + p, 1 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to divide, perhaps due to rounding error%%%{1,[0,1,2,0]%%%}
+%%%{1,[0,1,0,0]%%%} / %%%{2,[0,0,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 4.06, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec \left(f x+e\right)\right)^{p} \left(c-c \sec \left(f x+e\right)\right)}{a+a \sec \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^{p}}{a+\frac{a}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)),x)
```

```
[Out] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\left(g\sec\left(e+fx\right)\right)^{p}}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\left(g\sec\left(e+fx\right)\right)^{p}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x) + 1), x) + Integral((g*sec(
e + f*x))**p*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

3.177 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=226

$$-\frac{cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{c(5-4p)\sin(e+fx)(g\sec(e+fx))^p\,{}_2F}{3a^2f\sqrt{\sin^2(e+}}$$

```
[Out] -1/3*c*g*(3-4*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*se
c(f*x+e))^(-1+p)*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)+1/3*c*(5-4*p)*hyperg
eom([1/2, -1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a^2/f
/(sin(f*x+e)^2)^(1/2)-1/3*c*(5-4*p)*(g*sec(f*x+e))^p*tan(f*x+e)/a^2/f/(1+se
c(f*x+e))-2/3*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.41, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4020, 3787, 3772, 2643}

$$-\frac{cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{c(5-4p)\sin(e+fx)(g\sec(e+fx))^p\,{}_2F}{3a^2f\sqrt{\sin^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -(c*g*(3 - 4*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^2
]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) +
(c*(5 - 4*p)*Hypergeometric2F1[1/2, -p/2, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec
[e + f*x])^p*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (c*(5 - 4*p)*(g
*Sec[e + f*x])^p*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - (2*c*(g*Sec[e
 + f*x])^p*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\int \frac{(g\sec(e+fx))^p(ac(3-2p)-2ac(1-p)\sec(e+}{a+a\sec(e+fx)}}{3a^2}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{c(3-4p)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+fx))}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 13.33, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
 x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
```

```
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to divide, perhaps due to rounding error%%%{-1,[0,1,4,0]%%%
}+%%%{1,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%} Error: Bad Argument Value
```

maple [F] time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{p}\left(c-c\sec\left(fx+e\right)\right)}{\left(a+a\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a)^2,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\left(g\sec\left(e+fx\right)\right)^{p}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\left(g\sec\left(e+fx\right)\right)^{p}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
) + Integral((g*sec(e + f*x))**p*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e +
f*x) + 1), x))/a**2
```

3.178 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=104

$$\frac{2g\cot(e+fx)\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}{cf}-\frac{2\sqrt{a}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{cf}$$

```
[Out] -2*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))*a^(1/2)/c/f+2*g*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*
x+e))^(1/2)/c/f
```

Rubi [A] time = 0.28, antiderivative size = 143, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3964, 47, 63, 217, 203}

$$\frac{2ag^{3/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{g\sec(e+fx)}}{\sqrt{g}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{2ag\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x]),
x]
```

```
[Out] (-2*a*g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
 c*Sec[e + f*x])) + (2*a*g^(3/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqr
t[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{(acg\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{gx}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\left(ag^2\tan(e+fx)\right)\text{Sub}}{f\sqrt{a+a\sec(}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(2ag\tan(e+fx))\,\text{Sub}}{f\sqrt{a+a\sec(}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(2ag\tan(e+fx))\,\text{Sub}}{f\sqrt{a+a\sec(e}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{2ag^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{g\,\text{se}}}{\sqrt{g}\,\sqrt{c-c}}\right.}{\sqrt{c}\,f\sqrt{a+a\sec(e+f}}$$

Mathematica [A] time = 1.53, size = 162, normalized size = 1.56

$$\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,(g\sec(e+fx))^{3/2}\left(\sqrt{\sec(e+fx)}\,\sqrt{\sec(e+fx)+1}+\sqrt{\tan^2(e+fx)}\,\left(\log\right.\right.}{cf\sec^{\frac{3}{2}}(e+fx)(\sec(e+fx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e +
f*x]),x]
```

```
[Out] (2*Cot[(e + f*x)/2]*(g*Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt
[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (Log[1 + Sec[e + f*x]] - Log[Sqrt[S
ec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x
]^2]])*Sqrt[Tan[e + f*x]^2]))/(c*f*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^(3
/2))
```

fricas [A] time = 0.58, size = 340, normalized size = 3.27

$$\left[\frac{\sqrt{ag}\,g\log\left(\frac{ag\cos\left(fx+e\right)^{3}-7\,ag\cos\left(fx+e\right)^{2}+4\,\sqrt{ag}\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{g}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)+8\,ag}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}}\right)\sin\left(fx+e\right)+}{2\,cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2 + 4*sqrt(a
*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 + cos(f*x + e
)^2))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), -(sqrt(-a*g)*g*arctan(2*sqrt(-
a*g)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g))*sin(f*x +
 e) - 2*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(
f*x + e))/(c*f*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{a\sec\left(fx+e\right)+a}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(-sqrt(a*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e)
- c), x)
```

maple [B] time = 2.29, size = 236, normalized size = 2.27

$$-\frac{\left(2\sin\left(fx+e\right)\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}+\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\,\left(\cos\left(fx+e\right)+1+\sin\left(fx+e\right)\right)}{2}\right)\cos\left(fx+e\right)+\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*(2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)+arctanh(1/2*(1/(1+cos(f*x+e))
)^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))*cos(f*x+e)+arctanh(1/2*(1/(1+cos(f*x+e))
)^(1/2)*(-cos(f*x+e)-1+sin(f*x+e)))*cos(f*x+e)-arctanh(1/2*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))-arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(-
cos(f*x+e)-1+sin(f*x+e))))*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+
e))*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^4
```

maxima [B] time = 1.22, size = 979, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sqrt(2)*g*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*sqrt(2)*g*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) +
(g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(
2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (g*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)
*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) + (g*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin
(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arc
```

```
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 2))*sqrt(a)*sqrt(g)/((c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 - 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + c)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c-\frac{c}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x)
),x)
```

```
[Out] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{a\sec\left(e+fx\right)+a}}{\sec\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral((g*sec(e + f*x))**(3/2)*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) -
1), x)/c
```

3.179 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=81

$$\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{acf}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}\,cf}$$

```
[Out] -1/2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1
/2)/a^(1/2)+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] time = 0.23, antiderivative size = 116, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3964, 78, 63, 208}

$$-\frac{\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}-\frac{\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]))) - (ArcTan
h[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c
]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{x}{(a+ax)(c-cx)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(a\tan(e+fx))\,\text{Subst}}{2f\sqrt{a+a\sec(e}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{(a\tan(e+fx))\,\text{Subst}}{cf\sqrt{a+a\sec(}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-c\sec(}}{\sqrt{2}\,\sqrt{}}\right.}{\sqrt{2}\,\sqrt{c}\,f\sqrt{a+a\sec(e-}}$$

Mathematica [A] time = 0.47, size = 73, normalized size = 0.90

$$-\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{2}\,\sqrt{\sec(e+fx)-1}\,\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right)-2\right)}{2cf\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/2*(Cot[(e + f*x)/2]*(-2 + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]
]*Sqrt[-1 + Sec[e + f*x]]))/(c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 0.52, size = 260, normalized size = 3.21

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\,\sqrt{2}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{-\frac{1}{a}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)+3\,\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+4\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,c}{4\,acf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqr
t(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t
```

```
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assump
tions on a variableWarning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableUnable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
```

```
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
```

```
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at
zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_
nostep^2-1)]Evaluation time: 1.02index.cc index_m i_lex_is_greater Error: B
ad Argument Value
```

maple [B] time = 1.68, size = 204, normalized size = 2.52

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\ \ln\left(-\frac{-\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\ \sin\left(fx+e\right)+\cos\left(fx+e\right)-1}{\sin\left(fx+e\right)}\right)+2\sin\left(fx+e\right)\cos\left(f\right.\right.}{2cf\left(-1+\cos^{2}\left(fx+e\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/c/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)^2*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/sin(f*x+e))+2*sin(f*x+e)*cos(f*x+e)-(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e
)-1)/sin(f*x+e)))/(-1+cos(f*x+e)^2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec\left(fx+e\right)^{2}}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\sqrt{a+\frac{a}{\cos\left(e+fx\right)}}\,\left(c\,\cos\left(e+f\,x\right)-c\,\left(\frac{\cos\left(2\,e+2\,f\,x\right)}{2}+\frac{1}{2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
```

```
[Out] -int(1/((a + a/cos(e + f*x))^(1/2)*(c*cos(e + f*x) - c*(cos(2*e + 2*f*x)/2
+ 1/2))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)-\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sec(e + f*x)**2/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*s
ec(e + f*x) + a)), x)/c
```

3.180 $\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=140

$$\frac{\csc(e+fx)\sqrt{a\sec(e+fx)+a}}{acf\sqrt{\sec(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)\sqrt{\sec(e+fx)}}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf}-\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

```
[Out] -2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*arcta
nh(1/2*sin(f*x+e)*a^(1/2)*sec(f*x+e)^(1/2)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/
c/f*2^(1/2)/a^(1/2)+csc(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f/sec(f*x+e)^(1/2
)
```

Rubi [A] time = 0.28, antiderivative size = 213, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3964, 98, 157, 63, 217, 203, 93, 205}

$$-\frac{\sin(e+fx)\sec^{\frac{3}{2}}(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}+\frac{2\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tan^{-1}\left(\right.}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -((Sec[e + f*x]^(3/2)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x]))) + (2*ArcTan[(Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x
]]]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*
x]]) - (ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]
]]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{x^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int\right.}{cf\sqrt{a+a\sec(e}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int\right.}{f\sqrt{a+a\sec(e+}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(2\tan(e+fx))\,\text{Subst}}{f\sqrt{a+a\sec(e+}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec}}{\sqrt{c-c\sec(e}}\right.}{\sqrt{2}\sqrt{c}\,f\sqrt{a+a\sec(e}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sec(e+}}{\sqrt{c-c\sec(e+}}\right.}{\sqrt{c}\,f\sqrt{a+a\sec(e+f}}$$

Mathematica [B] time = 6.46, size = 724, normalized size = 5.17

$$\frac{\sin(e+fx)\sin^2\left(\frac{e}{2}+\frac{fx}{2}\right)\cos(e+fx)(\sec(e+fx)+1)^{3/2}\sqrt{\sec^2(e+fx)-1}\,\left(\log\left(-3\sec^2(e+fx)-2\sqrt{2}\sqrt{\sec(e}\right.\right.}{2f(\cos(e+fx)+1)\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*Sqrt[(1 + Cos[e + f*x])*Sec[e + f*x]]*Sqrt[1 + Sec[e +
f*x]]*((-2*Cot[e])/f + (Csc[e/2]*Csc[e/2 + (f*x)/2]*Sin[(f*x)/2])/f + (Sec[
e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/f)*Sin[e/2 + (f*x)/2]^2)/(Sqrt[a*(1 +
 Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e + f*x]*(Log[1 - 2*Sec[e + f*
```

```
x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
*Sqrt[-1 + Sec[e + f*x]^2]] - Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2
*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2
]])*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e/2 + (f*x)/2]^2
*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*Sqrt[2 - 2*Cos[e + f*x]^2]*Sqrt[1 -
Cos[e + f*x]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e +
 f*x]*(-8*Log[1 + Sec[e + f*x]] + 8*Log[Sqrt[Sec[e + f*x]] + Sec[e + f*x]^(
3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Sqrt[2]*(-Log[1
- 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 +
 Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Log[1 - 2*Sec[e + f*x] - 3*Sec[
e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 +
Sec[e + f*x]^2]]))*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e
/2 + (f*x)/2]^2*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*(1 - Cos[e + f*x]^2)*
Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x]))
```

fricas [A] time = 0.51, size = 462, normalized size = 3.30

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos\left(fx+e\right)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\cos\left(fx+e\right)}\sin\left(fx+e\right)}{\sqrt{a}}-2\cos\left(fx+e\right)-3}{\cos\left(fx+e\right)^2+2\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+2\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^3-7a\cos\ldots}{\ldots}\right)\ldots}{4acf\sin\left(fx+\ldots\right)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(a)*log(-(cos(f*x + e)^2 - 2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(cos(f*x + e))*sin(f*x + e)/sqrt(a) - 2*cos(f*x + e
) - 3)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 2*sqrt(a)*log(
(a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/sqrt(cos(f*x
 + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x + e)), -1
/2*(sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*sqrt(cos(f*x + e))/sin(f*x + e))*sin(f*x + e) + 2*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)
)*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))*sin(f*x + e) - 2*
```

```
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x
+ e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sec\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-sec(f*x + e)^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c
)), x)
```

maple [B] time = 2.88, size = 317, normalized size = 2.26

$$\frac{\left(\frac{1}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\left(\cos^{3}\left(fx+e\right)\right)\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos\left(fx+e\right)}}\ \left(\cos\left(fx+e\right)+1+\sin\left(fx+e\right)\right)\sqrt{2}}{4}\right)\cos\left(fx+e\right)-\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/c/f*(1/cos(f*x+e))^(5/2)*cos(f*x+e)^3*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)
*(2^(1/2)*arctan(1/4*(-2/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e))*2^
(1/2))*cos(f*x+e)-2^(1/2)*arctan(1/4*(-2/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+
1-sin(f*x+e))*2^(1/2))*cos(f*x+e)-2^(1/2)*arctan(1/4*(-2/(1+cos(f*x+e)))^(1
/2)*(cos(f*x+e)+1+sin(f*x+e))*2^(1/2))+2^(1/2)*arctan(1/4*(-2/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)+1-sin(f*x+e))*2^(1/2))-arctan(1/2*sin(f*x+e)*(-2/(1+co
s(f*x+e)))^(1/2))*cos(f*x+e)+sin(f*x+e)*(-2/(1+cos(f*x+e)))^(1/2)+arctan(1/
2*sin(f*x+e)*(-2/(1+cos(f*x+e)))^(1/2)))/(-2/(1+cos(f*x+e)))^(1/2)/sin(f*x+
e)^2/a
```

maxima [B] time = 1.06, size = 1310, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqr
t(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*log(2*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*lo
g(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2) + (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*
sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt
(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/
4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 1) + (cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*log(cos(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1) - 4*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 4*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))/((sqrt(2)*
c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
```

```
[Out] int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.181 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=116

$$\frac{g\cot(e+fx)\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}{acf}-\frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{2}\,\sqrt{a}\,cf}$$

```
[Out] -1/2*g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)+g*cot(f*x+e)*(g*sec(f*x+e
))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] time = 0.30, antiderivative size = 150, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3964, 94, 93, 205}

$$\frac{g^{3/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{g\sec(e+fx)}}{\sqrt{g}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{g\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*
Sec[e + f*x]))) + (g^(3/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(S
qrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(acg\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{gx}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\left(ag^2\tan(e+fx)\right)\text{Sub}}{2f\sqrt{a+a\,\text{se}}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\left(ag^2\tan(e+fx)\right)\text{Sub}}{f\sqrt{a+a\sec(e}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{g}}{\sqrt{g}\,\sqrt{c-c}}\right.}{\sqrt{2}\,\sqrt{c}\,f\sqrt{a+a\sec(e}}$$

Mathematica [B] time = 2.98, size = 236, normalized size = 2.03

$$-\frac{a\sin^3\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{5/2}\left(-4\sec(e+fx)+\frac{\sqrt{\tan^2(e+fx)}\left(\log\left(-3\sec^2(e+fx)-2\sec(e+fx)-2\sqrt{2}\right.\right.}{}\right.}{cfg(\sec(e+fx)-1)^2(a(\text{s}}$$

Antiderivative was successfully verified.

[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]

[Out] -((a*Cos[(e + f*x)/2]*(g*Sec[e + f*x])^(5/2)*Sin[(e + f*x)/2]^3*(-4 - 4*Sec[e + f*x] + ((Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]])*Sqrt[Tan[e + f*x]^2])/Sqrt[Sec[(e + f*x)/2]^2]))/(c*f*g*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*x]))^(3/2)))

fricas [A] time = 0.45, size = 330, normalized size = 2.84

$$\left[\frac{\sqrt{2}\,ag\sqrt{\frac{g}{a}}\,\log\left(-\frac{2\,\sqrt{2}\,\sqrt{\frac{g}{a}}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{g}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)\sin\left(fx+e\right)+g\cos\left(fx+e\right)^{2}-2\,g\cos\left(fx+e\right)-3\,g}{\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+4\,g\sqrt{\cdots}}{4\,acf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*a*g*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*a*g*sqrt(-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

```
[Out] integrate(-(g*sec(f*x + e))^(3/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

maple [A] time = 2.70, size = 152, normalized size = 1.31

$$\frac{\left(\cos\left(fx+e\right)\sqrt{2}\ \operatorname{arcsinh}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\sqrt{2}\ \operatorname{arcsinh}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2\sin\left(fx+e\right)\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\right)\left(\frac{g}{\cos\left(fx+e\right)}\right)}{2cf\left(\frac{1}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/c/f*(cos(f*x+e)*2^(1/2)*arcsinh((-1+cos(f*x+e))/sin(f*x+e))-2^(1/2)*arc
sinh((-1+cos(f*x+e))/sin(f*x+e))-2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2))*(g/
cos(f*x+e))^(3/2)*(-1+cos(f*x+e))*cos(f*x+e)^2*(a*(1+cos(f*x+e))/cos(f*x+e)
)^(1/2)/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^4/a
```

maxima [B] time = 1.09, size = 536, normalized size = 4.62

$$\frac{\left(4\,g\cos\left(\frac{1}{4}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)-4\,g\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (g*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*g*sin(1/4*arctan2(sin(
```

```
2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(e+fx\right)+a}\,\sec\left(e+fx\right)-\sqrt{a\sec\left(e+fx\right)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) -
sqrt(a*sec(e + f*x) + a)), x)/c
```

3.182 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=179

$$-\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,cf}+\frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,cf}+\frac{g^2\cot(e+fx)\sqrt{a\sec(e+fx)}}{acf}$$

```
[Out] -2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x
+e)*2^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2
)+g^2*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] time = 0.36, antiderivative size = 242, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3964, 98, 157, 63, 217, 203, 93, 205}

$$\frac{2g^{5/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^{5/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
c*Sec[e + f*x]))) + (2*g^(5/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[
g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]) - (g^(5/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Se
c[e + f*x]])/(Sqrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(acg\tan(e+fx))\,\text{Subst}\left(\int \frac{(gx)^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(g\tan(e+fx))\,\text{Subst}}{cf\sqrt{a+a\sec}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\left(g^3\tan(e+fx)\right)\text{Subs}}{f\sqrt{a+a\sec(e}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\left(2g^2\tan(e+fx)\right)\text{Sub}}{f\sqrt{a+a\sec(}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g}}{\sqrt{g}\sqrt{c-c}}\right.}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{2g^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{g\sec}}{\sqrt{g}\sqrt{c-c\sec}}\right.}{\sqrt{c}f\sqrt{a+a\sec(e+f}}$$

Mathematica [A] time = 2.29, size = 328, normalized size = 1.83

$$-\frac{\sin^3(e+fx)\sqrt{\sec(e+fx)+1}\,(g\sec(e+fx))^{5/2}\left(8\sqrt{\sec(e+fx)}\,\sqrt{\sec(e+fx)+1}+\sqrt{\tan^2(e+fx)}\,\left(16\log(\sec\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]

[Out] -1/8*((g*Sec[e + f*x])^(5/2)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]^3*(8*Sqrt[
Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (16*Log[1 + Sec[e + f*x]] - 16*Log[S
```

```
qrt[Sec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e
+ f*x]^2]] + Sqrt[2]*(Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]
*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 -
2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + S
ec[e + f*x]]*Sqrt[Tan[e + f*x]^2]]))*Sqrt[Tan[e + f*x]^2]))/(c*f*(-1 + Cos[
e + f*x])*(1 + Cos[e + f*x])^2*(-1 + Sec[e + f*x])*Sec[e + f*x]^(5/2)*Sqrt[
a*(1 + Sec[e + f*x])])
```

fricas [A] time = 0.60, size = 569, normalized size = 3.18

$$\left[\sqrt{2}\, a g^2 \sqrt{\frac{g}{a}} \log\left(\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - g\cos(fx+e)^2 + 2g\cos(fx+e) + 3g}{\cos(fx+e)^2 + 2\cos(fx+e) + 1}\right)\sin(fx+e) + 2ag^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g^2*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(
f*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
*sin(f*x + e) + 2*a*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 + 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g
/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(f*x + e)^3 + c
os(f*x + e)^2))*sin(f*x + e) + 4*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), -1/2*(sqrt(2)*a*
g^2*sqrt(-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*
a*g^2*sqrt(-g/a)*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(
f*x + e) - 2*g))*sin(f*x + e) - 2*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(g\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(g*sec(f*x + e))^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

maple [A] time = 2.95, size = 294, normalized size = 1.64

$$\frac{\left(\cos\left(fx+e\right)\sqrt{2}\ \operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\sqrt{2}\ \operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+2\sin\left(fx+e\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\ +2\operatorname{arctanh}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/c/f*(cos(f*x+e)*2^(1/2)*arcsinh((-1+cos(f*x+e))/sin(f*x+e))-2^(1/2)*arc
sinh((-1+cos(f*x+e))/sin(f*x+e))+2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)+2*ar
ctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))*cos(f*x+e)+2*
arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e)))*cos(f*x+e)
-2*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))-2*arctan
h(1/2*(1/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e))))*(g/cos(f*x+e))^
(5/2)*(-1+cos(f*x+e))^2*cos(f*x+e)^3*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(1
/(1+cos(f*x+e)))^(5/2)/sin(f*x+e)^6/a
```

maxima [B] time = 1.22, size = 1400, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g^2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g^2*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 4*g^2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (sqrt(
2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(
1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(
```

```
2*f*x + 2*e))) + 2) + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*
cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*g^2*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2)
 + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sq
rt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2
)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*l
og(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 2) + (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 - 2*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(c
os(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1) - (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(cos(1/4
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + sqrt(2)*c)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.183 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(tan(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3963, 2620, 29}

$$\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Tan[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[(
(-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e
 + f*x]]), Int[(g*Csc[e + f*x])^p*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b
, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && Intege
rQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\int \csc(e+fx)\sec(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.78, size = 94, normalized size = 2.04

$$\frac{4i\left(-1+e^{i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\tanh^{-1}\left(e^{2i(e+fx)}\right)}{f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^((2*I)*(e + f*x))]*Cos[(e + f*x)/2]
^2*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

fricas [B] time = 0.55, size = 255, normalized size = 5.54

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(2\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(2\,ac\cos\left(fx+e\right)^4-2\,ac\cos\left(fx+e\right)^2+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^4-\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}\right)}{2\,acf}, \sqrt{ac}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((2*cos(f*x + e)^3 - cos(f*x + e))*sqrt(-a*c)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))
```

```
+ (2*a*c*cos(f*x + e)^4 - 2*a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f
*x + e)^4 - cos(f*x + e)^2)*sin(f*x + e)))/(a*c*f), sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/((2*a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)))/(a*c*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
```

```
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableUnab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
```

>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discon
tinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>

```
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integra
tion of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Disc
ontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
```

```
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
```

```
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
```

```
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
```

```
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
```

```
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
```

```
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
```

to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si

```
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not che
ckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or
 sign assumes constant sign by intervals (correct if the argument is real):
Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_n
ostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) w
ere not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_n
ostep^2-1)]Warning, integration of abs or sign assumes constant sign by int
ervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%
%%{c,0%%%}+%%%{1,2%%%})Evaluation time: 2.15Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 1.98, size = 142, normalized size = 3.09

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\ln\left(-\frac{-\sin\left(fx+e\right)-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\cos\left(fx+e\right)\sqrt{}}{f\sin\left(fx+e\right)ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(ln(-(-1+cos(f*x+e))/sin(f*x+e))-l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-sin(f*x+e)-1+cos(f*x+e))/si
n(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c/a
```

maxima [A] time = 1.09, size = 56, normalized size = 1.22

$$-\frac{\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\,\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -(arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(c)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

3.184 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

```
[Out] 2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(c-d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^
(1/2)/f/(c-d)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3967, 208}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c - d]*Sqrt[a + a*S
ec[e + f*x]])])/(Sqrt[c - d]*Sqrt[d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{ac-ad-dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}\,f}$$

Mathematica [A] time = 0.26, size = 98, normalized size = 1.51

$$\frac{\sqrt{2}\sqrt{\cos(e+fx)}\,\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c-d}\sqrt{\cos(e+fx)}}\right)}{\sqrt{d}\,f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c - d]*Sqrt[Cos[e
 + f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/
(Sqrt[c - d]*Sqrt[d]*f)
```

fricas [B] time = 0.69, size = 357, normalized size = 5.49

$$\left[\frac{\sqrt{\frac{a}{cd-d^2}}\,\log\left(-\frac{\left(ac^2-8\,acd+8\,ad^2\right)\cos\left(fx+e\right)^3+ad^2+\left(ac^2-2\,acd\right)\cos\left(fx+e\right)^2+4\left(\left(c^2d-3\,cd^2+2\,d^3\right)\cos\left(fx+e\right)^2+\left(cd^2-d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a}{cd-d^2}}\sqrt{}}{c^2\cos\left(fx+e\right)^3+\left(c^2-2\,cd\right)\cos\left(fx+e\right)^2+d^2-\left(2\,cd-d^2\right)\cos\left(fx+e\right)}\right)}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(a/(c*d - d^2))*log(-((a*c^2 - 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 +
 a*d^2 + (a*c^2 - 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d - 3*c*d^2 + 2*d^3)*co
s(f*x + e)^2 + (c*d^2 - d^3)*cos(f*x + e))*sqrt(a/(c*d - d^2))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (6*a*c*d - 7*a*d^2)*cos(f*x + e)
```

```
)/(c^2*cos(f*x + e)^3 + (c^2 - 2*c*d)*cos(f*x + e)^2 + d^2 - (2*c*d - d^2)*
cos(f*x + e)))/f, -sqrt(-a/(c*d - d^2))*arctan(2*(c*d - d^2)*sqrt(-a/(c*d -
 d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((
a*c - 2*a*d)*cos(f*x + e)^2 + a*d + (a*c - a*d)*cos(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2*a*sqrt(-a)*sign(cos(f*x+exp(1)))*a
tan(1/2*(c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1)
)))^2+d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))
^2+a*c-3*a*d)/sqrt(2)/sqrt(-d^2+c*d)/a)/sqrt(-d^2+c*d)/a/f
```

maple [B] time = 1.66, size = 414, normalized size = 6.37

$$-\frac{\left(\ln\left(-\frac{2\left(\sqrt{-\frac{2d}{c+d}}\,\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\;c\sin\left(fx+e\right)+\sqrt{-\frac{2d}{c+d}}\,\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\;d\sin\left(fx+e\right)+\sqrt{(c+d)(c-d)}\;\cos\left(fx+e\right)-c\sin\left(fx+e\right)-d\sin\left(fx+e\right)-\sqrt{(c+d)(c}\cdots}{c\cos\left(fx+e\right)+d\cos\left(fx+e\right)-\sqrt{(c+d)(c-d)}\;\sin\left(fx+e\right)-c-d}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x)
```

```
[Out] -1/f*(ln(-2*((-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*c*sin(
f*x+e)+(-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)
+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/
2))/(c*cos(f*x+e)+d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c-d))-ln(-2*(
(-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*c*sin(f*x+e)+(-2*d/
(c+d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d)
)^(1/2)*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(c*cos(f*
x+e)+d*cos(f*x+e)+((c+d)*(c-d))^(1/2)*sin(f*x+e)-c-d)))*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(-2*d/(c+d))^(1/2)/((
c+d)*(c-d))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a \sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{d\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{d-c\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - d/cos(e + f*x))),x)
```

```
[Out] -int((a + a/cos(e + f*x))^(1/2)/(d - c*cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c-d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c - d*sec(e + f*x)), x)
```

3.185 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=236

$$\frac{a\left(12c^2+35cd+16d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{60f}+\frac{ad\left(24c^3+130c^2d+116cd^2+45d^3\right)\tan(e+fx)\sec(e+$$

```
[Out] 1/8*a*(8*c^4+16*c^3*d+24*c^2*d^2+12*c*d^3+3*d^4)*arctanh(sin(f*x+e))/f+1/30
*a*(12*c^4+95*c^3*d+112*c^2*d^2+80*c*d^3+16*d^4)*tan(f*x+e)/f+1/120*a*d*(24
*c^3+130*c^2*d+116*c*d^2+45*d^3)*sec(f*x+e)*tan(f*x+e)/f+1/60*a*(12*c^2+35*
c*d+16*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*a*(4*c+5*d)*(c+d*sec(f*x+e
))^3*tan(f*x+e)/f+1/5*a*(c+d*sec(f*x+e))^4*tan(f*x+e)/f
```

Rubi [A] time = 0.44, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(112c^2d^2+95c^3d+12c^4+80cd^3+16d^4\right)\tan(e+fx)}{30f}+\frac{a\left(24c^2d^2+16c^3d+8c^4+12cd^3+3d^4\right)\tanh^{-1}(\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]]
)/(8*f) + (a*(12*c^4 + 95*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Tan[e +
f*x])/(30*f) + (a*d*(24*c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Sec[e + f*x]*
Tan[e + f*x])/(120*f) + (a*(12*c^2 + 35*c*d + 16*d^2)*(c + d*Sec[e + f*x])^
2*Tan[e + f*x])/(60*f) + (a*(4*c + 5*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(20*f) + (a*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f}+\frac{1}{5}\int \sec(e+fx)(c$$

$$= \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{a(c+d\,\text{s}}{}$$

$$= \frac{a\left(12c^2+35cd+16d^2\right)(c+d\sec(e+fx))^2\tan(e+fx}{60f}$$

$$= \frac{ad\left(24c^3+130c^2d+116cd^2+45d^3\right)\sec(e+fx)\tan(e}{120f}$$

$$= \frac{ad\left(24c^3+130c^2d+116cd^2+45d^3\right)\sec(e+fx)\tan(e}{120f}$$

$$= \frac{a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\tanh^{-1}(\sin(e}{8f}$$

$$= \frac{a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\tanh^{-1}(\sin(e}{8f}$$

Mathematica [A] time = 1.74, size = 153, normalized size = 0.65

$$\frac{a\left(\tan(e+fx)\left(80d^2\left(3c^2+2cd+d^2\right)\tan^2(e+fx)+15d\left(16c^3+24c^2d+12cd^2+3d^3\right)\sec(e+fx)+30d^3(4c+d\right.\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(15*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f
*x]] + Tan[e + f*x]*(120*(c + d)^4 + 15*d*(16*c^3 + 24*c^2*d + 12*c*d^2 + 3
*d^3)*Sec[e + f*x] + 30*d^3*(4*c + d)*Sec[e + f*x]^3 + 80*d^2*(3*c^2 + 2*c*
d + d^2)*Tan[e + f*x]^2 + 24*d^4*Tan[e + f*x]^4)))/(120*f)
```

fricas [A] time = 0.44, size = 281, normalized size = 1.19

$$\frac{15\left(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\,ad^4\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(8\,ac^4+16\,ac^3d+24\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2
+ 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*a*d^4
 + 8*(15*a*c^4 + 60*a*c^3*d + 60*a*c^2*d^2 + 40*a*c*d^3 + 8*a*d^4)*cos(f*x
+ e)^4 + 15*(16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*a*c^2*d^2 + 10*a*c*d^3 + 2*a*d^4)*cos(f*x + e)^2 + 30*(4*a*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(8*a*c^4+16*a*c^3*d+24*a*c^2*d^2+12*a*c*d^3+3*a*d^4)/16*ln(ab
s(tan((f*x+exp(1))/2)-1))+(8*a*c^4+16*a*c^3*d+24*a*c^2*d^2+12*a*c*d^3+3*a*d
^4)/16*ln(abs(tan((f*x+exp(1))/2)+1))-(120*tan((f*x+exp(1))/2)^9*a*c^4+240*
tan((f*x+exp(1))/2)^9*a*c^3*d+360*tan((f*x+exp(1))/2)^9*a*c^2*d^2+180*tan((
f*x+exp(1))/2)^9*a*c*d^3+45*tan((f*x+exp(1))/2)^9*a*d^4-480*tan((f*x+exp(1)
)/2)^7*a*c^4-1440*tan((f*x+exp(1))/2)^7*a*c^3*d-1200*tan((f*x+exp(1))/2)^7*
a*c^2*d^2-1160*tan((f*x+exp(1))/2)^7*a*c*d^3-130*tan((f*x+exp(1))/2)^7*a*d^
4+720*tan((f*x+exp(1))/2)^5*a*c^4+2880*tan((f*x+exp(1))/2)^5*a*c^3*d+2400*t
an((f*x+exp(1))/2)^5*a*c^2*d^2+1600*tan((f*x+exp(1))/2)^5*a*c*d^3+464*tan((
f*x+exp(1))/2)^5*a*d^4-480*tan((f*x+exp(1))/2)^3*a*c^4-2400*tan((f*x+exp(1)
)/2)^3*a*c^3*d-2640*tan((f*x+exp(1))/2)^3*a*c^2*d^2-1400*tan((f*x+exp(1))/2
)^3*a*c*d^3-190*tan((f*x+exp(1))/2)^3*a*d^4+120*tan((f*x+exp(1))/2)*a*c^4+7
20*tan((f*x+exp(1))/2)*a*c^3*d+1080*tan((f*x+exp(1))/2)*a*c^2*d^2+780*tan((
f*x+exp(1))/2)*a*c*d^3+195*tan((f*x+exp(1))/2)*a*d^4)*1/120/(tan((f*x+exp(1
))/2)^2-1)^5)
```

maple [A] time = 1.61, size = 431, normalized size = 1.83

$$\frac{a\,c^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{4a\,c^3d\tan\left(fx+e\right)}{f}+\frac{3a\,c^2d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{f}+\frac{3a\,c^2d^2\ln\left(\sec\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x)
```

```
[Out] 1/f*a*c^4*ln(sec(f*x+e)+tan(f*x+e))+4/f*a*c^3*d*tan(f*x+e)+3/f*a*c^2*d^2*se
c(f*x+e)*tan(f*x+e)+3/f*a*c^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+8/3/f*a*c*d^3*t
```

```
an(f*x+e)+4/3/f*a*c*d^3*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a*d^4*tan(f*x+e)*sec(
f*x+e)^3+3/8/f*a*d^4*sec(f*x+e)*tan(f*x+e)+3/8/f*a*d^4*ln(sec(f*x+e)+tan(f*
x+e))+1/f*a*c^4*tan(f*x+e)+2/f*a*c^3*d*sec(f*x+e)*tan(f*x+e)+2/f*a*c^3*d*ln
(sec(f*x+e)+tan(f*x+e))+4/f*a*c^2*d^2*tan(f*x+e)+2/f*a*c^2*d^2*tan(f*x+e)*s
ec(f*x+e)^2+1/f*a*c*d^3*tan(f*x+e)*sec(f*x+e)^3+3/2/f*a*c*d^3*sec(f*x+e)*ta
n(f*x+e)+3/2/f*a*c*d^3*ln(sec(f*x+e)+tan(f*x+e))+8/15/f*a*d^4*tan(f*x+e)+1/
5/f*a*d^4*tan(f*x+e)*sec(f*x+e)^4+4/15/f*a*d^4*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.38, size = 379, normalized size = 1.61

$$\frac{480\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ac^{2}d^{2}+320\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)acd^{3}+16\left(3\,\tan\left(fx+e\right)^{5}+1\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*a*d^4 - 60*a*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*a*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*a*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

mupad [B] time = 5.51, size = 361, normalized size = 1.53

$$\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{2\left(4\,c^4+8\,c^3\,d+12\,c^2\,d^2+6\,c\,d^3+\frac{3\,d^4}{2}\right)}\right)\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{4\,f}-\frac{\left(2\,a\,c^4+4\,a\,c^3\,d+6\,\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*
d^2))/(2*(6*c*d^3 + 8*c^3*d + 4*c^4 + (3*d^4)/2 + 12*c^2*d^2)))*(12*c*d^3 +
```

```
 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/(4*f) - (tan(e/2 + (f*x)/2)^9*(2*a
*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3 + 4*a*c^3*d) - tan(e/2 + (f*x)
/2)^7*(8*a*c^4 + (13*a*d^4)/6 + 20*a*c^2*d^2 + (58*a*c*d^3)/3 + 24*a*c^3*d)
 - tan(e/2 + (f*x)/2)^3*(8*a*c^4 + (19*a*d^4)/6 + 44*a*c^2*d^2 + (70*a*c*d^
3)/3 + 40*a*c^3*d) + tan(e/2 + (f*x)/2)^5*(12*a*c^4 + (116*a*d^4)/15 + 40*a
*c^2*d^2 + (80*a*c*d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*(2*a*c^4 + (13
*a*d^4)/4 + 18*a*c^2*d^2 + 13*a*c*d^3 + 12*a*c^3*d))/(f*(5*tan(e/2 + (f*x)/
2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x
)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^4\sec\left(e+fx\right)dx+\int c^4\sec^2\left(e+fx\right)dx+\int d^4\sec^5\left(e+fx\right)dx+\int d^4\sec^6\left(e+fx\right)dx+\int 4cd^3\sec^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(c**4*sec(e + f*x), x) + Integral(c**4*sec(e + f*x)**2, x) + Int
egral(d**4*sec(e + f*x)**5, x) + Integral(d**4*sec(e + f*x)**6, x) + Integr
al(4*c*d**3*sec(e + f*x)**4, x) + Integral(4*c*d**3*sec(e + f*x)**5, x) + I
ntegral(6*c**2*d**2*sec(e + f*x)**3, x) + Integral(6*c**2*d**2*sec(e + f*x)
**4, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(4*c**3*d*sec(e +
 f*x)**3, x))
```

3.186 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=171

$$\frac{ad\left(6c^2+20cd+9d^2\right)\tan(e+fx)\sec(e+fx)}{24f}+\frac{a\left(3c^3+16c^2d+12cd^2+4d^3\right)\tan(e+fx)}{6f}+\frac{a\left(8c^3+12c^2d+12c\right.}{}$$

```
[Out] 1/8*a*(8*c^3+12*c^2*d+12*c*d^2+3*d^3)*arctanh(sin(f*x+e))/f+1/6*a*(3*c^3+16
*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f+1/24*a*d*(6*c^2+20*c*d+9*d^2)*sec(f*x+e
)*tan(f*x+e)/f+1/12*a*(3*c+4*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/4*a*(c+d*
sec(f*x+e))^3*tan(f*x+e)/f
```

Rubi [A] time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(16c^2d+3c^3+12cd^2+4d^3\right)\tan(e+fx)}{6f}+\frac{a\left(12c^2d+8c^3+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{ad\left(6c^2+20cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]])/(8*f) + (a*
(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(6*f) + (a*d*(6*c^2 + 2
0*c*d + 9*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + (a*(3*c + 4*d)*(c + d*Se
c[e + f*x])^2*Tan[e + f*x])/(12*f) + (a*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\int \sec(e+fx)$$

$$= \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{a(c+d}{}$$

$$= \frac{ad\left(6c^2+20cd+9d^2\right)\sec(e+fx)\tan(e+fx)}{24f}+\frac{a(3}{}$$

$$= \frac{ad\left(6c^2+20cd+9d^2\right)\sec(e+fx)\tan(e+fx)}{24f}+\frac{a(3}{}$$

$$= \frac{a\left(8c^3+12c^2d+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+$$

$$= \frac{a\left(8c^3+12c^2d+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+$$

Mathematica [A] time = 0.68, size = 103, normalized size = 0.60

$$\frac{a\left(3\left(8c^3+12c^2d+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(8d^2(3c+d)\tan^2(e+fx)+9d(2c+d)^2\sec(e\right.\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(3*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e +
 f*x]*(24*(c + d)^3 + 9*d*(2*c + d)^2*Sec[e + f*x] + 6*d^3*Sec[e + f*x]^3 +
 8*d^2*(3*c + d)*Tan[e + f*x]^2)))/(24*f)
```

fricas [A] time = 0.47, size = 211, normalized size = 1.23

$$\frac{3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*a*d^3 + 8*(3*a*c^3 + 9*a*c^2*d + 6*a*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*a*c^2*d + 4*a*c*d^2 + a*d^3)*cos(f*x +
 e)^2 + 8*(3*a*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-8*a*c^3-12*a*c^2*d-12*a*c*d^2-3*a*d^3)/16*ln(abs(tan((f*x+ex
p(1))/2)-1))-(-8*a*c^3-12*a*c^2*d-12*a*c*d^2-3*a*d^3)/16*ln(abs(tan((f*x+ex
p(1))/2)+1))+(-24*tan((f*x+exp(1))/2)^7*a*c^3-36*tan((f*x+exp(1))/2)^7*a*c^
2*d-36*tan((f*x+exp(1))/2)^7*a*c*d^2-9*tan((f*x+exp(1))/2)^7*a*d^3+72*tan((
```

```
f*x+exp(1))/2)^5*a*c^3+180*tan((f*x+exp(1))/2)^5*a*c^2*d+84*tan((f*x+exp(1)
)/2)^5*a*c*d^2+49*tan((f*x+exp(1))/2)^5*a*d^3-72*tan((f*x+exp(1))/2)^3*a*c^
3-252*tan((f*x+exp(1))/2)^3*a*c^2*d-156*tan((f*x+exp(1))/2)^3*a*c*d^2-31*ta
n((f*x+exp(1))/2)^3*a*d^3+24*tan((f*x+exp(1))/2)*a*c^3+108*tan((f*x+exp(1))
/2)*a*c^2*d+108*tan((f*x+exp(1))/2)*a*c*d^2+39*tan((f*x+exp(1))/2)*a*d^3)*1
/24/(tan((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.33, size = 290, normalized size = 1.70

$$\frac{a c^3 \ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{3a c^2 d \tan\left(fx+e\right)}{f}+\frac{3ac d^2 \sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{3ac d^2 \ln\left(\sec\left(fx\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*a*c^3*ln(sec(f*x+e)+tan(f*x+e))+3/f*a*c^2*d*tan(f*x+e)+3/2/f*a*c*d^2*se
c(f*x+e)*tan(f*x+e)+3/2/f*a*c*d^2*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*a*d^3*tan
(f*x+e)+1/3/f*a*d^3*tan(f*x+e)*sec(f*x+e)^2+1/f*a*c^3*tan(f*x+e)+3/2/f*a*c^
2*d*sec(f*x+e)*tan(f*x+e)+3/2/f*a*c^2*d*ln(sec(f*x+e)+tan(f*x+e))+2/f*a*c*d
^2*tan(f*x+e)+1/f*a*c*d^2*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a*d^3*tan(f*x+e)*se
c(f*x+e)^3+3/8/f*a*d^3*sec(f*x+e)*tan(f*x+e)+3/8/f*a*d^3*ln(sec(f*x+e)+tan(
f*x+e))
```

maxima [A] time = 0.62, size = 266, normalized size = 1.56

$$\frac{48\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)acd^2+16\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ad^3-3\,ad^3\left(\frac{2\left(3\,\sin\left(fx+e\right)^3-5\,\sin\left(fx+\right.\right.}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*a*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*a*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*a*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

mupad [B] time = 5.34, size = 255, normalized size = 1.49

$$\frac{\left(-2\,a\,c^3-3\,a\,c^2\,d-3\,a\,c\,d^2-\frac{3\,a\,d^3}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(6\,a\,c^3+15\,a\,c^2\,d+7\,a\,c\,d^2+\frac{49\,a\,d^3}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-6\,a\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(2*a*c^3 + (13*a*d^3)/4 + 9*a*c*d^2 + 9*a*c^2*d) - tan(
e/2 + (f*x)/2)^7*(2*a*c^3 + (3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) - tan(e/2
+ (f*x)/2)^3*(6*a*c^3 + (31*a*d^3)/12 + 13*a*c*d^2 + 21*a*c^2*d) + tan(e/2
+ (f*x)/2)^5*(6*a*c^3 + (49*a*d^3)/12 + 7*a*c*d^2 + 15*a*c^2*d))/(f*(6*tan(
e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/
2 + (f*x)/2)^8 + 1)) + (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^2 + 12*c^2*d +
8*c^3 + 3*d^3))/(2*(6*c*d^2 + 6*c^2*d + 4*c^3 + (3*d^3)/2)))*(12*c*d^2 + 12
*c^2*d + 8*c^3 + 3*d^3))/(4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^3\sec\left(e+fx\right)dx+\int c^3\sec^2\left(e+fx\right)dx+\int d^3\sec^4\left(e+fx\right)dx+\int d^3\sec^5\left(e+fx\right)dx+\int 3cd^2\sec^3\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(c**3*sec(e + f*x), x) + Integral(c**3*sec(e + f*x)**2, x) + Int
egral(d**3*sec(e + f*x)**4, x) + Integral(d**3*sec(e + f*x)**5, x) + Integr
al(3*c*d**2*sec(e + f*x)**3, x) + Integral(3*c*d**2*sec(e + f*x)**4, x) + I
ntegral(3*c**2*d*sec(e + f*x)**2, x) + Integral(3*c**2*d*sec(e + f*x)**3, x
))
```

3.187 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=108

$$\frac{2a\left(c^2+3cd+d^2\right)\tan(e+fx)}{3f}+\frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}+\frac{ad(}{}$$

```
[Out] 1/2*a*(2*c^2+2*c*d+d^2)*arctanh(sin(f*x+e))/f+2/3*a*(c^2+3*c*d+d^2)*tan(f*x
+e)/f+1/6*a*d*(2*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*a*(c+d*sec(f*x+e))^2*ta
n(f*x+e)/f
```

Rubi [A] time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2a\left(c^2+3cd+d^2\right)\tan(e+fx)}{3f}+\frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}+\frac{ad(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a*(c^2 + 3*c*d +
 d^2)*Tan[e + f*x])/(3*f) + (a*d*(2*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(6*
f) + (a*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}\int \sec(e+fx)(c \ldots$$

$$= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\sec(e+ \ldots}{\ldots}$$

$$= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\sec(e+ \ldots}{\ldots}$$

$$= \frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad(2c+3d) \ldots}{\ldots}$$

$$= \frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a\left(c^2+3cd \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.43, size = 75, normalized size = 0.69

$$\frac{a\left(3\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(2\left(3(c+d)^2+d^2\tan^2(e+fx)\right)+3d(2c+d)\sec(e+fx) \ldots\right.\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(3*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*d*(2*c
+ d)*Sec[e + f*x] + 2*(3*(c + d)^2 + d^2*Tan[e + f*x]^2))))/(6*f)
```

fricas [A] time = 0.48, size = 150, normalized size = 1.39

$$\frac{3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\cos\left(fx+e\right)^{3}\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\cos\left(fx+e\right)^{3}\log\left(-\sin\left(f\right.\right.}{12\,f\cos\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
a*d^2 + 2*(3*a*c^2 + 6*a*c*d + 2*a*d^2)*cos(f*x + e)^2 + 3*(2*a*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(2*a*c^2+2*a*c*d+a*d^2)/4*ln(abs(tan((f*x+exp(1))/2)-1))+(2*a
*c^2+2*a*c*d+a*d^2)/4*ln(abs(tan((f*x+exp(1))/2)+1))+(-6*tan((f*x+exp(1))/2
)^5*a*c^2-6*tan((f*x+exp(1))/2)^5*a*c*d-3*tan((f*x+exp(1))/2)^5*a*d^2+12*ta
n((f*x+exp(1))/2)^3*a*c^2+24*tan((f*x+exp(1))/2)^3*a*c*d+4*tan((f*x+exp(1))
/2)^3*a*d^2-6*tan((f*x+exp(1))/2)*a*c^2-18*tan((f*x+exp(1))/2)*a*c*d-9*tan(
(f*x+exp(1))/2)*a*d^2)*1/6/(tan((f*x+exp(1))/2)^2-1)^3)
```

maple [A] time = 1.11, size = 174, normalized size = 1.61

$$\frac{a\,c^{2}\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{2acd\tan\left(fx+e\right)}{f}+\frac{a\,d^{2}\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{a\,d^{2}\ln\left(\sec\left(fx+e\right)+\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*a*c^2*ln(sec(f*x+e)+tan(f*x+e))+2/f*a*c*d*tan(f*x+e)+1/2/f*a*d^2*sec(f*
x+e)*tan(f*x+e)+1/2/f*a*d^2*ln(sec(f*x+e)+tan(f*x+e))+a*c^2*tan(f*x+e)/f+1/
f*a*c*d*sec(f*x+e)*tan(f*x+e)+1/f*a*c*d*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*a*d
^2*tan(f*x+e)+1/3/f*a*d^2*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.47, size = 165, normalized size = 1.53

$$\frac{4\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ad^{2}-6\,acd\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*d^2 - 6*a*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*a*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

mupad [B] time = 4.76, size = 196, normalized size = 1.81

$$\frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(2\,c^2+2\,c\,d+d^2\right)}{4\,c^2+4\,c\,d+2\,d^2}\right)\left(2\,c^2+2\,c\,d+d^2\right)}{f}-\frac{\left(2\,a\,c^2+2\,a\,c\,d+a\,d^2\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-4\,a\,c^2-8\,a\,c\,d-\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,\tan\left(\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a*atanh((2*tan(e/2 + (f*x)/2)*(2*c*d + 2*c^2 + d^2))/(4*c*d + 4*c^2 + 2*d^
2))*(2*c*d + 2*c^2 + d^2))/f - (tan(e/2 + (f*x)/2)*(2*a*c^2 + 3*a*d^2 + 6*a
*c*d) + tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 + 2*a*c*d) - tan(e/2 + (f*x)/
2)^3*(4*a*c^2 + (4*a*d^2)/3 + 8*a*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(
e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^{2}\sec\left(e+fx\right)dx+\int c^{2}\sec^{2}\left(e+fx\right)dx+\int d^{2}\sec^{3}\left(e+fx\right)dx+\int d^{2}\sec^{4}\left(e+fx\right)dx+\int 2cd\sec^{2}\left(e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(c**2*sec(e + f*x), x) + Integral(c**2*sec(e + f*x)**2, x) + Int
egral(d**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**4, x) + Integr
al(2*c*d*sec(e + f*x)**2, x) + Integral(2*c*d*sec(e + f*x)**3, x))
```

3.188 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=56

$$\frac{a(c+d)\tan(e+fx)}{f}+\frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*c+d)*arctanh(sin(f*x+e))/f+a*(c+d)*tan(f*x+e)/f+1/2*a*d*sec(f*x+e)
*tan(f*x+e)/f
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{a(c+d)\tan(e+fx)}{f}+\frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*ArcTanh[Sin[e + f*x]])/(2*f) + (a*(c + d)*Tan[e + f*x])/f + (a
*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$
\begin{aligned}
\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx &= \frac{ad\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\int \sec(e+fx)(a(2c+\\
&= \frac{ad\sec(e+fx)\tan(e+fx)}{2f}+(a(c+d))\int \sec^2(e+fx\\
&= \frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\sec(e+fx)\tan(e}{2f}\\
&= \frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a(c+d)\tan(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.34

$$
\frac{ac\tan(e+fx)}{f}+\frac{ac\tanh^{-1}(\sin(e+fx))}{f}+\frac{ad\tan(e+fx)}{f}+\frac{ad\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (a*d*ArcTanh[Sin[e + f*x]])/(2*f) + (a*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (a*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

fricas [A] time = 0.46, size = 96, normalized size = 1.71

$$
\frac{(2\,ac+ad)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+ad)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(ad+2\right(}{4\,f\cos\left(fx+e\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + a*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + a*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a*d + 2*(a*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-2*a*c-a*d)/4*ln(abs(tan((f*x+exp(1))/2)-1))-(-2*a*c-a*d)/4*l
n(abs(tan((f*x+exp(1))/2)+1))-(2*tan((f*x+exp(1))/2)^3*a*c+tan((f*x+exp(1))
/2)^3*a*d-2*tan((f*x+exp(1))/2)*a*c-3*tan((f*x+exp(1))/2)*a*d)*1/2/(tan((f*
x+exp(1))/2)^2-1)^2)
```

maple [A] time = 0.96, size = 86, normalized size = 1.54

$$\frac{ca\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{da\tan\left(fx+e\right)}{f}+\frac{ac\tan\left(fx+e\right)}{f}+\frac{ad\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{da\ln\left(\sec\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*c*a*ln(sec(f*x+e)+tan(f*x+e))+1/f*d*a*tan(f*x+e)+a*c*tan(f*x+e)/f+1/2*a
*d*sec(f*x+e)*tan(f*x+e)/f+1/2/f*d*a*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.38, size = 88, normalized size = 1.57

$$-\frac{ad\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-4\,ac}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(a*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*a*c*tan(f
*x + e) - 4*a*d*tan(f*x + e))/f
```

mupad [B] time = 2.57, size = 111, normalized size = 1.98

$$\frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(2\,c+d)}{4\,c+2\,d}\right)(2\,c+d)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,a\,c+a\,d)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,a\,c+3\,a\,d)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a*atanh((2*tan(e/2 + (f*x)/2)*(2*c + d))/(4*c + 2*d))*(2*c + d))/f - (tan(
e/2 + (f*x)/2)^3*(2*a*c + a*d) - tan(e/2 + (f*x)/2)*(2*a*c + 3*a*d))/(f*(ta
n(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sec\left(e+fx\right)dx+\int c\sec^2\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\int d\sec^3\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(c*sec(e + f*x), x) + Integral(c*sec(e + f*x)**2, x) + Integral(
d*sec(e + f*x)**2, x) + Integral(d*sec(e + f*x)**3, x))
```

3.189 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{a\tanh^{-1}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

```
[Out] a*arctanh(sin(f*x+e))/d/f-2*a*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^
(1/2))*(c-d)^(1/2)/d/f/(c+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{a\tanh^{-1}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/(d*f) - (2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Tan
[(e + f*x)/2])/Sqrt[c + d]])/(d*Sqrt[c + d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\int \sec(e+fx)\,dx}{d} + \frac{(-ac+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d}$$

$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{(a(c-d))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2}$$

$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{(2a(c-d))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e\right.\right.}{d^2 f}$$

$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}\,f}$$

Mathematica [A] time = 0.19, size = 107, normalized size = 1.55

$$\frac{a\left(\frac{2(c-d)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*((2*(c - d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c
^2 - d^2] - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]))/(d*f)
```

fricas [A] time = 0.53, size = 255, normalized size = 3.70

$$\left[\frac{a\sqrt{\frac{c-d}{c+d}}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2-2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\,\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)+a\log\left(\sin\left(fx+e\right)+1\right)}{2\,df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*(a*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*
sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)
) + a*log(sin(f*x + e) + 1) - a*log(-sin(f*x + e) + 1))/(d*f), -1/2*(2*a*sq
rt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((
c - d)*sin(f*x + e))) - a*log(sin(f*x + e) + 1) + a*log(-sin(f*x + e) + 1))
/(d*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-a*1/2/d*ln(abs(tan((f*x+exp(1))/2)-1))+a*1/2/d*ln(abs(tan((f*
x+exp(1))/2)+1))+(2*a*c-2*a*d)/d*1/2/sqrt(-c^2+d^2)*(atan((c*tan((f*x+exp(1
))/2)-d*tan((f*x+exp(1))/2))/sqrt(-c^2+d^2))+pi*sign(2*c-2*d)*floor((f*x+ex
p(1))/2/pi+1/2)))
```

maple [B] time = 0.70, size = 135, normalized size = 1.96

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c}{fd\sqrt{(c+d)\,(c-d)}}+\frac{2a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{f\sqrt{(c+d)\,(c-d)}}-\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fd}+\frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{fd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] -2/f*a/d/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))
^(1/2))*c+2/f*a/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)
*(c-d))^(1/2))-1/f*a/d*ln(tan(1/2*e+1/2*f*x)-1)+1/f*a/d*ln(tan(1/2*e+1/2*f*
x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.16, size = 195, normalized size = 2.83

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c+d\right)}+\frac{2\,a\,\left(\mathrm{atanh}\left(\frac{d^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^2\,d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c^2-d^2\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}\,\left(d^2+c\,d\right)}\right)\,\sqrt{c^2 -}\right.}{d\,f\,\left(c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (2*a*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*a*(atan
h((d^3*sin(e/2 + (f*x)/2) - c^3*sin(e/2 + (f*x)/2) + c*d^2*sin(e/2 + (f*x)/
2) - c^2*d*sin(e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2)*(c^2 - d^2))/(cos(e/2
+ (f*x)/2)*(c^2 - d^2)^(1/2)*(c*d + d^2)))*(c^2 - d^2)^(1/2) + c*atanh(sin(
e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*(c + d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**
2/(c + d*sec(e + f*x)), x))
```

3.190 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,(c+d)^{3/2}}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

```
[Out] 2*a*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c+d)^(3/2)/f/(c-d)
^(1/2)+a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,(c+d)^{3/2}}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c +
 d)^(3/2)*f) + (a*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx &= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} - \frac{\int \frac{a(c-d)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c+d} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d(c+d)} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+ \right.\right.}{d(c+d)f} \\
&= \frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,(c+d)^{3/2}f} + \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 75, normalized size = 0.95

$$
\frac{a\left(\frac{\sin(e+fx)}{c\cos(e+fx)+d} - \frac{2\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}\right)}{f(c+d)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*((-2*ArcTanh[((-c + d)*Tan[(e + f*x)/2]])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^
2] + Sin[e + f*x]/(d + c*Cos[e + f*x])))/((c + d)*f)
```

fricas [B] time = 0.47, size = 357, normalized size = 4.52

$$\left[\frac{\left(ac\cos\left(fx+e\right)+ad\right)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)+2\,\left(ac\right.}{2\left(\left(c^4+c^3d-c^2d^2-cd^3\right)f\cos\left(fx+e\right)+\left(c^3d+c^2d^2-cd^3-d^4\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*cos(f*x + e) + a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c
^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
 + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(
a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*cos(f*x + e
) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f), ((a*c*cos(f*x + e) + a*d)*sqrt(-c^2
 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x
+ e))) + (a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*c
os(f*x + e) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f)]
```

giac [B] time = 0.34, size = 143, normalized size = 1.81

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,c-2\,d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)a}{\sqrt{-c^2+d^2}\left(c+d\right)}+\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)\left(c+d\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a/(sqrt(-c^2 + d^2)
*(c + d)) + a*tan(1/2*f*x + 1/2*e)/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f
*x + 1/2*e)^2 - c - d)*(c + d)))/f
```

maple [A] time = 0.72, size = 105, normalized size = 1.33

$$\frac{4a\left(-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}+\frac{\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 4/f*a*(-1/2*tan(1/2*e+1/2*f*x)/(c+d)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*
f*x)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d
)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 1.91, size = 85, normalized size = 1.08

$$\frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\left(\left(d-c\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+c+d\right)}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,{\left(c+d\right)}^{3/2}\,\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*a*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
 + (2*a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)
^(3/2)*(c - d)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{\sec^2\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**2)
, x) + Integral(sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f
*x)**2), x))
```

3.191 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{5/2}}+\frac{a(c-2d)\tan(e+fx)}{2f(c-d)(c+d)^2(c+d\sec(e+fx))}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

```
[Out] a*(2*c-d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c-d)^(3/2)/(
c+d)^(5/2)/f+1/2*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+1/2*a*(c-2*d)*tan(
f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.27, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{5/2}}+\frac{a(c-2d)\tan(e+fx)}{2f(c-d)(c+d)^2(c+d\sec(e+fx))}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(2*c - d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^
(3/2)*(c + d)^(5/2)*f) + (a*Tan[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])
^2) + (a*(c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2a(c-d)-a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \cdots$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \cdots$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \cdots$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \cdots$$

$$= \frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f} + \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \cdots$$

Mathematica [A] time = 1.24, size = 167, normalized size = 1.27

$$\frac{a(\cos(e+fx)+1)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{c^2-d^2}\sin(e+fx)\left(\left(2c^2-2cd-d^2\right)\cos(e+fx)+d(c-2d)\right)-2(2c-d)(\right.}{4f(c-d)(c+d)^2\sqrt{c^2-d^2}\,(c\cos(e+fx)+d)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(-2*(2*c - d)*ArcTanh[((-c + d)*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2 + Sqrt[c^2 - d^2]*(
(c - 2*d)*d + (2*c^2 - 2*c*d - d^2)*Cos[e + f*x])*Sin[e + f*x]))/(4*(c - d)
*(c + d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^2)
```

fricas [B] time = 0.52, size = 736, normalized size = 5.62

$$\left[\frac{\left(2\,acd^2-ad^3+\left(2\,ac^3-ac^2d\right)\cos\left(fx+e\right)^2+2\left(2\,ac^2d-acd^2\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)}{\right.}\right.}{4\left(\left(c^7+c^6d-2\,c^5d^2-2\,c^4d^3+c^3d^4+c^2d^5\right)f\cos\left(fx+e\right)^2+2\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*a*c*d^2 - a*d^3 + (2*a*c^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*
d - a*c*d^2)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e
) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a*c^
3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2
+ 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c^6*d - 2*c^5*d^2
- 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6*d + c^5*d^2 - 2*
c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^5*d^2 + c^4*d^3
- 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f), 1/2*((2*a*c*d^2 - a*d^3 + (2*a*c
^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3
*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c
^6*d - 2*c^5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6
*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c
^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f)]
```

giac [B] time = 0.42, size = 274, normalized size = 2.09

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(2ac-ad)}{(c^3+c^2d-cd^2-d^3)\sqrt{-c^2+d^2}}-\frac{2ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{(c^3+c^2d-cd^2-d^3)\left(c\tan\left(\frac{1}{2}fx+\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*a*c - a*d)/((c^3
+ c^2*d - c*d^2 - d^3)*sqrt(-c^2 + d^2)) - (2*a*c^2*tan(1/2*f*x + 1/2*e)^3
- 3*a*c*d*tan(1/2*f*x + 1/2*e)^3 + a*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*t
an(1/2*f*x + 1/2*e) + a*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x + 1/
2*e))/((c^3 + c^2*d - c*d^2 - d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*
x + 1/2*e)^2 - c - d)^2))/f
```

maple [A] time = 0.74, size = 178, normalized size = 1.36

$$\frac{4a\left(\frac{-\frac{(2c-d)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{4(c^2+2cd+d^2)}+\frac{(2c-3d)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4(c+d)(c-d)}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^2}+\frac{(2c-d)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{4(c^3+c^2d-c\,d^2-d^3)\sqrt{(c+d)(c-d)}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 4/f*a*((-1/4*(2*c-d)/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3+1/4*(2*c-3*d)/(c+
d)/(c-d)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d
-c-d)^2+1/4*(2*c-d)/(c^3+c^2*d-c*d^2-d^3)/((c+d)*(c-d))^(1/2)*arctanh(tan(1
/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 3.78, size = 171, normalized size = 1.31

$$\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)(2\,c-d)}{f\,(c+d)^{5/2}\,(c-d)^{3/2}}-\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3(2\,a\,c-a\,d)}{(c+d)^2}-\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(2\,c-3\,d)}{(c+d)\,(c-d)}}{f\left(2\,c\,d-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(2\,c^2-2\,d^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(c^2-2\,c\,d+d^2\right)+c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] (a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(2*c - d))/(f*(c
 + d)^(5/2)*(c - d)^(3/2)) - ((tan(e/2 + (f*x)/2)^3*(2*a*c - a*d))/(c + d)^
2 - (a*tan(e/2 + (f*x)/2)*(2*c - 3*d))/((c + d)*(c - d)))/(f*(2*c*d - tan(e
/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2)
+ c^2 + d^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx+\int\frac{\sec^2\left(e+fx\right.}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**2/(c**3 + 3*c**
2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

3.192 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=189

$$\frac{a\left(2c^2-2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{7/2}}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sec(e+fx))}+\frac{a(2c-3d)\tan(e+}{6f(c-d)(c+d)^2(c+d\sec}$$

```
[Out] a*(2*c^2-2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c-
d)^(5/2)/(c+d)^(7/2)/f+1/3*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3+1/6*a*(2
*c-3*d)*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))^2+1/6*a*(c-4*d)*(2*c-d)
*tan(f*x+e)/(c-d)^2/(c+d)^3/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.45, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{a\left(2c^2-2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{7/2}}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sec(e+fx))}+\frac{a(2c-3d)\tan(e+}{6f(c-d)(c+d)^2(c+d\sec}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2]])/Sqrt[c + d]
])/((c - d)^(5/2)*(c + d)^(7/2)*f) + (a*Tan[e + f*x])/(3*(c + d)*f*(c + d*S
ec[e + f*x])^3) + (a*(2*c - 3*d)*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c +
d*Sec[e + f*x])^2) + (a*(c - 4*d)*(2*c - d)*Tan[e + f*x])/(6*(c - d)^2*(c +
 d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3a(c-d)-2a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx}{3\left(c^2-d^2\right)}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} +$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} +$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} +$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} +$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} +$$

$$= \frac{a\left(2c^2-2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f} + \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+f}$$

Mathematica [A] time = 3.35, size = 247, normalized size = 1.31

$$-\frac{a(\cos(e+fx)+1)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(6\left(2c^2-2cd+d^2\right)(c\cos(e+fx)+d)^3\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)-\frac{1}{2}\sqrt{c^2-}\right.}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -1/12*(a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(6*(2*c^2 - 2*c*d + d^2)*Arc
Tanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3 -
(Sqrt[c^2 - d^2]*(6*c^4 - 12*c^3*d + 2*c^2*d^2 - 15*c*d^3 + 10*d^4 + 6*d*(2
*c^3 - 7*c^2*d + 2*c*d^2 + d^3)*Cos[e + f*x] + (6*c^4 - 12*c^3*d - 2*c^2*d^
2 + 3*c*d^3 + 2*d^4)*Cos[2*(e + f*x)])*Sin[e + f*x])/2))/((c - d)^2*(c + d)
^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

fricas [B] time = 0.54, size = 1278, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d + a*c^3*d^
2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*x + e)^2
+ 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^
5 - 4*a*d^6 + (6*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*
d^4 - 3*a*c*d^5 - 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*
a*c^2*d^4 - 2*a*c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d -
 3*c^8*d^2 - 3*c^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f
*x + e)^3 + 3*(c^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*
d^6 - c^3*d^7 - c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^
4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (
c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 -
 d^10)*f), 1/6*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d +
 a*c^3*d^2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*
x + e)^2 + 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(-c^2
+ d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
 e))) + (2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6 + (6
*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*d^4 - 3*a*c*d^5
- 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*
c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d - 3*c^8*d^2 - 3*c
^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f*x + e)^3 + 3*(c
^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*d^6 - c^3*d^7 -
c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 +
3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (c^7*d^3 + c^6*d^
4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f)]
```

giac [B] time = 0.39, size = 468, normalized size = 2.48

$$-\frac{\frac{3\left(2\,ac^2-2\,acd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^5+c^4d-2\,c^3d^2-2\,c^2d^3+cd^4+d^5\right)\sqrt{-c^2+d^2}}+\frac{6\,ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-18\,ac^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+21\,ac^2d^2\,\mathrm{t}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2
*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-
c^2 + d^2)))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(-c^2
 + d^2)) + (6*a*c^4*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^3*d*tan(1/2*f*x + 1/2*e
)^5 + 21*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c*d^3*tan(1/2*f*x + 1/2*e)
^5 + 3*a*d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*tan(1/2*f*x + 1/2*e)^3 + 24*
a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 8*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*
c*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^4*tan
(1/2*f*x + 1/2*e) - 6*a*c^3*d*tan(1/2*f*x + 1/2*e) - 21*a*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 9*a*d^4*tan(1/2*f*x + 1/2*e))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c
^2*d^3 + c*d^4 + d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2
- c - d)^3))/f
```

maple [A] time = 0.78, size = 271, normalized size = 1.43

$$\frac{4a\left(\frac{-\frac{\left(2c^2-2cd+d^2\right)\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{4\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{\left(3c^2-6cd+d^2\right)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3(c-d)\left(c^2+2cd+d^2\right)}-\frac{\left(2c^2-6cd+3d^2\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4(c+d)\left(c^2-2cd+d^2\right)}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^3}+\frac{\left(2c^2-2cd+d^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{4\left(c^5+c^4d-2c^3d^2-2c^2d^3+c\,d^4+d^5\right)\sqrt{(c+d)(c-d)}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 4/f*a*((-1/4*(2*c^2-2*c*d+d^2)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*e+1/2*f*x)
^5+1/3*(3*c^2-6*c*d+d^2)/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3-1/4*(2*
c^2-6*c*d+3*d^2)/(c+d)/(c^2-2*c*d+d^2)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f
*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^3+1/4*(2*c^2-2*c*d+d^2)/(c^5+c^4*d-2*c^
3*d^2-2*c^2*d^3+c*d^4+d^5)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(
c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 5.28, size = 321, normalized size = 1.70

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(2ac^2-2acd+ad^2\right)}{(c+d)^3}+\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2c^2-6cd+3d^2\right)}{(c+d)\left(c^2-2cd+d^2\right)}-\frac{4a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3c\right.}{3(c+d)^2(c}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-3c^3-3c^2d+3cd^2+3d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(-3c^3+3c^2d+3cd^2-3d^3\right)+3cd^2+3c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 - 2*a*c*d))/(c + d)^3 + (a*tan(e/2
+ (f*x)/2)*(2*c^2 - 6*c*d + 3*d^2))/((c + d)*(c^2 - 2*c*d + d^2)) - (4*a*ta
n(e/2 + (f*x)/2)^3*(3*c^2 - 6*c*d + d^2))/(3*(c + d)^2*(c - d)))/(f*(tan(e/
2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(
3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/
2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (a*atanh((tan(e/2 + (f*x
)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c
^2 - 2*c*d + d^2))/(f*(c + d)^(7/2)*(c - d)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec\left(e+fx\right)}{c^4+4c^3d\sec\left(e+fx\right)+6c^2d^2\sec^2\left(e+fx\right)+4cd^3\sec^3\left(e+fx\right)+d^4\sec^4\left(e+fx\right)}\,dx+\int\frac{}{c^4+4c^3d\sec\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
+ f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral
(sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**
2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

3.193 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=327

$$-\frac{a^2\left(4c^2-48cd-55d^2\right)\tan(e+fx)(c+d\sec(e+fx))^3}{120df}-\frac{a^2\left(4c^3-48c^2d-123cd^2-64d^3\right)\tan(e+fx)(c+d\sec}{120df}$$

```
[Out] 1/16*a^2*(24*c^4+64*c^3*d+84*c^2*d^2+48*c*d^3+11*d^4)*arctanh(sin(f*x+e))/f
-1/60*a^2*(4*c^5-48*c^4*d-311*c^3*d^2-448*c^2*d^3-288*c*d^4-64*d^5)*tan(f*x
+e)/d/f-1/240*a^2*(8*c^4-96*c^3*d-438*c^2*d^2-464*c*d^3-165*d^4)*sec(f*x+e)
*tan(f*x+e)/f-1/120*a^2*(4*c^3-48*c^2*d-123*c*d^2-64*d^3)*(c+d*sec(f*x+e))^
2*tan(f*x+e)/d/f-1/120*a^2*(4*c^2-48*c*d-55*d^2)*(c+d*sec(f*x+e))^3*tan(f*x
+e)/d/f-1/30*a^2*(c-12*d)*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f+1/6*a^2*(c+d*se
c(f*x+e))^5*tan(f*x+e)/d/f
```

Rubi [A] time = 0.42, antiderivative size = 371, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 100, 153, 147, 50, 63, 217, 203}

$$\frac{a^2\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\tan(e+fx)}{16f}+\frac{a^3\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\tan(e+fx}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Tan[e + f*x])/(16
*f) + (a^3*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*ArcTan[Sqrt
[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((24*c^4 + 64*c^3*d + 84*c^2*
d^2 + 48*c*d^3 + 11*d^4)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d
*(9*c + 2*d)*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
0*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(6*f)
 + (d*(a + a*Sec[e + f*x])^2*(2*(52*c^3 + 56*c^2*d + 48*c*d^2 + 9*d^3) + d*
(48*c^2 + 32*c*d + 19*d^2)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^4}{\sqrt{a-ax}}\,dx, x, \sec(e\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan}{30f}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan}{30f}$$

$$= \frac{\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\left(a^2+a^2\sec\right.}{48f}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+f}{16f}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+f}{16f}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+f}{16f}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+f}{16f}$$

Mathematica [A] time = 2.03, size = 460, normalized size = 1.41

$$-\frac{a^2(\cos(e+fx)+1)^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\left(240\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\cos^6(e+fx)\right(1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -1/15360*(a^2*(1 + Cos[e + f*x])^2*Sec[(e + f*x)/2]^4*Sec[e + f*x]^6*(240*(
24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Cos[e + f*x]^6*(Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]) - 2*(360*c^4 + 2880*c^3*d + 5220*c^2*d^2 + 4080*c*d^3 + 1255*d^4 + 32*(7
5*c^4 + 310*c^3*d + 480*c^2*d^2 + 336*c*d^3 + 88*d^4)*Cos[e + f*x] + 20*(24
*c^4 + 192*c^3*d + 324*c^2*d^2 + 240*c*d^3 + 55*d^4)*Cos[2*(e + f*x)] + 120
0*c^4*Cos[3*(e + f*x)] + 4640*c^3*d*Cos[3*(e + f*x)] + 6720*c^2*d^2*Cos[3*(
e + f*x)] + 4032*c*d^3*Cos[3*(e + f*x)] + 896*d^4*Cos[3*(e + f*x)] + 120*c^
4*Cos[4*(e + f*x)] + 960*c^3*d*Cos[4*(e + f*x)] + 1260*c^2*d^2*Cos[4*(e + f
*x)] + 720*c*d^3*Cos[4*(e + f*x)] + 165*d^4*Cos[4*(e + f*x)] + 240*c^4*Cos[
5*(e + f*x)] + 800*c^3*d*Cos[5*(e + f*x)] + 960*c^2*d^2*Cos[5*(e + f*x)] +
576*c*d^3*Cos[5*(e + f*x)] + 128*d^4*Cos[5*(e + f*x)])*Sin[e + f*x]))/f
```

fricas [A] time = 0.47, size = 387, normalized size = 1.18

$$\frac{15\left(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(24\,a^2c^4+64\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*(24*a^2*c^4 + 64*a^2*c^3*
d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^6*log(-sin(f*x
 + e) + 1) + 2*(40*a^2*d^4 + 32*(15*a^2*c^4 + 50*a^2*c^3*d + 60*a^2*c^2*d^2
 + 36*a^2*c*d^3 + 8*a^2*d^4)*cos(f*x + e)^5 + 15*(8*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^4 + 64*(5*a^2*c^
3*d + 15*a^2*c^2*d^2 + 9*a^2*c*d^3 + 2*a^2*d^4)*cos(f*x + e)^3 + 10*(36*a^2
*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^2 + 96*(2*a^2*c*d^3 + a^
2*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-11*a^2*d^4-48*a^2*d^3*c-84*a^2*d^2*c^2-64*a^2*d*c^3-24*a^2*c
```

```
^4)/32*ln(abs(tan((f*x+exp(1))/2)-1))-(-11*a^2*d^4-48*a^2*d^3*c-84*a^2*d^2*
c^2-64*a^2*d*c^3-24*a^2*c^4)/32*ln(abs(tan((f*x+exp(1))/2)+1))-(165*tan((f*
x+exp(1))/2)^11*a^2*d^4+720*tan((f*x+exp(1))/2)^11*a^2*d^3*c+1260*tan((f*x+
exp(1))/2)^11*a^2*d^2*c^2+960*tan((f*x+exp(1))/2)^11*a^2*d*c^3+360*tan((f*x
+exp(1))/2)^11*a^2*c^4-935*tan((f*x+exp(1))/2)^9*a^2*d^4-4080*tan((f*x+exp(
1))/2)^9*a^2*d^3*c-7140*tan((f*x+exp(1))/2)^9*a^2*d^2*c^2-5440*tan((f*x+exp
(1))/2)^9*a^2*d*c^3-2040*tan((f*x+exp(1))/2)^9*a^2*c^4+1986*tan((f*x+exp(1)
)/2)^7*a^2*d^4+10272*tan((f*x+exp(1))/2)^7*a^2*d^3*c+15480*tan((f*x+exp(1))
/2)^7*a^2*d^2*c^2+13440*tan((f*x+exp(1))/2)^7*a^2*d*c^3+4560*tan((f*x+exp(1
))/2)^7*a^2*c^4-3006*tan((f*x+exp(1))/2)^5*a^2*d^4-11232*tan((f*x+exp(1))/2
)^5*a^2*d^3*c-19080*tan((f*x+exp(1))/2)^5*a^2*d^2*c^2-17280*tan((f*x+exp(1)
)/2)^5*a^2*d*c^3-5040*tan((f*x+exp(1))/2)^5*a^2*c^4+1305*tan((f*x+exp(1))/2
)^3*a^2*d^4+7440*tan((f*x+exp(1))/2)^3*a^2*d^3*c+13980*tan((f*x+exp(1))/2)^
3*a^2*d^2*c^2+11200*tan((f*x+exp(1))/2)^3*a^2*d*c^3+2760*tan((f*x+exp(1))/2
)^3*a^2*c^4-795*tan((f*x+exp(1))/2)*a^2*d^4-3120*tan((f*x+exp(1))/2)*a^2*d^
3*c-4500*tan((f*x+exp(1))/2)*a^2*d^2*c^2-2880*tan((f*x+exp(1))/2)*a^2*d*c^3
-600*tan((f*x+exp(1))/2)*a^2*c^4)*1/240/(tan((f*x+exp(1))/2)^2-1)^6)
```

maple [A] time = 1.96, size = 602, normalized size = 1.84

$$\frac{20a^2c^3d\tan\left(fx+e\right)}{3f}+\frac{2a^2c\,d^3\tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{f}+\frac{12a^2c\,d^3\tan\left(fx+e\right)\left(\sec^2\left(fx+e\right)\right)}{5f}+\frac{3a^2c\,d^3\sec\left(f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x)
```

```
[Out] 20/3/f*a^2*c^3*d*tan(f*x+e)+4/f*a^2*c^2*d^2*tan(f*x+e)*sec(f*x+e)^2+2/f*a^2
*c*d^3*tan(f*x+e)*sec(f*x+e)^3+3/f*a^2*c*d^3*sec(f*x+e)*tan(f*x+e)+12/5/f*a
^2*c*d^3*tan(f*x+e)*sec(f*x+e)^2+1/2*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f+4/f*a^
2*c^3*d*sec(f*x+e)*tan(f*x+e)+21/4/f*a^2*c^2*d^2*sec(f*x+e)*tan(f*x+e)+4/5/
f*a^2*c*d^3*tan(f*x+e)*sec(f*x+e)^4+3/2/f*a^2*c^2*d^2*tan(f*x+e)*sec(f*x+e)
^3+4/3/f*a^2*c^3*d*tan(f*x+e)*sec(f*x+e)^2+2*a^2*c^4*tan(f*x+e)/f+3/2/f*a^2
*c^4*ln(sec(f*x+e)+tan(f*x+e))+11/16/f*a^2*d^4*ln(sec(f*x+e)+tan(f*x+e))+16
/15/f*a^2*d^4*tan(f*x+e)+21/4/f*a^2*c^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+11/24
/f*a^2*d^4*tan(f*x+e)*sec(f*x+e)^3+11/16/f*a^2*d^4*sec(f*x+e)*tan(f*x+e)+4/
f*a^2*c^3*d*ln(sec(f*x+e)+tan(f*x+e))+3/f*a^2*c*d^3*ln(sec(f*x+e)+tan(f*x+e
))+2/5/f*a^2*d^4*tan(f*x+e)*sec(f*x+e)^4+8/15/f*a^2*d^4*tan(f*x+e)*sec(f*x+
e)^2+24/5/f*a^2*c*d^3*tan(f*x+e)+8/f*a^2*c^2*d^2*tan(f*x+e)+1/6/f*a^2*d^4*t
an(f*x+e)*sec(f*x+e)^5
```

maxima [B] time = 0.48, size = 683, normalized size = 2.09

$$640\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^3d+1920\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^2d^2+128\left(3\,\tan\left(fx+e\right)^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3*d + 1920*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a^2*c^2*d^2 + 128*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^
3 + 15*tan(f*x + e))*a^2*c*d^3 + 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*
c*d^3 + 64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*d^4
 - 5*a^2*d^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(
sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
+ e) + 1) + 15*log(sin(f*x + e) - 1)) - 180*a^2*c^2*d^2*(2*(3*sin(f*x + e)^
3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
 + e) + 1) + 3*log(sin(f*x + e) - 1)) - 240*a^2*c*d^3*(2*(3*sin(f*x + e)^3
- 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
 e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a^2*d^4*(2*(3*sin(f*x + e)^3 - 5*s
in(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^
2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 960*a^2*c^3*d*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 720*a^2*c^2*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^2*c^4*log(sec(f*x + e) + tan(f
*x + e)) + 960*a^2*c^4*tan(f*x + e) + 1920*a^2*c^3*d*tan(f*x + e))/f
```

mupad [B] time = 5.35, size = 484, normalized size = 1.48

$$\frac{\left(-3\,a^2\,c^4-8\,a^2\,c^3\,d-\frac{21\,a^2\,c^2\,d^2}{2}-6\,a^2\,c\,d^3-\frac{11\,a^2\,d^4}{8}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\left(17\,a^2\,c^4+\frac{136\,a^2\,c^3\,d}{3}+\frac{119\,a^2\,c^2\,d^2}{2}+34\,a^2\,c\,\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(5*a^2*c^4 + (53*a^2*d^4)/8 + 26*a^2*c*d^3 + 24*a^2*c^3
*d + (75*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^11*(3*a^2*c^4 + (11*a^2*d^4)/
8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*c^2*d^2)/2) + tan(e/2 + (f*x)/2)^9*
(17*a^2*c^4 + (187*a^2*d^4)/24 + 34*a^2*c*d^3 + (136*a^2*c^3*d)/3 + (119*a^
2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^3*(23*a^2*c^4 + (87*a^2*d^4)/8 + 62*a^2*
c*d^3 + (280*a^2*c^3*d)/3 + (233*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^7*(38
*a^2*c^4 + (331*a^2*d^4)/20 + (428*a^2*c*d^3)/5 + 112*a^2*c^3*d + 129*a^2*c
^2*d^2) + tan(e/2 + (f*x)/2)^5*(42*a^2*c^4 + (501*a^2*d^4)/20 + (468*a^2*c*
d^3)/5 + 144*a^2*c^3*d + 159*a^2*c^2*d^2))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*
```

```
tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 -
6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*atanh((tan(e/2
 + (f*x)/2)*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2))/(4*(12*c*
d^3 + 16*c^3*d + 6*c^4 + (11*d^4)/4 + 21*c^2*d^2)))*(48*c*d^3 + 64*c^3*d +
24*c^4 + 11*d^4 + 84*c^2*d^2))/(8*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int c^4 \sec \left(e + fx\right) dx + \int 2c^4 \sec^2 \left(e + fx\right) dx + \int c^4 \sec^3 \left(e + fx\right) dx + \int d^4 \sec^5 \left(e + fx\right) dx + \int 2d^4 \sec^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(c**4*sec(e + f*x), x) + Integral(2*c**4*sec(e + f*x)**2, x)
+ Integral(c**4*sec(e + f*x)**3, x) + Integral(d**4*sec(e + f*x)**5, x) + I
ntegral(2*d**4*sec(e + f*x)**6, x) + Integral(d**4*sec(e + f*x)**7, x) + In
tegral(4*c*d**3*sec(e + f*x)**4, x) + Integral(8*c*d**3*sec(e + f*x)**5, x)
 + Integral(4*c*d**3*sec(e + f*x)**6, x) + Integral(6*c**2*d**2*sec(e + f*x
)**3, x) + Integral(12*c**2*d**2*sec(e + f*x)**4, x) + Integral(6*c**2*d**2
*sec(e + f*x)**5, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(8*c
**3*d*sec(e + f*x)**3, x) + Integral(4*c**3*d*sec(e + f*x)**4, x))
```

3.194 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=242

$$\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2\left(c^2-10cd-12d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{20df}-\frac{a^2\left(2\cdots\right.}{\cdots}$$

```
[Out] 3/8*a^2*(2*c+d)*(2*c^2+3*c*d+2*d^2)*arctanh(sin(f*x+e))/f-1/10*a^2*(c^4-10*
c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)*tan(f*x+e)/d/f-1/40*a^2*(2*c^3-20*c^2*d-5
7*c*d^2-30*d^3)*sec(f*x+e)*tan(f*x+e)/f-1/20*a^2*(c^2-10*c*d-12*d^2)*(c+d*s
ec(f*x+e))^2*tan(f*x+e)/d/f-1/20*a^2*(c-10*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)
/d/f+1/5*a^2*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f
```

Rubi [A] time = 0.34, antiderivative size = 277, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 100, 147, 50, 63, 217, 203}

$$\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f}+\frac{3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*Tan[e + f*x])/(8*f) + (3*a^3*(2*c
 + d)*(2*c^2 + 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Se
c[e + f*x])]]*Tan[e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
 + f*x]]) + ((2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*(a^2 + a^2*Sec[e + f*x])*Tan[
e + f*x])/(8*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e +
f*x])/(5*f) + (d*(a + a*Sec[e + f*x])^2*(2*(8*c^2 + 5*c*d + 2*d^2) + d*(7*c
 + 2*d)*Sec[e + f*x])*Tan[e + f*x])/(20*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
```

```
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^3}{\sqrt{a-ax}}\,dx,x,\sec(\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx}{5f}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx}{5f}$$

$$= \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan}{8f}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{3a^3(2c}{}$$

Mathematica [A] time = 1.39, size = 326, normalized size = 1.35

$$-\frac{a^2(\cos(e+fx)+1)^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\left(120\left(4c^3+8c^2d+7cd^2+2d^3\right)\cos^5(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -1/1280*(a^2*(1 + Cos[e + f*x])^2*Sec[(e + f*x)/2]^4*Sec[e + f*x]^5*(120*(4
*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3)*Cos[e + f*x]^5*(Log[Cos[(e + f*x)/2] - Si
n[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(120*c^3 +
380*c^2*d + 400*c*d^2 + 152*d^3 + 5*(12*c^3 + 72*c^2*d + 87*c*d^2 + 34*d^3)
*Cos[e + f*x] + 16*(10*c^3 + 30*c^2*d + 30*c*d^2 + 9*d^3)*Cos[2*(e + f*x)]
+ 20*c^3*Cos[3*(e + f*x)] + 120*c^2*d*Cos[3*(e + f*x)] + 105*c*d^2*Cos[3*(e
 + f*x)] + 30*d^3*Cos[3*(e + f*x)] + 40*c^3*Cos[4*(e + f*x)] + 100*c^2*d*Co
s[4*(e + f*x)] + 80*c*d^2*Cos[4*(e + f*x)] + 24*d^3*Cos[4*(e + f*x)])*Sin[e
 + f*x]))/f
```

fricas [A] time = 0.45, size = 294, normalized size = 1.21

$$15\left(4a^2c^3+8a^2c^2d+7a^2cd^2+2a^2d^3\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(4a^2c^3+8a^2c^2d+7a^2cd^2+2a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^5
*log(sin(f*x + e) + 1) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*
d^3)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^2*d^3 + 8*(10*a^2*c^3 +
 25*a^2*c^2*d + 20*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^4 + 5*(4*a^2*c^3 + 2
4*a^2*c^2*d + 21*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^3 + 8*(5*a^2*c^2*d + 1
0*a^2*c*d^2 + 3*a^2*d^3)*cos(f*x + e)^2 + 10*(3*a^2*c*d^2 + 2*a^2*d^3)*cos(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(6*a^2*d^3+21*a^2*d^2*c+24*a^2*d*c^2+12*a^2*c^3)/16*ln(abs(ta
n((f*x+exp(1))/2)-1))+(6*a^2*d^3+21*a^2*d^2*c+24*a^2*d*c^2+12*a^2*c^3)/16*l
n(abs(tan((f*x+exp(1))/2)+1))-(30*tan((f*x+exp(1))/2)^9*a^2*d^3+105*tan((f*
x+exp(1))/2)^9*a^2*d^2*c+120*tan((f*x+exp(1))/2)^9*a^2*d*c^2+60*tan((f*x+ex
p(1))/2)^9*a^2*c^3-140*tan((f*x+exp(1))/2)^7*a^2*d^3-490*tan((f*x+exp(1))/2
```

```
)^7*a^2*d^2*c-560*tan((f*x+exp(1))/2)^7*a^2*d*c^2-280*tan((f*x+exp(1))/2)^7
*a^2*c^3+288*tan((f*x+exp(1))/2)^5*a^2*d^3+800*tan((f*x+exp(1))/2)^5*a^2*d^
2*c+1120*tan((f*x+exp(1))/2)^5*a^2*d*c^2+480*tan((f*x+exp(1))/2)^5*a^2*c^3-
180*tan((f*x+exp(1))/2)^3*a^2*d^3-790*tan((f*x+exp(1))/2)^3*a^2*d^2*c-1040*
tan((f*x+exp(1))/2)^3*a^2*d*c^2-360*tan((f*x+exp(1))/2)^3*a^2*c^3+130*tan((
f*x+exp(1))/2)*a^2*d^3+375*tan((f*x+exp(1))/2)*a^2*d^2*c+360*tan((f*x+exp(1
))/2)*a^2*d*c^2+100*tan((f*x+exp(1))/2)*a^2*c^3)*1/40/(tan((f*x+exp(1))/2)^
2-1)^5)
```

maple [A] time = 1.62, size = 420, normalized size = 1.74

$$\frac{3a^2c^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{5a^2c^2d\tan\left(fx+e\right)}{f}+\frac{21a^2c\,d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{21a^2c\,d^2\ln\left(\mathrm{se}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 3/2/f*a^2*c^3*ln(sec(f*x+e)+tan(f*x+e))+5/f*a^2*c^2*d*tan(f*x+e)+21/8/f*a^2
*c*d^2*sec(f*x+e)*tan(f*x+e)+21/8/f*a^2*c*d^2*ln(sec(f*x+e)+tan(f*x+e))+6/5
/f*a^2*d^3*tan(f*x+e)+3/5/f*a^2*d^3*tan(f*x+e)*sec(f*x+e)^2+2*a^2*c^3*tan(f
*x+e)/f+3/f*a^2*c^2*d*sec(f*x+e)*tan(f*x+e)+3/f*a^2*c^2*d*ln(sec(f*x+e)+tan
(f*x+e))+4/f*a^2*c*d^2*tan(f*x+e)+2/f*a^2*c*d^2*tan(f*x+e)*sec(f*x+e)^2+1/2
/f*a^2*d^3*tan(f*x+e)*sec(f*x+e)^3+3/4/f*a^2*d^3*sec(f*x+e)*tan(f*x+e)+3/4/
f*a^2*d^3*ln(sec(f*x+e)+tan(f*x+e))+1/2/f*c^3*a^2*sec(f*x+e)*tan(f*x+e)+1/f
*a^2*c^2*d*tan(f*x+e)*sec(f*x+e)^2+3/4/f*a^2*c*d^2*tan(f*x+e)*sec(f*x+e)^3+
1/5/f*a^2*d^3*tan(f*x+e)*sec(f*x+e)^4
```

maxima [B] time = 0.41, size = 469, normalized size = 1.94

$$\frac{240\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^2d+480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2cd^2+16\left(3\,\tan\left(fx+e\right)^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2*d + 480*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a^2*c*d^2 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
15*tan(f*x + e))*a^2*d^3 + 80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d^3 - 4
5*a^2*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a
^2*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
```

```
e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 60*a^2*c^
3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 360*a^2*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin
(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 180*a^2*c*d^2*(2*sin(f*x + e)/(si
n(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^
2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^3*tan(f*x + e) + 720*a^2
*c^2*d*tan(f*x + e))/f
```

mupad [B] time = 5.49, size = 394, normalized size = 1.63

$$\frac{3\,a^2\,\mathrm{atanh}\left(\frac{3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c+d)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{2\,\left(6\,c^3+12\,c^2\,d+\frac{21\,c\,d^2}{2}+3\,d^3\right)}\right)\,(2\,c+d)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{4\,f}-\frac{\left(3\,a^2\,c^3+6\,a^2\,c^2\,d+\frac{21\,a^2\,c\,d^2}{4}+\frac{3\,a^2\,d^3}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (3*a^2*atanh((3*tan(e/2 + (f*x)/2)*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2))/(2*((
21*c*d^2)/2 + 12*c^2*d + 6*c^3 + 3*d^3)))*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2)
)/(4*f) - (tan(e/2 + (f*x)/2)^9*(3*a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)
/4 + 6*a^2*c^2*d) - tan(e/2 + (f*x)/2)^7*(14*a^2*c^3 + 7*a^2*d^3 + (49*a^2*
c*d^2)/2 + 28*a^2*c^2*d) - tan(e/2 + (f*x)/2)^3*(18*a^2*c^3 + 9*a^2*d^3 + (
79*a^2*c*d^2)/2 + 52*a^2*c^2*d) + tan(e/2 + (f*x)/2)^5*(24*a^2*c^3 + (72*a^
2*d^3)/5 + 40*a^2*c*d^2 + 56*a^2*c^2*d) + tan(e/2 + (f*x)/2)*(5*a^2*c^3 + (
13*a^2*d^3)/2 + (75*a^2*c*d^2)/4 + 18*a^2*c^2*d))/(f*(5*tan(e/2 + (f*x)/2)^
2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2
)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^3\sec\left(e+fx\right)dx+\int 2c^3\sec^2\left(e+fx\right)dx+\int c^3\sec^3\left(e+fx\right)dx+\int d^3\sec^4\left(e+fx\right)dx+\int 2d^3\sec^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(c**3*sec(e + f*x), x) + Integral(2*c**3*sec(e + f*x)**2, x)
+ Integral(c**3*sec(e + f*x)**3, x) + Integral(d**3*sec(e + f*x)**4, x) + I
ntegral(2*d**3*sec(e + f*x)**5, x) + Integral(d**3*sec(e + f*x)**6, x) + In
tegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(6*c*d**2*sec(e + f*x)**4, x)
 + Integral(3*c*d**2*sec(e + f*x)**5, x) + Integral(3*c**2*d*sec(e + f*x)**
2, x) + Integral(6*c**2*d*sec(e + f*x)**3, x) + Integral(3*c**2*d*sec(e + f
*x)**4, x))
```

3.195 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=176

$$\frac{a^2\left(12c^2+16cd+7d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2\left(c^3-8c^2d-20cd^2-8d^3\right)\tan(e+fx)}{6df}-\frac{a^2\left(2c(c-8d)-21d^2\right)\mathrm{ta}}{24}$$

```
[Out] 1/8*a^2*(12*c^2+16*c*d+7*d^2)*arctanh(sin(f*x+e))/f-1/6*a^2*(c^3-8*c^2*d-20
*c*d^2-8*d^3)*tan(f*x+e)/d/f-1/24*a^2*(2*c*(c-8*d)-21*d^2)*sec(f*x+e)*tan(f
*x+e)/f-1/12*a^2*(c-8*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/d/f+1/4*a^2*(c+d*sec
(f*x+e))^3*tan(f*x+e)/d/f
```

Rubi [A] time = 0.26, antiderivative size = 234, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 90, 80, 50, 63, 217, 203}

$$\frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f}+\frac{a^3\left(12c^2+16cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{\left(12c^2+16cd+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*Tan[e + f*x])/(8*f) + (a^3*(12*c^2 + 16*c*d
+ 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e
+ f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d*(5*c +
 2*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + ((12*c^2 + 16*c*d + 7*d
^2)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f*x])
^2*(c + d*Sec[e + f*x])*Tan[e + f*x])/(4*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^2}{\sqrt{a-ax}}\,dx,x,\sec(\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{d(a+}{}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{a^3\left(12c^2+16c\right.}{4f\sqrt{a}}$$

Mathematica [B] time = 1.00, size = 479, normalized size = 2.72

$$-\frac{a^2\sec^4(e+fx)\left(12\left(12c^2+16cd+7d^2\right)\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] -1/192*(a^2*Sec[e + f*x]^4*(108*c^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 144*c*d*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 63*d^2*Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]] + 12*(12*c^2 + 16*c*d + 7*d^2)*Cos[2*(e + f*x)]
*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e
```

```
+ f*x)/2]]) + 3*(12*c^2 + 16*c*d + 7*d^2)*Cos[4*(e + f*x)]*(Log[Cos[(e + f*
x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 108
*c^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 144*c*d*Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]] - 63*d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 2
4*c^2*Sin[e + f*x] - 96*c*d*Sin[e + f*x] - 90*d^2*Sin[e + f*x] - 96*c^2*Sin
[2*(e + f*x)] - 224*c*d*Sin[2*(e + f*x)] - 128*d^2*Sin[2*(e + f*x)] - 24*c^
2*Sin[3*(e + f*x)] - 96*c*d*Sin[3*(e + f*x)] - 42*d^2*Sin[3*(e + f*x)] - 48
*c^2*Sin[4*(e + f*x)] - 80*c*d*Sin[4*(e + f*x)] - 32*d^2*Sin[4*(e + f*x)]))
/f
```

fricas [A] time = 0.45, size = 209, normalized size = 1.19

$$\frac{3\left(12\,a^{2}c^{2}+16\,a^{2}cd+7\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-3\left(12\,a^{2}c^{2}+16\,a^{2}cd+7\,a^{2}d^{2}\right)\cos\left(fx+e\right)^{4}\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(sin(f*x +
e) + 1) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(-sin(f
*x + e) + 1) + 2*(6*a^2*d^2 + 16*(3*a^2*c^2 + 5*a^2*c*d + 2*a^2*d^2)*cos(f*
x + e)^3 + 3*(4*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^2 + 16*(a^2*
c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-7*a^2*d^2-16*a^2*d*c-12*a^2*c^2)/16*ln(abs(tan((f*x+exp(1))/
2)-1))-(-7*a^2*d^2-16*a^2*d*c-12*a^2*c^2)/16*ln(abs(tan((f*x+exp(1))/2)+1))
+(-21*tan((f*x+exp(1))/2)^7*a^2*d^2-48*tan((f*x+exp(1))/2)^7*a^2*d*c-36*tan
((f*x+exp(1))/2)^7*a^2*c^2+77*tan((f*x+exp(1))/2)^5*a^2*d^2+176*tan((f*x+ex
p(1))/2)^5*a^2*d*c+132*tan((f*x+exp(1))/2)^5*a^2*c^2-83*tan((f*x+exp(1))/2)
^3*a^2*d^2-272*tan((f*x+exp(1))/2)^3*a^2*d*c-156*tan((f*x+exp(1))/2)^3*a^2*
c^2+75*tan((f*x+exp(1))/2)*a^2*d^2+144*tan((f*x+exp(1))/2)*a^2*d*c+60*tan((
f*x+exp(1))/2)*a^2*c^2)*1/24/(tan((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.42, size = 268, normalized size = 1.52

$$\frac{3a^2c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{10a^2cd\tan\left(fx+e\right)}{3f}+\frac{7a^2d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{7a^2d^2\ln\left(\sec\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 3/2/f*a^2*c^2*ln(sec(f*x+e)+tan(f*x+e))+10/3/f*a^2*c*d*tan(f*x+e)+7/8/f*a^2
*d^2*sec(f*x+e)*tan(f*x+e)+7/8/f*a^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+2*a^2*c^
2*tan(f*x+e)/f+2/f*a^2*c*d*sec(f*x+e)*tan(f*x+e)+2/f*a^2*c*d*ln(sec(f*x+e)+
tan(f*x+e))+4/3/f*a^2*d^2*tan(f*x+e)+2/3/f*a^2*d^2*tan(f*x+e)*sec(f*x+e)^2+
1/2*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+2/3/f*a^2*c*d*tan(f*x+e)*sec(f*x+e)^2+1
/4/f*a^2*d^2*tan(f*x+e)*sec(f*x+e)^3
```

maxima [A] time = 0.63, size = 324, normalized size = 1.84

$$\frac{32\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2cd+32\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2d^2-3\,a^2d^2\left(\frac{2\left(3\,\sin\left(fx+e\right)^3-5\,\sin\left(\right.\right.}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c*d + 32*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a^2*d^2 - 3*a^2*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(
sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(si
n(f*x + e) - 1)) - 12*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*d*(2*sin(f*x + e)/(sin(
f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*a^2*d
^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f
*x + e) - 1)) + 48*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)) + 96*a^2*c^2*ta
n(f*x + e) + 96*a^2*c*d*tan(f*x + e))/f
```

mupad [B] time = 5.46, size = 237, normalized size = 1.35

$$\frac{\left(-3\,a^2\,c^2-4\,a^2\,c\,d-\frac{7\,a^2\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(11\,a^2\,c^2+\frac{44\,a^2\,c\,d}{3}+\frac{77\,a^2\,d^2}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-13\,a^2\,c^2-\frac{68\,a^2\,c\,d}{3}\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(5*a^2*c^2 + (25*a^2*d^2)/4 + 12*a^2*c*d) - tan(e/2 + (
f*x)/2)^7*(3*a^2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^5*(1
1*a^2*c^2 + (77*a^2*d^2)/12 + (44*a^2*c*d)/3) - tan(e/2 + (f*x)/2)^3*(13*a^
2*c^2 + (83*a^2*d^2)/12 + (68*a^2*c*d)/3))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*t
an(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) +
 (a^2*atanh((tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(2*(8*c*d + 6*c^
2 + (7*d^2)/2)))*(16*c*d + 12*c^2 + 7*d^2))/(4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^2\sec\left(e+fx\right)dx+\int 2c^2\sec^2\left(e+fx\right)dx+\int c^2\sec^3\left(e+fx\right)dx+\int d^2\sec^3\left(e+fx\right)dx+\int 2d^2\sec^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(c**2*sec(e + f*x), x) + Integral(2*c**2*sec(e + f*x)**2, x)
+ Integral(c**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**3, x) + I
ntegral(2*d**2*sec(e + f*x)**4, x) + Integral(d**2*sec(e + f*x)**5, x) + In
tegral(2*c*d*sec(e + f*x)**2, x) + Integral(4*c*d*sec(e + f*x)**3, x) + Int
egral(2*c*d*sec(e + f*x)**4, x))
```

3.196 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=103

$$\frac{2a^2(3c+2d)\tan(e+fx)}{3f}+\frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2(3c+2d)\tan(e+fx)\sec(e+fx)}{6f}+\frac{d\tan(e+fx)}{}$$

```
[Out] 1/2*a^2*(3*c+2*d)*arctanh(sin(f*x+e))/f+2/3*a^2*(3*c+2*d)*tan(f*x+e)/f+1/6*
a^2*(3*c+2*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4001, 3788, 3767, 8, 4046, 3770}

$$\frac{2a^2(3c+2d)\tan(e+fx)}{3f}+\frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2(3c+2d)\tan(e+fx)\sec(e+fx)}{6f}+\frac{d\tan(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*(3*c + 2*d)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a^2*(3*c + 2*d)*Tan[e +
f*x])/(3*f) + (a^2*(3*c + 2*d)*Sec[e + f*x]*Tan[e + f*x])/(6*f) + (d*(a + a
*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx = \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f} + \frac{1}{3}(3c+2d)\int \sec($$

$$= \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f} + \frac{1}{3}(3c+2d)\int \sec($$

$$= \frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+}{}$$

$$= \frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{2a^2(3c+2d)\tan(e}{3f}$$

Mathematica [B] time = 6.36, size = 481, normalized size = 4.67

$$a^2\cos^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^2(c+d\sec(e+fx))\left(\frac{4(6c+5d)\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)} + \frac{}{(\mathrm{si}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]^3*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(c + d*Sec[e +
f*x])*(-6*(3*c + 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 6*(3*c + 2
*d)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*d*Sin[(f*x)/2])/((Cos[e/2
] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + ((3*c + 7*d)*Cos[e
/2] - (3*c + 5*d)*Sin[e/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2) + (4*(6*c + 5*d)*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])) + (2*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/
2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - ((3*c + 7*d)*Cos[e/2] + (3*c
 + 5*d)*Sin[e/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^2) + (4*(6*c + 5*d)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(48*f*(d + c*Cos[e + f*x]))
```

fricas [A] time = 0.42, size = 138, normalized size = 1.34

$$\frac{3\left(3\,a^2c+2\,a^2d\right)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\left(3\,a^2c+2\,a^2d\right)\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)+}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(3*a^2*c + 2*a^2*d)*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*(3*a^2
*c + 2*a^2*d)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*a^2*d + 2*(6*a^2
*c + 5*a^2*d)*cos(f*x + e)^2 + 3*(a^2*c + 2*a^2*d)*cos(f*x + e))*sin(f*x +
e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(3*a^2*c+2*a^2*d)/4*ln(abs(tan((f*x+exp(1))/2)-1))+(3*a^2*c+2
*a^2*d)/4*ln(abs(tan((f*x+exp(1))/2)+1))+(-9*tan((f*x+exp(1))/2)^5*a^2*c-6*
tan((f*x+exp(1))/2)^5*a^2*d+24*tan((f*x+exp(1))/2)^3*a^2*c+16*tan((f*x+exp(
1))/2)^3*a^2*d-15*tan((f*x+exp(1))/2)*a^2*c-18*tan((f*x+exp(1))/2)*a^2*d)*1
/6/(tan((f*x+exp(1))/2)^2-1)^3)
```

maple [A] time = 1.16, size = 141, normalized size = 1.37

$$\frac{3a^2c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{5a^2d\tan\left(fx+e\right)}{3f}+\frac{2a^2c\tan\left(fx+e\right)}{f}+\frac{a^2d\sec\left(fx+e\right)\tan\left(fx+e\right)}{f}+\frac{a^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x)
```

```
[Out] 3/2/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))+5/3/f*a^2*d*tan(f*x+e)+2*a^2*c*tan(f*
x+e)/f+1/f*a^2*d*sec(f*x+e)*tan(f*x+e)+1/f*a^2*d*ln(sec(f*x+e)+tan(f*x+e))+
1/2*a^2*c*sec(f*x+e)*tan(f*x+e)/f+1/3/f*a^2*d*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.53, size = 167, normalized size = 1.62

$$\frac{4\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2d-3a^2c\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-6a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d - 3*a^2*c*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 6*a^
2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a^2*c*log(sec(f*x + e) + tan(f*x + e)) + 24*a^2*c*tan(f
*x + e) + 12*a^2*d*tan(f*x + e))/f
```

mupad [B] time = 4.52, size = 161, normalized size = 1.56

$$\frac{2a^2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3c}{2}+d\right)}{6c+4d}\right)\left(\frac{3c}{2}+d\right)}{f}-\frac{\left(3a^2c+2a^2d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\left(-8a^2c-\frac{16a^2d}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+\left(5a\right.}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (2*a^2*atanh((4*tan(e/2 + (f*x)/2)*((3*c)/2 + d))/(6*c + 4*d))*((3*c)/2 + d
))/f - (tan(e/2 + (f*x)/2)*(5*a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^5*(3*a^
2*c + 2*a^2*d) - tan(e/2 + (f*x)/2)^3*(8*a^2*c + (16*a^2*d)/3))/(f*(3*tan(e
/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c\sec\left(e+fx\right)dx+\int 2c\sec^2\left(e+fx\right)dx+\int c\sec^3\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\int 2d\sec^3\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(c*sec(e + f*x), x) + Integral(2*c*sec(e + f*x)**2, x) + Inte
gral(c*sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**2, x) + Integral(2*d*
sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**4, x))
```

3.197 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2(c-2d)\tanh^{-1}(\sin(e+fx))}{d^2f}+\frac{2a^2(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2f\sqrt{c+d}}+\frac{a^2\tan(e+fx)}{df}$$

```
[Out] -a^2*(c-2*d)*arctanh(sin(f*x+e))/d^2/f+2*a^2*(c-d)^(3/2)*arctanh((c-d)^(1/2
)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/d^2/f/(c+d)^(1/2)+a^2*tan(f*x+e)/d/f
```

Rubi [B] time = 0.25, antiderivative size = 208, normalized size of antiderivative = 2.19, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 102, 157, 63, 217, 203, 93, 205}

$$-\frac{2a^3(c-2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2a^3(c-d)^{3/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^2f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2\tan(e+}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Tan[e + f*x])/(d*f) - (2*a^3*(c - 2*d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]
/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*(c - d)^(3/2)*ArcTan[(Sqrt[c + d]*Sqrt[a
 + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(
d^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} + \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{-a^3d+a^3(c-2d)x}{\sqrt{a-ax}\,\sqrt{a+ax}\,(c+dx)}\,dx,x,\sec(e+\right.}{df\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} + \frac{\left(a^4(c-2d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,\sqrt{a+ax}}\,dx,x,\text{se}\right.}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{\left(2a^3(c-2d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-\right.}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-d)^{3/2}\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2\sqrt{c+d}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \cdots$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-2d)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-\cdots}{d^2\sqrt{c\cdots}}$$

Mathematica [C] time = 2.02, size = 329, normalized size = 3.46

$$\frac{a^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^2(c\cos(e+fx)+d)\left(-\frac{2i(c-d)^2(\cos(e)-i\sin(e))\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\cdots\right.}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-\cdots}}\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x]
)^2*((c - 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - (c - 2*d)*Log[Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]] - ((2*I)*(c - d)^2*ArcTan[((I*Cos[e] + Si
n[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos
[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
```

```
*Sin[e])^2]) + (d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])) + (d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(4*d^2*f*(c + d*Sec[e + f*x]))
```

fricas [A] time = 0.63, size = 398, normalized size = 4.19

$$\left[\frac{2\,a^2 d \sin\left(fx+e\right) - \left(a^2 c - a^2 d\right)\sqrt{\frac{c-d}{c+d}}\,\cos\left(fx+e\right)\log\left(\frac{2\,cd\cos\left(fx+e\right) - \left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2 - 2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c}{c}}}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*a^2*d*sin(f*x + e) - (a^2*c - a^2*d)*sqrt((c - d)/(c + d))*cos(f*x
+ e)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d
+ (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^
2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (a^2*c - 2*a^2*d)*cos
(f*x + e)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f
*x + e) + 1))/(d^2*f*cos(f*x + e)), 1/2*(2*a^2*d*sin(f*x + e) + 2*(a^2*c -
a^2*d)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c
 + d))/((c - d)*sin(f*x + e)))*cos(f*x + e) - (a^2*c - 2*a^2*d)*cos(f*x + e
)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f*x + e)
+ 1))/(d^2*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-2*a^2*c^2+4*a^2*c*d-2*a^2*d^2)*1/2/d^2/sqrt(-c^2+d^2)*(atan(
(c*tan((f*x+exp(1))/2)-d*tan((f*x+exp(1))/2))/sqrt(-c^2+d^2))+pi*sign(2*c-2
*d)*floor((f*x+exp(1))/2/pi+1/2))-(-a^2*c+2*a^2*d)*1/2/d^2*ln(abs(tan((f*x+
exp(1))/2)-1))+(-a^2*c+2*a^2*d)*1/2/d^2*ln(abs(tan((f*x+exp(1))/2)+1))-tan(
(f*x+exp(1))/2)*a^2/d/(tan((f*x+exp(1))/2)^2-1))
```

maple [B] time = 0.62, size = 291, normalized size = 3.06

$$\frac{2a^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c^2}{f\,d^2\sqrt{(c+d)\,(c-d)}}-\frac{4a^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c}{fd\sqrt{(c+d)\,(c-d)}}+\frac{2a^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{f\sqrt{(c+d)\,(c-d)}}-\frac{a^2}{fd\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f*a^2/d^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-
d))^(1/2))*c^2-4/f*a^2/d/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-
d)/((c+d)*(c-d))^(1/2))*c+2/f*a^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2
*f*x)*(c-d)/((c+d)*(c-d))^(1/2))-1/f*a^2/d/(tan(1/2*e+1/2*f*x)-1)+1/f*a^2/d
^2*ln(tan(1/2*e+1/2*f*x)-1)*c-2/f*a^2/d*ln(tan(1/2*e+1/2*f*x)-1)-1/f*a^2/d/
(tan(1/2*e+1/2*f*x)+1)-1/f*a^2/d^2*ln(tan(1/2*e+1/2*f*x)+1)*c+2/f*a^2/d*ln(
tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.59, size = 529, normalized size = 5.57

$$\frac{2\,a^2\,\left(\frac{\sin\left(e+f\,x\right)}{2}+2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}+\frac{2\,a^2\,\left(\frac{c\,\sin\left(e+f\,x\right)}{2}+c\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{d\,f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}-\frac{2\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (2*a^2*(sin(e + f*x)/2 + 2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 +
(f*x)/2))))/(f*cos(e + f*x)*(c + d)) + (2*a^2*((c*sin(e + f*x))/2 + c*cos(e
```

```
 + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*cos(e + f*x)*(c
 + d)) - (2*a^2*(c^2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/
2)) + cos(e + f*x)*atan(((2*c*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 -
 d^4)^(3/2) - 2*c^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2
) + 5*d^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - c*d^4*
sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 4*c^4*d*sin(e/2
+ (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - 9*c^2*d^3*sin(e/2 + (f*x
)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 3*c^3*d^2*sin(e/2 + (f*x)/2)*(
2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2))*1i)/(d*cos(e/2 + (f*x)/2)*(c + d)*(8*
c*d^4 + 3*c^4*d - 5*d^5 + 2*c^2*d^3 - 8*c^3*d^2)))*((c + d)*(c - d)^3)^(1/2
)*1i))/(d^2*f*cos(e + f*x)*(c + d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{\sec^3\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(2*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**3/(c + d*sec(e + f
*x)), x))
```

3.198 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=117

$$-\frac{2a^2\sqrt{c-d}\,(c+2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2f(c+d)^{3/2}}-\frac{a^2(c-d)\tan(e+fx)}{df(c+d)(c+d\sec(e+fx))}+\frac{a^2\tanh^{-1}(\sin(e+fx))}{d^2f}$$

```
[Out] a^2*arctanh(sin(f*x+e))/d^2/f-2*a^2*(c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1
/2*f*x)/(c+d)^(1/2))*(c-d)^(1/2)/d^2/(c+d)^(3/2)/f-a^2*(c-d)*tan(f*x+e)/d/(
c+d)/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.97, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 98, 157, 63, 217, 203, 93, 205}

$$\frac{2a^3\sqrt{c-d}\,(c+2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{d^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{a^2(c-d)\tan(e+fx)}{df(c+d)(c+d\sec(e+fx))}+\frac{2a^3\tan(e+fx)\tan^{-1}}{d^2f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*Sq
rt[c - d]*(c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^2*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*(c - d)*Tan[e + f*x])/(d*(
c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{-2a^3d-a^3(c+d)x}{\sqrt{a-ax}\,\sqrt{a+ax}\,(c+}\right.}{d(c+d)f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,\sqrt{a+ax}}\,d\right.}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\,\text{se}}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{\left(2a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx,x\right.}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a}}$$

$$= \frac{2a^3\sqrt{c-d}\,(c+2d)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{a^2(c-d}{d(c+d)f(c}$$

$$= \frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\sqrt{c-d}\,(c+2d)\tan^{-1}\left(\right.}{d^2(c+d)^{3/2}f\sqrt{a-a\sec}}$$

Mathematica [C] time = 1.51, size = 312, normalized size = 2.67

$$\frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^2(c\cos(e+fx)+d)\left(\frac{2\left(c^2+cd-2d^2\right)(\sin(e)+i\cos(e))(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{f}{2}\right.\right.}{\sqrt{c^2-d^2}\,\sqrt{(\mathrm{co}}}\right.}{(c+d)\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(-((d + c
*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (d + c*Cos[e + f
*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*(c^2 + c*d - 2*d^2)*ArcT
```

```
an[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x])*(I*Cos[e] + Sin
[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((c - d)*d*(d
*Sin[e] - c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2
]))))/(4*d^2*f*(c + d*Sec[e + f*x])^2)
```

fricas [B] time = 0.67, size = 567, normalized size = 4.85

$$\left[\frac{\left(a^{2}cd + 2\,a^{2}d^{2} + \left(a^{2}c^{2} + 2\,a^{2}cd\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\,\log\left(\frac{2\,cd\cos\left(fx+e\right) - \left(c^{2}-2\,d^{2}\right)\cos\left(fx+e\right)^{2} - 2\left(c^{2}+cd+\left(cd+d^{2}\right)\cos\left(fx+e\right)\right)\cdots}{c^{2}\cos\left(fx+e\right)^{2}+2\,cd\cos\left(fx+e\right)+d^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e))*sqrt((c -
d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2
 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c
^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (a^2*c*d + a^2
*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(sin(f*x + e) + 1) - (a^2*c*d +
 a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(a^
2*c*d - a^2*d^2)*sin(f*x + e))/((c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 +
 d^4)*f), -1/2*(2*(a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e)
)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d)
)/((c - d)*sin(f*x + e))) - (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*
x + e))*log(sin(f*x + e) + 1) + (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*co
s(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^2*c*d - a^2*d^2)*sin(f*x + e))/((
c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 + d^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-a^2*1/2/d^2*ln(abs(tan((f*x+exp(1))/2)-1))+a^2*1/2/d^2*ln(abs
(tan((f*x+exp(1))/2)+1))+(-2*a^2*c^2-2*a^2*c*d+4*a^2*d^2)*1/2/(-c*d^2-d^3)/
```

```
sqrt(-c^2+d^2)*(atan((c*tan((f*x+exp(1))/2)-d*tan((f*x+exp(1))/2))/sqrt(-c^
2+d^2))+pi*sign(2*c-2*d)*floor((f*x+exp(1))/2/pi+1/2))+(-tan((f*x+exp(1))/2
)*a^2*c+tan((f*x+exp(1))/2)*a^2*d)/(-c*d-d^2)/(tan((f*x+exp(1))/2)^2*c-tan(
(f*x+exp(1))/2)^2*d-c-d))
```

maple [B] time = 0.82, size = 330, normalized size = 2.82

$$\frac{2a^2c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{fd\,(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c^2}{f\,d^2\,(c+d)\sqrt{(c+d)\,(c-d)}}-\frac{2a^2c\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\right.}{\sqrt{(c+d}}\right.}{fd\,(c+d)\sqrt{(c+d)\,(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 2/f*a^2/d*c/(c+d)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*
f*x)^2*d-c-d)-2/f*a^2/d^2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f
*x)*(c-d)/((c+d)*(c-d))^(1/2))*c^2-2/f*a^2/d*c/(c+d)/((c+d)*(c-d))^(1/2)*ar
ctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))-2/f*a^2/(c+d)*tan(1/2*e
+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)+4/f*a^2/(c+d)
/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))-
1/f*a^2/d^2*ln(tan(1/2*e+1/2*f*x)-1)+1/f*a^2/d^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 4.79, size = 2563, normalized size = 21.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (a^2*atan(((a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2
*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x
)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d
^2 + (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*
d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2))*1i)/d^2 - (a^2*((a^2*((32*
(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 +
 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)
*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*
d^3 + d^4 + c^2*d^2))*1i)/d^2)/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*c*d^3 + a^
6*c^3*d + 3*a^6*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((a^2*((32*(3*a^
2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^
4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^
5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a
^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 +
 d^4 + c^2*d^2)))/d^2 + (a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^
2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*t
an(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 +
c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d
^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2)))/d^2))*2i)/(d
^2*f) + (a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*
a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) + (a^2*
((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c
^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*
tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6
 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*
d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*((c + d)^3*(c - d))^(1/2)*(c
 + 2*d)*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*((32*tan(e/2 + (f*
x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/
(2*c*d^3 + d^4 + c^2*d^2) - (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(
3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*
c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/
2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*
(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d
^2))*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c
^3*d^2))/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*c*d^3 + a^6*c^3*d + 3*a^6*c^2*d^
2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5
*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2
*d^2) + (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(3*a^2*d^8 - 2*a^2*c*
d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^
3) - (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*(2*c*d^
8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*(3*c*d^4 + d^5 + 3*c
^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*((c + d)^3*(c -
 d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) - (a^2*((32*tan
(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*
```

```
c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2
*d)*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4
*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c
 - d))^(1/2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 +
 c^2*d^2)*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d
^3 + c^3*d^2))*((c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*
d^3 + c^3*d^2)))*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*2i)/(f*(3*c*d^4 + d^5
+ 3*c^2*d^3 + c^3*d^2)) - (2*a^2*tan(e/2 + (f*x)/2)*(c - d))/(d*f*(c + d)*(
c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{}{c^2+2cd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(2*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) + d*
*2*sec(e + f*x)**2), x))
```

3.199 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=130

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,(c+d)^{5/2}}+\frac{3a^2\tan(e+fx)}{2f(c+d)^2(c+d\sec(e+fx))}+\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c+d)(c+d\sec(e+fx))^2}$$

```
[Out] 3*a^2*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c+d)^(5/2)/f/(c-
d)^(1/2)+1/2*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+3/2
*a^2*tan(f*x+e)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 94, 93, 205}

$$-\frac{3a^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}\,(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{3a^2\tan(e+fx)}{2f(c+d)^2(c+d\sec(e+fx))}+\frac{\tan(e+fx)\left(a^2\sec(\right.}{2f(c+d)(c+d\,\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (-3*a^3*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(5/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])^2) + (3*a^2*Tan[e + f*x])/(2*(c + d)
^2*f*(c + d*Sec[e + f*x]))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\left(3a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}\,(c+\cdots}\right.}{2(c+d)f\sqrt{a-a\sec(e+fx)}\,\sqrt{\cdots}}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))} - \frac{\left(3a\cdots\right.}{\cdots}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))} - \frac{\left(3a\cdots\right.}{\cdots}$$

$$= -\frac{3a^3\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}\,(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^2+a^2\,\text{se}\cdots\right.}{2(c+d)f\cdots}$$

Mathematica [C] time = 1.25, size = 249, normalized size = 1.92

$$\frac{a^2 \sec^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(\sec(e+fx)+1)^2(c\cos(e+fx)+d)\left(-\frac{6i(\cos(e)-i\sin(e))(c\cos(e+fx)+d)^2\tan^{-1}\left(\frac{(\sin(e)+i\cos(e\ldots}{\sqrt{c^2\ldots}}\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))\ldots}}\right.}{8f(c+d)^2(c+d\sec\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Sec[e + f*x]*(1 + Sec[e + f*x]
)^2*(((-6*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f
*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x]
)^2*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((
c - d)*(c + d)*Sec[e]*(-(d*Sin[e]) + c*Sin[f*x]))/c^2 + ((d + c*Cos[e + f*x
])*Sec[e]*((c^2 - 4*c*d - 2*d^2)*Sin[e] + c*(4*c + d)*Sin[f*x]))/c^2))/(8*(
c + d)^2*f*(c + d*Sec[e + f*x])^3)
```

fricas [B] time = 0.50, size = 622, normalized size = 4.78

$$\left[\frac{3\left(a^2c^2\cos\left(fx+e\right)^2+2\,a^2cd\cos\left(fx+e\right)+a^2d^2\right)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx\ldots\right.\right.}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2\ldots}\right.}{4\left(\left(c^6+2\,c^5d-2\,c^3d^3-c^2d^4\right)f\cos\left(fx+e\right)^2+2\left(c^5d+2\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e) + a^2*d^2)*sqrt(c^
2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^
2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)
^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*
a^2*d^3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin
(f*x + e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5
*d + 2*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 -
 2*c*d^5 - d^6)*f), 1/2*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e)
 + a^2*d^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/
((c^2 - d^2)*sin(f*x + e))) + (a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^
3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin(f*x +
 e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5*d + 2
```

```
*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 - 2*c*d
^5 - d^6)*f)]
```

giac [A] time = 3.14, size = 220, normalized size = 1.69

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^2}{\left(c^2+2cd+d^2\right)\sqrt{-c^2+d^2}}+\frac{3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-5a^2d\,\mathrm{t}}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2\left(c^2+2cd+d^2\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^2/((c^2 + 2*c*d +
 d^2)*sqrt(-c^2 + d^2)) + (3*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*d*tan(1/2
*f*x + 1/2*e)^3 - 5*a^2*c*tan(1/2*f*x + 1/2*e) - 5*a^2*d*tan(1/2*f*x + 1/2*
e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2*(c^2 +
 2*c*d + d^2)))/f
```

maple [A] time = 0.77, size = 167, normalized size = 1.28

$$\frac{8a^2\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{8(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}+\frac{3\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{8(c+d)\sqrt{(c+d)(c-d)}}}{c+d}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 8/f*a^2*(1/4*tan(1/2*e+1/2*f*x)/(c+d)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2
*f*x)^2*d-c-d)^2+3/4/(c+d)*(-1/2*tan(1/2*e+1/2*f*x)/(c+d)/(tan(1/2*e+1/2*f*
x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(ta
n(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 3.57, size = 158, normalized size = 1.22

$$\frac{\frac{5\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{c+d}-\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(a^2\,c-a^2\,d\right)}{{\left(c+d\right)}^2}}{f\,\left(2\,c\,d-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-2\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(c^2-2\,c\,d+d^2\right)+c^2+d^2\right)}+\frac{3\,a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,{\left(c+d\right)}^{5/2}\,\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((5*a^2*tan(e/2 + (f*x)/2))/(c + d) - (3*tan(e/2 + (f*x)/2)^3*(a^2*c - a^2*
d))/(c + d)^2)/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 +
 (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (3*a^2*atanh((tan(e/2 + (f*
x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)^(5/2)*(c - d)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx+\int\frac{2\sec^2\left(e+\right.}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(2*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

3.200 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=213

$$\frac{a^2(3c-2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{7/2}}+\frac{a^2(3c-2d)\tan(e+fx)}{2f(c-d)(c+d)^3(c+d\sec(e+fx))}+\frac{(3c-2d)\tan(e+fx)\left(a^2\sec(e+fx\right.}{6f(c-d)(c+d)^2(c+d\sec(e+f}$$

```
[Out] a^2*(3*c-2*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c-d)^(3/
2)/(c+d)^(7/2)/f-1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f
*x+e))^3+1/6*(3*c-2*d)*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d
*sec(f*x+e))^2+1/2*a^2*(3*c-2*d)*tan(f*x+e)/(c-d)/(c+d)^3/f/(c+d*sec(f*x+e)
)
```

Rubi [A] time = 0.28, antiderivative size = 268, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3987, 96, 94, 93, 205}

$$-\frac{a^3(3c-2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2(3c-2d)\tan(e+fx)}{2f(c-d)(c+d)^3(c+d\sec(e+fx))}+\frac{(3c-2d)\tan}{6f(c-d)(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -((a^3*(3*c - 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(3/2)*(c + d)^(7/2)*f*
Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])) - (d*(a + a*Sec[e + f*x
])^2*Tan[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((3*c - 2*d)*
(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sec[e
+ f*x])^2) + (a^2*(3*c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^3*f*(c + d*S
ec[e + f*x]))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^4}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} - \frac{\left(a^2(3c-2d)\tan(e+fx)\right)\text{Subst}\left(\int\right.}{3\left(c^2-d^2\right)f\sqrt{a-a\sec(e+}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan}{6(c-d)(c+d)^2f(c+d\sec(e+}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan}{6(c-d)(c+d)^2f(c+d\sec(e+}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan}{6(c-d)(c+d)^2f(c+d\sec(e+}$$

$$= -\frac{a^3(3c-2d)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{(c-d)^{3/2}(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{d(a+a\,\text{se}}{3\left(c^2-d^2\right.}$$

Mathematica [**A**] time = 4.78, size = 211, normalized size = 0.99

$$\frac{a^2(c-d)^2\left(24(3c-2d)(c\cos(e+fx)+d)^3\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right) - 2\sqrt{c^2-d^2}\,\sin(e+fx)\left(12c^3-5c^2d+6\left(c^3\right.\right.\right.}{24f(d-c)^3(c+d)^3\sqrt{c^2-d^2}\,(c\,\text{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(c - d)^2*(24*(3*c - 2*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2
 - d^2]]*(d + c*Cos[e + f*x])^3 - 2*Sqrt[c^2 - d^2]*(12*c^3 - 5*c^2*d + 6*c
*d^2 - 22*d^3 + 6*(c^3 + 6*c^2*d - 7*c*d^2 - 2*d^3)*Cos[e + f*x] + (12*c^3
- 7*c^2*d - 6*c*d^2 - 2*d^3)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(24*(-c + d)^
3*(c + d)^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

fricas [**B**] time = 0.55, size = 1234, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^4 - 2*a^2*c^3*d)*cos(f*x + e)^
3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x + e)^2 + 3*(3*a^2*c^2*d^2 - 2*a
^2*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^4
*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a^2*c^
5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2*d^5)
*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2*d^3
+ 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d - c^
7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 + 3*(c^
8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)*f*co
s(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c
^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6
 - c^2*d^7 + 2*c*d^8 + d^9)*f), 1/6*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^
4 - 2*a^2*c^3*d)*cos(f*x + e)^3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x +
 e)^2 + 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arct
an(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (a^
2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a
^2*c^5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2
*d^5)*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2
*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d
 - c^7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 +
3*(c^8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)
*f*cos(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6
+ 2*c^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^
3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f)]
```

giac [B] time = 1.11, size = 420, normalized size = 1.97

$$-\frac{\frac{3\left(3\,a^2c-2\,a^2d\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^4+2\,c^3d-2\,cd^3-d^4\right)\sqrt{-c^2+d^2}}+\frac{9\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,a^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+21\,a^2cd^2\,\mathrm{ta}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d
) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^
2)))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*sqrt(-c^2 + d^2)) + (9*a^2*c^3*tan(1/
```

```
2*f*x + 1/2*e)^5 - 24*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 21*a^2*c*d^2*tan(1
/2*f*x + 1/2*e)^5 - 6*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c^3*tan(1/2*f
*x + 1/2*e)^3 + 16*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^2*tan(1/2*
f*x + 1/2*e)^3 - 16*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^3*tan(1/2*f*x
 + 1/2*e) + 12*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - 21*a^2*c*d^2*tan(1/2*f*x +
1/2*e) - 18*a^2*d^3*tan(1/2*f*x + 1/2*e))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*
(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

maple [A] time = 0.83, size = 228, normalized size = 1.07

$$\frac{8a^2\left(-\frac{\frac{(3c-2d)(c-d)\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{8(c+d)(c-d)}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^3}+\frac{(3c-2d)\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c+d)(c-d)}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 8/f*a^2*(-(1/8*(3*c-2*d)*(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*e+1/2*f*x)
^5-1/3*(3*c-2*d)/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3+1/8*(5*c-6*d)/(c+d)/(
c-d)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d
)^3+1/8*(3*c-2*d)/(c^4+2*c^3*d-2*c*d^3-d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan
(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 5.09, size = 286, normalized size = 1.34

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(3\,a^2\,c^2-5\,a^2\,c\,d+2\,a^2\,d^2\right)}{{(c+d)}^3}-\frac{8\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,a^2\,c-2\,a^2\,d\right)}{3\,{(c+d)}^2}+\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{(c+d)\,(c}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d^2+3\,c^2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(3*a^2*c^2 + 2*a^2*d^2 - 5*a^2*c*d))/(c + d)^3 - (8*
tan(e/2 + (f*x)/2)^3*(3*a^2*c - 2*a^2*d))/(3*(c + d)^2) + (a^2*tan(e/2 + (f
*x)/2)*(5*c - 6*d))/((c + d)*(c - d)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 -
3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3
- 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 -
3*c^2*d + c^3 - d^3))) + (2*a^2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c
 + d)^(1/2))*((3*c)/2 - d))/(f*(c + d)^(7/2)*(c - d)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c^4+4c^3d\sec\left(e+fx\right)+6c^2d^2\sec^2\left(e+fx\right)+4cd^3\sec^3\left(e+fx\right)+d^4\sec^4\left(e+fx\right)}\,dx+\int \frac{}{c^4+4c^3d\,\mathrm{se}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(2*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(se
c(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 +
 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

3.201 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=276

$$\frac{a^2\left(12c^2-16cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4f(c-d)^{5/2}(c+d)^{9/2}}+\frac{a^2\left(2c^2+16cd-21d^2\right)\tan(e+fx)}{24df(c-d)(c+d)^3(c+d\sec(e+fx))^2}+\frac{a^2\left(2c^3+16c^2d-59c\right.}{24df(c-d)^2(c+a}$$

```
[Out] 1/4*a^2*(12*c^2-16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^
(1/2))/(c-d)^(5/2)/(c+d)^(9/2)/f-1/4*a^2*(c-d)*tan(f*x+e)/d/(c+d)/f/(c+d*se
c(f*x+e))^4+1/12*a^2*(c+8*d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^3+1/24
*a^2*(2*c^2+16*c*d-21*d^2)*tan(f*x+e)/(c-d)/d/(c+d)^3/f/(c+d*sec(f*x+e))^2+
1/24*a^2*(2*c^3+16*c^2*d-59*c*d^2+32*d^3)*tan(f*x+e)/(c-d)^2/d/(c+d)^4/f/(c
+d*sec(f*x+e))
```

Rubi [A] time = 0.56, antiderivative size = 330, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 151, 12, 93, 205}

$$-\frac{a^3\left(12c^2-16cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{5/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{a^2\left(16c^2d+2c^3-59cd^2+32d^3\right)\tan(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sec(e+fx))}+\frac{}{24}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] -(a^3*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(4*(c - d)^(5/2)*(
c + d)^(9/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*(c
 - d)*Tan[e + f*x])/(4*d*(c + d)*f*(c + d*Sec[e + f*x])^4) + (a^2*(c + 8*d)
*Tan[e + f*x])/(12*d*(c + d)^2*f*(c + d*Sec[e + f*x])^3) + (a^2*(2*c^2 + 16
*c*d - 21*d^2)*Tan[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sec[e + f*x])
^2) + (a^2*(2*c^3 + 16*c^2*d - 59*c*d^2 + 32*d^3)*Tan[e + f*x])/(24*(c - d)
^2*d*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^5}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{-8a^3d-a^3(c+}{\sqrt{a-ax}\,\sqrt{a+ax}\,(}\right.}{4d(c+d)f\sqrt{a-a\sec(e+fx)}\,\sqrt{}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{\mathrm{ta}}{}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{}{2}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{}{2}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{}{2}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{}{2}$$

$$= -\frac{a^3\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{4(c-d)^{5/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{a^2}{4d(c+}$$

Mathematica [A] time = 9.06, size = 322, normalized size = 1.17

$$a^2\left(\frac{\sin(e+fx)\left(48c^5\cos(3(e+fx))+24c^5-68c^4d\cos(3(e+fx))+192c^4d-16c^3d^2\cos(3(e+fx))-446c^3d^2+5c^2d^3\cos(3(e+fx))+128c^2d^3+\left(144c^5-172c^4d+2\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*((-24*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sq
rt[c^2 - d^2]])/Sqrt[c^2 - d^2] + ((24*c^5 + 192*c^4*d - 446*c^3*d^2 + 128*
c^2*d^3 - 148*c*d^4 + 160*d^5 + (144*c^5 - 172*c^4*d + 208*c^3*d^2 - 785*c^
2*d^3 + 368*c*d^4 + 102*d^5)*Cos[e + f*x] + 2*(12*c^5 + 96*c^4*d - 227*c^3*
d^2 + 32*c^2*d^3 + 44*c*d^4 + 16*d^5)*Cos[2*(e + f*x)] + 48*c^5*Cos[3*(e +
f*x)] - 68*c^4*d*Cos[3*(e + f*x)] - 16*c^3*d^2*Cos[3*(e + f*x)] + 5*c^2*d^3
*Cos[3*(e + f*x)] + 16*c*d^4*Cos[3*(e + f*x)] + 6*d^5*Cos[3*(e + f*x)])*Sin
[e + f*x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)^2*(c + d)^4*f)
```

fricas [B] time = 0.59, size = 1908, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(3*(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*
c^5*d + 7*a^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 +
7*a^2*c^3*d^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*
c^2*d^4)*cos(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)
*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(
f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 -
d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^2*c^5*d^2 +
16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^
7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c
^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6 - 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c
^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2 - 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16
*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 6
4*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2
*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^12 + 2*c^11*d - 2*c^10
*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x +
 e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*
d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c
^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*
x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c
^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6
*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f), 1/24*(3*(1
2*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*c^5*d + 7*a
^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 7*a^2*c^3*d
^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*c^2*d^4)*co
s(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x +
e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
d^2)*sin(f*x + e))) + (2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16
*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*
```

```
a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6
- 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2
- 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2
*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16
*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*s
in(f*x + e))/((c^12 + 2*c^11*d - 2*c^10*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6
*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x + e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c
^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x
 + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c
^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 -
2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(
f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2
*d^10 - 2*c*d^11 - d^12)*f)]
```

giac [B] time = 0.65, size = 739, normalized size = 2.68

$$\frac{3\left(12\,a^2c^2-16\,a^2cd+7\,a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^6+2\,c^5d-c^4d^2-4\,c^3d^3-c^2d^4+2\,cd^5+d^6\right)\sqrt{-c^2+d^2}}-\frac{36\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-156\,a^2c^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^6 + 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 +
2*c*d^5 + d^6)*sqrt(-c^2 + d^2)) - (36*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 156
*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 273*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^7
- 243*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 111*a^2*c*d^4*tan(1/2*f*x + 1/2*
e)^7 - 21*a^2*d^5*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^5*tan(1/2*f*x + 1/2*e)
^5 + 308*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 121*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e)^5 - 231*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 253*a^2*c*d^4*tan(1/2*f*
x + 1/2*e)^5 - 77*a^2*d^5*tan(1/2*f*x + 1/2*e)^5 + 156*a^2*c^5*tan(1/2*f*x
+ 1/2*e)^3 - 116*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 345*a^2*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 199*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 189*a^2*c*d^4*ta
n(1/2*f*x + 1/2*e)^3 - 83*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^5*tan(1
/2*f*x + 1/2*e) - 36*a^2*c^4*d*tan(1/2*f*x + 1/2*e) + 177*a^2*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 147*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 81*a^2*c*d^4*tan(1
/2*f*x + 1/2*e) - 75*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 + 2*c^5*d - c^4*d^
2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(
1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

maple [A] time = 0.92, size = 352, normalized size = 1.28

$$8a^2\left(-\frac{\frac{(12c^2-16cd+7d^2)(c-d)\left(\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11(12c^2-16cd+7d^2)\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{96(c^3+3c^2d+3c\,d^2+d^3)}+\frac{(156c^2-272cd+83d^2)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{96(c-d)(c^2+2cd+d^2)}-\frac{(20c^2-48cd+25d^2)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{32(c+d)(c^2-2cd+d^2)}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^4}+\frac{(1}{32(c^6+}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x)
```

```
[Out] 8/f*a^2*(-(1/32*(12*c^2-16*c*d+7*d^2)*(c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+
d^4)*tan(1/2*e+1/2*f*x)^7-11/96*(12*c^2-16*c*d+7*d^2)/(c^3+3*c^2*d+3*c*d^2+
d^3)*tan(1/2*e+1/2*f*x)^5+1/96*(156*c^2-272*c*d+83*d^2)/(c-d)/(c^2+2*c*d+d^
2)*tan(1/2*e+1/2*f*x)^3-1/32*(20*c^2-48*c*d+25*d^2)/(c+d)/(c^2-2*c*d+d^2)*t
an(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^4+1/
32*(12*c^2-16*c*d+7*d^2)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6
)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 5.24, size = 438, normalized size = 1.59

$$\frac{\frac{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(12\,a^2\,c^2-16\,a^2\,c\,d+7\,a^2\,d^2\right)}{12\,(c+d)^3}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(12\,a^2\,c^3-28\,a^2\,c^2\,d+23\,a^2\,c\,d^2-\right.}{4\,(c+d)^4}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(6\,c^4-12\,c^2\,d^2+6\,d^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(-4\,c^4-8\,c^3\,d+8\,c\,d^3+4\,d^4\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(4\,c^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

```
[Out] ((11*tan(e/2 + (f*x)/2)^5*(12*a^2*c^2 + 7*a^2*d^2 - 16*a^2*c*d))/(12*(c + d
)^3) - (tan(e/2 + (f*x)/2)^7*(12*a^2*c^3 - 7*a^2*d^3 + 23*a^2*c*d^2 - 28*a^
2*c^2*d))/(4*(c + d)^4) - (a^2*tan(e/2 + (f*x)/2)^3*(156*c^2 - 272*c*d + 83
*d^2))/(12*(c + d)^2*(c - d)) + (a^2*tan(e/2 + (f*x)/2)*(20*c^2 - 48*c*d +
25*d^2))/(4*(c + d)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 +
 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*
d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3*d + 4*c^4 - 4*d^4) + tan(e/2 +
 (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d
 + c^4 + d^4 + 6*c^2*d^2)) + (a^2*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^
2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(12*c^2 - 16*c*d + 7*d^2
))/(4*f*(c + d)^(9/2)*(c - d)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c^5+5c^4d\sec\left(e+fx\right)+10c^3d^2\sec^2\left(e+fx\right)+10c^2d^3\sec^3\left(e+fx\right)+5cd^4\sec^4\left(e+fx\right)+d^5\sec^5\left(e+fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(2*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(sec(e + f*x)*
*3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d
**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x))
```

3.202 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=288

$$\frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f}+\frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(40c^3 \ldots}{}$$

```
[Out] 1/16*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*arctanh(sin(f*x+e))/f+1/16*a^3*(
40*c^3+90*c^2*d+78*c*d^2+23*d^3)*tan(f*x+e)/f+1/48*(40*c^3+90*c^2*d+78*c*d^
2+23*d^3)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f+1/30*a*(3*c+8*d)*(a+a*sec(f*x+e
))^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/6*a*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+
e))^3*tan(f*x+e)/f+1/120*a*(a+a*sec(f*x+e))^2*(8*c^3+148*c^2*d+132*c*d^2+42
*d^3+d*(6*c^2+62*c*d+31*d^2)*sec(f*x+e))*tan(f*x+e)/f
```

Rubi [A] time = 0.43, antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 100, 147, 50, 63, 217, 203}

$$\frac{a^3\left(90c^2d+40c^3+78cd^2+23d^3\right)\tan(e+fx)}{16f}+\frac{a^4\left(90c^2d+40c^3+78cd^2+23d^3\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+\ldots}}{\sqrt{a(\sec(e+fx\ldots}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Tan[e + f*x])/(16*f) + (a^4*(4
0*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(a + a*Sec[e
 + f*x])^2*Tan[e + f*x])/(120*f) + ((40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)
*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d*(a + a*Sec[e + f*x])^3*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(6*f) + (d*(a + a*Sec[e + f*x])^3*(70*
c^2 + 54*c*d + 19*d^2 + 4*d*(8*c + 3*d)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
```

```
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x, \sec(\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx}{6f}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx}{6f}$$

$$= \frac{a\left(40c^3+90c^2d+78cd^2+23d^3\right)(a+a\sec(e+fx))}{120f}$$

$$= \frac{a\left(40c^3+90c^2d+78cd^2+23d^3\right)(a+a\sec(e+fx))}{120f}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a\left(\right.}{}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a\left(\right.}{}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a\left(\right.}{}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a^4}{}$$

Mathematica [A] time = 2.69, size = 380, normalized size = 1.32

$$-\frac{a^3(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\left(240\left(40c^3+90c^2d+78cd^2+23d^3\right)\cos^6(e+fx)\left(\log\left(\cos\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -1/30720*(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^6*(240*(
40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Cos[e + f*x]^6*(Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(1080*
c^3 + 4770*c^2*d + 5670*c*d^2 + 2275*d^3 + 16*(305*c^3 + 945*c^2*d + 984*c*
d^2 + 344*d^3)*Cos[e + f*x] + 20*(72*c^3 + 306*c^2*d + 342*c*d^2 + 115*d^3)
*Cos[2*(e + f*x)] + 2360*c^3*Cos[3*(e + f*x)] + 6840*c^2*d*Cos[3*(e + f*x)]
 + 6384*c*d^2*Cos[3*(e + f*x)] + 1904*d^3*Cos[3*(e + f*x)] + 360*c^3*Cos[4*
(e + f*x)] + 1350*c^2*d*Cos[4*(e + f*x)] + 1170*c*d^2*Cos[4*(e + f*x)] + 34
5*d^3*Cos[4*(e + f*x)] + 440*c^3*Cos[5*(e + f*x)] + 1080*c^2*d*Cos[5*(e + f
*x)] + 912*c*d^2*Cos[5*(e + f*x)] + 272*d^3*Cos[5*(e + f*x)])*Sin[e + f*x])
)/f
```

fricas [A] time = 0.46, size = 337, normalized size = 1.17

$$\frac{15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd^2+23\,a^3d^3\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x +
 e)^6*log(sin(f*x + e) + 1) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2
+ 23*a^3*d^3)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(40*a^3*d^3 + 16*(5
5*a^3*c^3 + 135*a^3*c^2*d + 114*a^3*c*d^2 + 34*a^3*d^3)*cos(f*x + e)^5 + 15
*(24*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^4 + 1
6*(5*a^3*c^3 + 45*a^3*c^2*d + 57*a^3*c*d^2 + 17*a^3*d^3)*cos(f*x + e)^3 + 1
0*(18*a^3*c^2*d + 54*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^2 + 144*(a^3*c*d^
2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-23*a^3*d^3-78*a^3*d^2*c-90*a^3*d*c^2-40*a^3*c^3)/32*ln(abs(t
an((f*x+exp(1))/2)-1))-(-23*a^3*d^3-78*a^3*d^2*c-90*a^3*d*c^2-40*a^3*c^3)/3
2*ln(abs(tan((f*x+exp(1))/2)+1))-(345*tan((f*x+exp(1))/2)^11*a^3*d^3+1170*t
an((f*x+exp(1))/2)^11*a^3*d^2*c+1350*tan((f*x+exp(1))/2)^11*a^3*d*c^2+600*t
an((f*x+exp(1))/2)^11*a^3*c^3-1955*tan((f*x+exp(1))/2)^9*a^3*d^3-6630*tan((
f*x+exp(1))/2)^9*a^3*d^2*c-7650*tan((f*x+exp(1))/2)^9*a^3*d*c^2-3400*tan((f
*x+exp(1))/2)^9*a^3*c^3+4554*tan((f*x+exp(1))/2)^7*a^3*d^3+15444*tan((f*x+e
xp(1))/2)^7*a^3*d^2*c+17820*tan((f*x+exp(1))/2)^7*a^3*d*c^2+7920*tan((f*x+e
xp(1))/2)^7*a^3*c^3-5814*tan((f*x+exp(1))/2)^5*a^3*d^3-17964*tan((f*x+exp(1
))/2)^5*a^3*d^2*c-22500*tan((f*x+exp(1))/2)^5*a^3*d*c^2-9360*tan((f*x+exp(1
))/2)^5*a^3*c^3+3165*tan((f*x+exp(1))/2)^3*a^3*d^3+12570*tan((f*x+exp(1))/2
)^3*a^3*d^2*c+15390*tan((f*x+exp(1))/2)^3*a^3*d*c^2+5560*tan((f*x+exp(1))/2
)^3*a^3*c^3-1575*tan((f*x+exp(1))/2)*a^3*d^3-4590*tan((f*x+exp(1))/2)*a^3*d
^2*c-4410*tan((f*x+exp(1))/2)*a^3*d*c^2-1320*tan((f*x+exp(1))/2)*a^3*c^3)*1
/240/(tan((f*x+exp(1))/2)^2-1)^6)
```

maple [A] time = 1.95, size = 523, normalized size = 1.82

$$\frac{23a^3d^3\tan\left(fx+e\right)\left(\sec^3\left(fx+e\right)\right)}{24f}+\frac{23a^3d^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{16f}+\frac{3a^3c^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{a^3d^3\ \mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 23/24/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^3+23/16/f*a^3*d^3*sec(f*x+e)*tan(f*x+
e)+3/2*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+1/6/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^
5+45/8/f*a^3*c^2*d*sec(f*x+e)*tan(f*x+e)+9/4/f*a^3*c*d^2*tan(f*x+e)*sec(f*x
+e)^3+34/15/f*a^3*d^3*tan(f*x+e)+17/15/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^2+3/
5/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^4+1/3/f*a^3*c^3*tan(f*x+e)*sec(f*x+e)^2+3
/4/f*a^3*c^2*d*tan(f*x+e)*sec(f*x+e)^3+38/5/f*a^3*c*d^2*tan(f*x+e)+19/5/f*a
^3*c*d^2*tan(f*x+e)*sec(f*x+e)^2+3/f*a^3*c^2*d*tan(f*x+e)*sec(f*x+e)^2+3/5/
f*a^3*c*d^2*tan(f*x+e)*sec(f*x+e)^4+9/f*a^3*c^2*d*tan(f*x+e)+39/8/f*a^3*c*d
^2*ln(sec(f*x+e)+tan(f*x+e))+45/8/f*a^3*c^2*d*ln(sec(f*x+e)+tan(f*x+e))+39/
8/f*a^3*c*d^2*sec(f*x+e)*tan(f*x+e)+5/2/f*a^3*c^3*ln(sec(f*x+e)+tan(f*x+e))
+11/3*a^3*c^3*tan(f*x+e)/f+23/16/f*a^3*d^3*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [B] time = 0.58, size = 701, normalized size = 2.43

$$\frac{160\left(\tan\left(fx+e\right)^3+3\ \tan\left(fx+e\right)\right)a^3c^3+1440\left(\tan\left(fx+e\right)^3+3\ \tan\left(fx+e\right)\right)a^3c^2d+96\left(3\ \tan\left(fx+e\right)^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/480*(160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 + 1440*(tan(f*x + e)^3
 + 3*tan(f*x + e))*a^3*c^2*d + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 1
5*tan(f*x + e))*a^3*c*d^2 + 1440*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c*d^
2 + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*d^3 + 1
60*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^3 - 5*a^3*d^3*(2*(15*sin(f*x + e
)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
 - 1)) - 90*a^3*c^2*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) -
1)) - 270*a^3*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) - 90*a^3*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 3
60*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 1080*a^3*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1)
 - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a^3*c*d^2*(2*sin(f*
x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)
) + 480*a^3*c^3*log(sec(f*x + e) + tan(f*x + e)) + 1440*a^3*c^3*tan(f*x + e
) + 1440*a^3*c^2*d*tan(f*x + e))/f
```

mupad [B] time = 5.24, size = 411, normalized size = 1.43

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{4\left(10\,c^3+\frac{45\,c^2\,d}{2}+\frac{39\,c\,d^2}{2}+\frac{23\,d^3}{4}\right)}\right)\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{8\,f}-\frac{\left(5\,a^3\,c^3+\frac{45\,a^3\,c^2\,d}{4}+\frac{39\,a^3\,c\,d^2}{4}+23\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (a^3*atanh((tan(e/2 + (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(4*
((39*c*d^2)/2 + (45*c^2*d)/2 + 10*c^3 + (23*d^3)/4)))*(78*c*d^2 + 90*c^2*d
+ 40*c^3 + 23*d^3))/(8*f) - (tan(e/2 + (f*x)/2)^11*(5*a^3*c^3 + (23*a^3*d^3
)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*((85*a^3*
c^3)/3 + (391*a^3*d^3)/24 + (221*a^3*c*d^2)/4 + (255*a^3*c^2*d)/4) + tan(e/
2 + (f*x)/2)^3*((139*a^3*c^3)/3 + (211*a^3*d^3)/8 + (419*a^3*c*d^2)/4 + (51
3*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^7*(66*a^3*c^3 + (759*a^3*d^3)/20 + (12
87*a^3*c*d^2)/10 + (297*a^3*c^2*d)/2) - tan(e/2 + (f*x)/2)^5*(78*a^3*c^3 +
```

```
(969*a^3*d^3)/20 + (1497*a^3*c*d^2)/10 + (375*a^3*c^2*d)/2) - tan(e/2 + (f*
x)/2)*(11*a^3*c^3 + (105*a^3*d^3)/8 + (153*a^3*c*d^2)/4 + (147*a^3*c^2*d)/4
))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x
)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x
)/2)^12 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c^3\sec\left(e+fx\right)dx+\int 3c^3\sec^2\left(e+fx\right)dx+\int 3c^3\sec^3\left(e+fx\right)dx+\int c^3\sec^4\left(e+fx\right)dx+\int d^3\sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(c**3*sec(e + f*x), x) + Integral(3*c**3*sec(e + f*x)**2, x)
+ Integral(3*c**3*sec(e + f*x)**3, x) + Integral(c**3*sec(e + f*x)**4, x) +
 Integral(d**3*sec(e + f*x)**4, x) + Integral(3*d**3*sec(e + f*x)**5, x) +
Integral(3*d**3*sec(e + f*x)**6, x) + Integral(d**3*sec(e + f*x)**7, x) + I
ntegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(9*c*d**2*sec(e + f*x)**4, x
) + Integral(9*c*d**2*sec(e + f*x)**5, x) + Integral(3*c*d**2*sec(e + f*x)*
*6, x) + Integral(3*c**2*d*sec(e + f*x)**2, x) + Integral(9*c**2*d*sec(e +
f*x)**3, x) + Integral(9*c**2*d*sec(e + f*x)**4, x) + Integral(3*c**2*d*sec
(e + f*x)**5, x))
```

3.203 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=257

$$\frac{a^3\left(20c^2+30cd+13d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3\left(2c^2-15cd+76d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{60d^2f}+\frac{a^3\left(4c^3-\right.}{}$$

```
[Out] 1/8*a^3*(20*c^2+30*c*d+13*d^2)*arctanh(sin(f*x+e))/f+1/30*a^3*(2*c^4-15*c^3
*d+72*c^2*d^2+180*c*d^3+76*d^4)*tan(f*x+e)/d^2/f+1/120*a^3*(4*c^3-30*c^2*d+
146*c*d^2+195*d^3)*sec(f*x+e)*tan(f*x+e)/d/f+1/60*a^3*(2*c^2-15*c*d+76*d^2)
*(c+d*sec(f*x+e))^2*tan(f*x+e)/d^2/f-1/20*a^3*(2*c-11*d)*(c+d*sec(f*x+e))^3
*tan(f*x+e)/d^2/f+1/5*(a^3+a^3*sec(f*x+e))*(c+d*sec(f*x+e))^3*tan(f*x+e)/d/
f
```

Rubi [A] time = 0.30, antiderivative size = 273, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 90, 80, 50, 63, 217, 203}

$$\frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f}+\frac{a^4\left(20c^2+30cd+13d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{\left(20c^2+30cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*Tan[e + f*x])/(8*f) + (a^4*(20*c^2 + 30*c*d
 + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[
e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(20*
c^2 + 30*c*d + 13*d^2)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + (3*d*(
2*c + d)*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + ((20*c^2 + 30*c*d +
13*d^2)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f
*x])^3*(c + d*Sec[e + f*x])*Tan[e + f*x])/(5*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^2}{\sqrt{a-ax}}\,dx, x, \sec(e\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f}$$

$$= \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{d(a+a}{}$$

$$= \frac{a\left(20c^2+30cd+13d^2\right)(a+a\sec(e+fx))^2\tan(e+f}{60f}$$

$$= \frac{a\left(20c^2+30cd+13d^2\right)(a+a\sec(e+fx))^2\tan(e+f}{60f}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a\left(20c^2+30c\right.}{}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a\left(20c^2+30c\right.}{}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a\left(20c^2+30c\right.}{}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a^4\left(20c^2+30\right.}{4f\sqrt{a}}$$

Mathematica [A] time = 2.69, size = 433, normalized size = 1.68

$$-\frac{a^3(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\left(240\left(20c^2+30cd+13d^2\right)\cos^5(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] -1/15360*(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^5*(240*(
20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - Sec[e]*(80*(34*c^2 +
60*c*d + 29*d^2)*Sin[f*x] - 240*(7*c^2 + 10*c*d + 3*d^2)*Sin[2*e + f*x] + 3
60*c^2*Sin[e + 2*f*x] + 1140*c*d*Sin[e + 2*f*x] + 750*d^2*Sin[e + 2*f*x] +
360*c^2*Sin[3*e + 2*f*x] + 1140*c*d*Sin[3*e + 2*f*x] + 750*d^2*Sin[3*e + 2*
f*x] + 1840*c^2*Sin[2*e + 3*f*x] + 3360*c*d*Sin[2*e + 3*f*x] + 1520*d^2*Sin
[2*e + 3*f*x] - 360*c^2*Sin[4*e + 3*f*x] - 240*c*d*Sin[4*e + 3*f*x] + 180*c
^2*Sin[3*e + 4*f*x] + 450*c*d*Sin[3*e + 4*f*x] + 195*d^2*Sin[3*e + 4*f*x] +
 180*c^2*Sin[5*e + 4*f*x] + 450*c*d*Sin[5*e + 4*f*x] + 195*d^2*Sin[5*e + 4*
f*x] + 440*c^2*Sin[4*e + 5*f*x] + 720*c*d*Sin[4*e + 5*f*x] + 304*d^2*Sin[4*
e + 5*f*x])))/f
```

fricas [A] time = 0.47, size = 245, normalized size = 0.95

$$15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\cos\left(fx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(sin(f*x
 + e) + 1) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(-
sin(f*x + e) + 1) + 2*(24*a^3*d^2 + 8*(55*a^3*c^2 + 90*a^3*c*d + 38*a^3*d^2
)*cos(f*x + e)^4 + 15*(12*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^3
 + 8*(5*a^3*c^2 + 30*a^3*c*d + 19*a^3*d^2)*cos(f*x + e)^2 + 30*(2*a^3*c*d +
 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(13*a^3*d^2+30*a^3*d*c+20*a^3*c^2)/16*ln(abs(tan((f*x+exp(1))
/2)-1))+(13*a^3*d^2+30*a^3*d*c+20*a^3*c^2)/16*ln(abs(tan((f*x+exp(1))/2)+1)
)-(195*tan((f*x+exp(1))/2)^9*a^3*d^2+450*tan((f*x+exp(1))/2)^9*a^3*d*c+300*
```

```
tan((f*x+exp(1))/2)^9*a^3*c^2-910*tan((f*x+exp(1))/2)^7*a^3*d^2-2100*tan((f
*x+exp(1))/2)^7*a^3*d*c-1400*tan((f*x+exp(1))/2)^7*a^3*c^2+1664*tan((f*x+ex
p(1))/2)^5*a^3*d^2+3840*tan((f*x+exp(1))/2)^5*a^3*d*c+2560*tan((f*x+exp(1))
/2)^5*a^3*c^2-1330*tan((f*x+exp(1))/2)^3*a^3*d^2-3660*tan((f*x+exp(1))/2)^3
*a^3*d*c-2120*tan((f*x+exp(1))/2)^3*a^3*c^2+765*tan((f*x+exp(1))/2)*a^3*d^2
+1470*tan((f*x+exp(1))/2)*a^3*d*c+660*tan((f*x+exp(1))/2)*a^3*c^2)*1/120/(t
an((f*x+exp(1))/2)^2-1)^5)
```

maple [A] time = 1.72, size = 342, normalized size = 1.33

$$\frac{5a^3c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{6a^3cd\tan\left(fx+e\right)}{f}+\frac{13a^3d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{13a^3d^2\ln\left(\sec\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 5/2/f*a^3*c^2*ln(sec(f*x+e)+tan(f*x+e))+6/f*a^3*c*d*tan(f*x+e)+13/8/f*a^3*d
^2*sec(f*x+e)*tan(f*x+e)+13/8/f*a^3*d^2*ln(sec(f*x+e)+tan(f*x+e))+11/3*a^3*
c^2*tan(f*x+e)/f+15/4/f*a^3*c*d*sec(f*x+e)*tan(f*x+e)+15/4/f*a^3*c*d*ln(sec
(f*x+e)+tan(f*x+e))+38/15/f*a^3*d^2*tan(f*x+e)+19/15/f*a^3*d^2*tan(f*x+e)*s
ec(f*x+e)^2+3/2*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+2/f*a^3*c*d*tan(f*x+e)*sec(
f*x+e)^2+3/4/f*a^3*d^2*tan(f*x+e)*sec(f*x+e)^3+1/3/f*a^3*c^2*tan(f*x+e)*sec
(f*x+e)^2+1/2/f*a^3*c*d*tan(f*x+e)*sec(f*x+e)^3+1/5/f*a^3*d^2*tan(f*x+e)*se
c(f*x+e)^4
```

maxima [A] time = 0.43, size = 459, normalized size = 1.79

$$\frac{80\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3c^2+480\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3cd+16\left(3\tan\left(fx+e\right)^5+10\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 480*(tan(f*x + e)^3 +
 3*tan(f*x + e))*a^3*c*d + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^3*d^2 + 240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^2 - 30*a^
3*c*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 45*a^3*d^2
*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2
+ 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 180*a^3*c^2*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
```

```
) - 1)) - 360*a^3*c*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 60*a^3*d^2*(2*sin(f*x + e)/(sin(f*x + e)
^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^3*c^2*log(
sec(f*x + e) + tan(f*x + e)) + 720*a^3*c^2*tan(f*x + e) + 480*a^3*c*d*tan(f
*x + e))/f
```

mupad [B] time = 5.52, size = 287, normalized size = 1.12

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(20\,c^2+30\,c\,d+13\,d^2\right)}{2\left(10\,c^2+15\,c\,d+\frac{13\,d^2}{2}\right)}\right)\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\,f}-\frac{\left(5\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-\frac{70}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a^3*atanh((tan(e/2 + (f*x)/2)*(30*c*d + 20*c^2 + 13*d^2))/(2*(15*c*d + 10*
c^2 + (13*d^2)/2)))*(30*c*d + 20*c^2 + 13*d^2))/(4*f) - (tan(e/2 + (f*x)/2)
*(11*a^3*c^2 + (51*a^3*d^2)/4 + (49*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^9*(5*a
^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^7*((70*a^3*c
^2)/3 + (91*a^3*d^2)/6 + 35*a^3*c*d) - tan(e/2 + (f*x)/2)^3*((106*a^3*c^2)/
3 + (133*a^3*d^2)/6 + 61*a^3*c*d) + tan(e/2 + (f*x)/2)^5*((128*a^3*c^2)/3 +
 (416*a^3*d^2)/15 + 64*a^3*c*d))/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 +
(f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (
f*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c^2\sec\left(e+fx\right)dx+\int 3c^2\sec^2\left(e+fx\right)dx+\int 3c^2\sec^3\left(e+fx\right)dx+\int c^2\sec^4\left(e+fx\right)dx+\int d^2\sec\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(c**2*sec(e + f*x), x) + Integral(3*c**2*sec(e + f*x)**2, x)
+ Integral(3*c**2*sec(e + f*x)**3, x) + Integral(c**2*sec(e + f*x)**4, x) +
 Integral(d**2*sec(e + f*x)**3, x) + Integral(3*d**2*sec(e + f*x)**4, x) +
Integral(3*d**2*sec(e + f*x)**5, x) + Integral(d**2*sec(e + f*x)**6, x) + I
ntegral(2*c*d*sec(e + f*x)**2, x) + Integral(6*c*d*sec(e + f*x)**3, x) + In
tegral(6*c*d*sec(e + f*x)**4, x) + Integral(2*c*d*sec(e + f*x)**5, x))
```

3.204 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=125

$$\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}+\frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3a^3(4c+3d)\tan(e+fx)}{8f}$$

```
[Out] 5/8*a^3*(4*c+3*d)*arctanh(sin(f*x+e))/f+a^3*(4*c+3*d)*tan(f*x+e)/f+3/8*a^3*
(4*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/f+1/1
2*a^3*(4*c+3*d)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}+\frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3a^3(4c+3d)\tan(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*(4*c + 3*d)*ArcTanh[Sin[e + f*x]])/(8*f) + (a^3*(4*c + 3*d)*Tan[e +
f*x])/f + (3*a^3*(4*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (d*(a + a*S
ec[e + f*x])^3*Tan[e + f*x])/(4*f) + (a^3*(4*c + 3*d)*Tan[e + f*x]^3)/(12*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx = \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}(4c+3d)\int \text{se}$$

$$= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}(4c+3d)\int \Big(a$$

$$= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\left(a^3(4c+3d)\right)$$

$$= \frac{a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{4f}+\frac{3a^3(4c+3d)\sec(}{8}$$

$$= \frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3(4c+3d)\tan(}{f}$$

Mathematica [B] time = 1.37, size = 273, normalized size = 2.18

$$-\frac{a^3(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\left(120(4c+3d)\cos^4(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

```
[Out] -1/1536*(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^4*(120*(4
*c + 3*d)*Cos[e + f*x]^4*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]]) - Sec[e]*(-24*(11*c + 9*d)*Sin[e] + (36
*c + 69*d)*Sin[f*x] + 36*c*Sin[2*e + f*x] + 69*d*Sin[2*e + f*x] + 280*c*Sin
[e + 2*f*x] + 264*d*Sin[e + 2*f*x] - 72*c*Sin[3*e + 2*f*x] - 24*d*Sin[3*e +
 2*f*x] + 36*c*Sin[2*e + 3*f*x] + 45*d*Sin[2*e + 3*f*x] + 36*c*Sin[4*e + 3*
f*x] + 45*d*Sin[4*e + 3*f*x] + 88*c*Sin[3*e + 4*f*x] + 72*d*Sin[3*e + 4*f*x
])))/f
```

fricas [A] time = 0.43, size = 161, normalized size = 1.29

$$\frac{15\left(4\,a^3c+3\,a^3d\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\left(4\,a^3c+3\,a^3d\right)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*(4*a^3*c + 3*a^3*d)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*(4*a
^3*c + 3*a^3*d)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*(6*a^3*d + 8*(11*
a^3*c + 9*a^3*d)*cos(f*x + e)^3 + 9*(4*a^3*c + 5*a^3*d)*cos(f*x + e)^2 + 8*
(a^3*c + 3*a^3*d)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-20*a^3*c-15*a^3*d)/16*ln(abs(tan((f*x+exp(1))/2)-1))-(-20*a^
3*c-15*a^3*d)/16*ln(abs(tan((f*x+exp(1))/2)+1))+(-60*tan((f*x+exp(1))/2)^7*
a^3*c-45*tan((f*x+exp(1))/2)^7*a^3*d+220*tan((f*x+exp(1))/2)^5*a^3*c+165*ta
n((f*x+exp(1))/2)^5*a^3*d-292*tan((f*x+exp(1))/2)^3*a^3*c-219*tan((f*x+exp(
1))/2)^3*a^3*d+132*tan((f*x+exp(1))/2)*a^3*c+147*tan((f*x+exp(1))/2)*a^3*d)
*1/24/(tan((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.39, size = 188, normalized size = 1.50

$$\frac{5a^3c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+\frac{3a^3d\tan\left(fx+e\right)}{f}+\frac{11a^3c\tan\left(fx+e\right)}{3f}+\frac{15a^3d\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x)
```

```
[Out] 5/2/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+3/f*a^3*d*tan(f*x+e)+11/3*a^3*c*tan(f
*x+e)/f+15/8/f*a^3*d*sec(f*x+e)*tan(f*x+e)+15/8/f*a^3*d*ln(sec(f*x+e)+tan(f
*x+e))+3/2*a^3*c*sec(f*x+e)*tan(f*x+e)/f+1/f*a^3*d*tan(f*x+e)*sec(f*x+e)^2+
1/3/f*a^3*c*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a^3*d*tan(f*x+e)*sec(f*x+e)^3
```

maxima [B] time = 0.42, size = 262, normalized size = 2.10

$$\frac{16\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)a^{3}c+48\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)a^{3}d-3a^{3}d\left(\frac{2\left(3\sin\left(fx+e\right)^{3}-5\sin\left(fx+e\ldots}{\sin\left(fx+e\right)^{4}-2\sin\left(fx+e\right)^{2}+\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c + 48*(tan(f*x + e)^3 + 3*t
an(f*x + e))*a^3*d - 3*a^3*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*
x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x
+ e) - 1)) - 36*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 36*a^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a^3*c*log(sec(f
*x + e) + tan(f*x + e)) + 144*a^3*c*tan(f*x + e) + 48*a^3*d*tan(f*x + e))/f
```

mupad [B] time = 5.31, size = 203, normalized size = 1.62

$$\frac{\left(-5a^{3}c-\frac{15a^{3}d}{4}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}+\left(\frac{55a^{3}c}{3}+\frac{55a^{3}d}{4}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}+\left(-\frac{73a^{3}c}{3}-\frac{73a^{3}d}{4}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}+\left(11a^{3}c\ldots\right.}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(11*a^3*c + (49*a^3*d)/4) - tan(e/2 + (f*x)/2)^7*(5*a^3
*c + (15*a^3*d)/4) + tan(e/2 + (f*x)/2)^5*((55*a^3*c)/3 + (55*a^3*d)/4) - t
an(e/2 + (f*x)/2)^3*((73*a^3*c)/3 + (73*a^3*d)/4))/(f*(6*tan(e/2 + (f*x)/2)
^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8
 + 1)) + (5*a^3*atanh((5*tan(e/2 + (f*x)/2)*(4*c + 3*d))/(2*(10*c + (15*d)/
2)))*(4*c + 3*d))/(4*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c\sec\left(e+fx\right)dx+\int 3c\sec^2\left(e+fx\right)dx+\int 3c\sec^3\left(e+fx\right)dx+\int c\sec^4\left(e+fx\right)dx+\int d\sec^2\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(c*sec(e + f*x), x) + Integral(3*c*sec(e + f*x)**2, x) + Inte
gral(3*c*sec(e + f*x)**3, x) + Integral(c*sec(e + f*x)**4, x) + Integral(d*
sec(e + f*x)**2, x) + Integral(3*d*sec(e + f*x)**3, x) + Integral(3*d*sec(e
 + f*x)**4, x) + Integral(d*sec(e + f*x)**5, x))
```

3.205 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=153

$$\frac{a^3\left(c^2-3cd+3d^2\right)\tanh^{-1}(\sin(e+fx))}{d^3f}-\frac{2a^3(c-d)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3f\sqrt{c+d}}-\frac{a^3(c-3d)\tan(e+fx)}{d^2f}+\frac{a^3\tan}{}$$

```
[Out] 1/2*a^3*arctanh(sin(f*x+e))/d/f+a^3*(c^2-3*c*d+3*d^2)*arctanh(sin(f*x+e))/d
^3/f-2*a^3*(c-d)^(5/2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/
d^3/f/(c+d)^(1/2)-a^3*(c-3*d)*tan(f*x+e)/d^2/f+1/2*a^3*sec(f*x+e)*tan(f*x+e
)/d/f
```

Rubi [A] time = 0.32, antiderivative size = 257, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3987, 102, 154, 157, 63, 217, 203, 93, 205}

$$\frac{a^4\left(2c^2-6cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f}+\frac{2a^4(c-d)^{5/2}\tan(e+fx)\tan}{d^3f\sqrt{c+d}\sqrt{a-a\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] -(a^3*(2*c - 5*d)*Tan[e + f*x])/(2*d^2*f) + (a^4*(2*c^2 - 6*c*d + 7*d^2)*Ar
cTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^
3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*(c - d)^(5/
2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Se
c[e + f*x]])]*Tan[e + f*x])/(d^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]]) + ((a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(2*d*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{a+ax}\left(-a^3\right.}{\sqrt{a}}\right.}{2df\sqrt{a-a\sec(e+f}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} - \frac{\tan(e}{}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(a^5(c\right.}{}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(2a^5(c\right.}{}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{2a^4(c-d)^{5/2}\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan}{d^3\sqrt{c+d}\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{a^4\left(2c^2-6cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e}}$$

Mathematica [C] time = 2.41, size = 419, normalized size = 2.74

$$\frac{a^3\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(-2\left(2c^2-6cd+7d^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*
x])^3*(-2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (8*(
c - d)^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/
2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(I*Cos[e] + Sin[e]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + d^2/(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])) - d^2/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]))))/(32*d^3*f*(c + d*Sec[e + f*x]))
```

fricas [A] time = 0.89, size = 532, normalized size = 3.48

$$\left[\frac{2\left(a^3c^2-2\,a^3cd+a^3d^2\right)\sqrt{\frac{c-d}{c+d}}\,\cos\left(fx+e\right)^2\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2-2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\,\sin\left(fx\right.}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt((c - d)/(c + d))*cos(f*x + e)^
2*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (
c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/
(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a^3*d^2 - 2*(a^3*c*d
 - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f*x + e)^2), -1/4*(4*(
a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x +
```

```
e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e)))*cos(f*x + e)^2 - (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*
(a^3*d^2 - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f
*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((2*a^3*c^3-6*a^3*c^2*d+6*a^3*c*d^2-2*a^3*d^3)*1/2/d^3/sqrt(-c^
2+d^2)*(atan((c*tan((f*x+exp(1))/2)-d*tan((f*x+exp(1))/2))/sqrt(-c^2+d^2))+
pi*sign(2*c-2*d)*floor((f*x+exp(1))/2/pi+1/2))+(-2*a^3*c^2+6*a^3*c*d-7*a^3*
d^2)*1/4/d^3*ln(abs(tan((f*x+exp(1))/2)-1))-(-2*a^3*c^2+6*a^3*c*d-7*a^3*d^2
)*1/4/d^3*ln(abs(tan((f*x+exp(1))/2)+1))-(-2*tan((f*x+exp(1))/2)^3*a^3*c+5*
tan((f*x+exp(1))/2)^3*a^3*d+2*tan((f*x+exp(1))/2)*a^3*c-7*tan((f*x+exp(1))/
2)*a^3*d)*1/2/d^2/(tan((f*x+exp(1))/2)^2-1)^2)
```

maple [B] time = 0.67, size = 491, normalized size = 3.21

$$-\frac{2a^3 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c^3}{f\,d^3\sqrt{(c+d)\,(c-d)}}+\frac{6a^3 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c^2}{f\,d^2\sqrt{(c+d)\,(c-d)}}-\frac{6a^3 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)c}{f\,d\sqrt{(c+d)\,(c-d)}}+\frac{2a^3 \operatorname{arctanh}\left(\cdots\right.}{f\sqrt{(c+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x)
```

```
[Out] -2/f*a^3/d^3/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c
-d))^(1/2))*c^3+6/f*a^3/d^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*
(c-d)/((c+d)*(c-d))^(1/2))*c^2-6/f*a^3/d/((c+d)*(c-d))^(1/2)*arctanh(tan(1/
2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*c+2/f*a^3/((c+d)*(c-d))^(1/2)*arcta
nh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))+1/2/f*a^3/d/(tan(1/2*e+1/2
*f*x)-1)^2+1/f*a^3/d^2/(tan(1/2*e+1/2*f*x)-1)*c-5/2/f*a^3/d/(tan(1/2*e+1/2*
f*x)-1)-1/f*a^3/d^3*ln(tan(1/2*e+1/2*f*x)-1)*c^2+3/f*a^3/d^2*ln(tan(1/2*e+1
/2*f*x)-1)*c-7/2/f*a^3/d*ln(tan(1/2*e+1/2*f*x)-1)-1/2/f*a^3/d/(tan(1/2*e+1/
2*f*x)+1)^2+1/f*a^3/d^2/(tan(1/2*e+1/2*f*x)+1)*c-5/2/f*a^3/d/(tan(1/2*e+1/2
```

```
*f*x)+1)+1/f*a^3/d^3*ln(tan(1/2*e+1/2*f*x)+1)*c^2-3/f*a^3/d^2*ln(tan(1/2*e+
1/2*f*x)+1)*c+7/2/f*a^3/d*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.89, size = 1902, normalized size = 12.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (atanh((18824*a^9*c^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (
16680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/
d^4 - 11560*a^9*c*d) - (16680*a^9*c^3*tan(e/2 + (f*x)/2))/(2968*a^9*d^3 - 1
6680*a^9*c^3 - 11560*a^9*c*d^2 + 18824*a^9*c^2*d + (8608*a^9*c^4)/d - (2480
*a^9*c^5)/d^2 + (320*a^9*c^6)/d^3) + (8608*a^9*c^4*tan(e/2 + (f*x)/2))/(860
8*a^9*c^4 + 2968*a^9*d^4 - 11560*a^9*c*d^3 - 16680*a^9*c^3*d + 18824*a^9*c^
2*d^2 - (2480*a^9*c^5)/d + (320*a^9*c^6)/d^2) - (2480*a^9*c^5*tan(e/2 + (f*
x)/2))/(2968*a^9*d^5 - 2480*a^9*c^5 - 11560*a^9*c*d^4 + 8608*a^9*c^4*d + 18
824*a^9*c^2*d^3 - 16680*a^9*c^3*d^2 + (320*a^9*c^6)/d) + (320*a^9*c^6*tan(e
/2 + (f*x)/2))/(320*a^9*c^6 + 2968*a^9*d^6 - 11560*a^9*c*d^5 - 2480*a^9*c^5
*d + 18824*a^9*c^2*d^4 - 16680*a^9*c^3*d^3 + 8608*a^9*c^4*d^2) + (2968*a^9*
d^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16680*a^9*c^3)/d +
 (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^4 - 11560*a^9*c*
d) - (11560*a^9*c*d*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16
680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^
4 - 11560*a^9*c*d))*(2*a^3*c^2 + 7*a^3*d^2 - 6*a^3*c*d))/(d^3*f) - ((tan(e/
2 + (f*x)/2)*(2*a^3*c - 7*a^3*d))/d^2 - (a^3*tan(e/2 + (f*x)/2)^3*(2*c - 5*
d))/d^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) - (a^3*ata
n(((a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^
6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 -
492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 + (a^3*((c + d)*(c - d)^5)^(1/2)*((
8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^4
```

```
*d^6))/d^6 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8 -
 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d)))*1i)/(d^3*(c + d))
+ (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6
*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 - 4
92*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 - (a^3*((c + d)*(c - d)^5)^(1/2)*((8
*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^4*
d^6))/d^6 + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8 -
16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d)))*1i)/(d^3*(c + d)))/
((16*(4*a^9*c^8 + 35*a^9*d^8 - 219*a^9*c*d^7 - 42*a^9*c^7*d + 592*a^9*c^2*d
^6 - 904*a^9*c^3*d^5 + 855*a^9*c^4*d^4 - 515*a^9*c^5*d^3 + 194*a^9*c^6*d^2)
)/d^6 - (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 -
53*a^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d
^4 - 492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 + (a^3*((c + d)*(c - d)^5)^(1/
2)*((8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^
3*c^4*d^6))/d^6 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*
d^8 - 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d))))/(d^3*(c + d)
) + (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a
^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 -
 492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 - (a^3*((c + d)*(c - d)^5)^(1/2)*(
(8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^
4*d^6))/d^6 + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8
- 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d))))/(d^3*(c + d))))*
((c + d)*(c - d)^5)^(1/2)*2i)/(d^3*f*(c + d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{3\sec^3\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{\sec^4\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f*x)**3/(c + d*sec(e +
 f*x)), x) + Integral(sec(e + f*x)**4/(c + d*sec(e + f*x)), x))
```

3.206 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^3(2c-3d)\tanh^{-1}(\sin(e+fx))}{d^3f}+\frac{2a^3(c-d)^{3/2}(2c+3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3f(c+d)^{3/2}}+\frac{2a^3c\tan(e+fx)}{d^2f(c+d)}-\frac{(c-d)\,\mathrm{ta}}{df(}$$

```
[Out] -a^3*(2*c-3*d)*arctanh(sin(f*x+e))/d^3/f+2*a^3*(c-d)^(3/2)*(2*c+3*d)*arctan
h((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/d^3/(c+d)^(3/2)/f+2*a^3*c*tan
(f*x+e)/d^2/(c+d)/f-(c-d)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*se
c(f*x+e))
```

Rubi [A] time = 0.35, antiderivative size = 274, normalized size of antiderivative = 1.70, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3987, 98, 154, 157, 63, 217, 203, 93, 205}

$$\frac{2a^3c\tan(e+fx)}{d^2f(c+d)}-\frac{2a^4(2c-3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2a^4(c-d)^{3/2}(2c+3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+}}{\sqrt{c-}}\right.}{d^3f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*c*Tan[e + f*x])/(d^2*(c + d)*f) - (2*a^4*(2*c - 3*d)*ArcTan[Sqrt[a -
 a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^3*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*(c - d)^(3/2)*(2*c + 3*d
)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec
[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(
d*(c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.}{d(c+d)f\sqrt{a-a\sec}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\tan(e+f}{d^2}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{\left(a^5(2c-\right.}{}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\left(2a^4(2c-\right.}{}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(c-d)^{3/2}(2c+3d)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(}{d^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+f}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(2c-3d)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{2a^4}{}$$

Mathematica [C] time = 4.21, size = 455, normalized size = 2.83

$$a^3\cos(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(-\frac{2i(2c+3d)(c-d)^2(\cos(e)-i\sin(e))(c\cos(e+fx)+d)\tan^{-}\cdots}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x]
)^3*((2*c - 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + (-2*c + 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]] - ((2*I)*(c - d)^2*(2*c + 3*d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e]
+ (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^
2])]*(d + c*Cos[e + f*x])*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqr
t[(Cos[e] - I*Sin[e])^2]) + ((c - d)^2*d*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c
+ d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (d*(d + c*Cos[e + f*x])
*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
) + (d*(d + c*Cos[e + f*x])*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]))))/(8*d^3*f*(c + d*Sec[e + f*x])^2)
```

fricas [B] time = 0.92, size = 859, normalized size = 5.34

$$\left[-\frac{\left(\left(2a^3c^3+a^3c^2d-3a^3cd^2\right)\cos\left(fx+e\right)^2+\left(2a^3c^2d+a^3cd^2-3a^3d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2cd\cos\left(fx+e\right)-\left(c\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d
 + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*co
s(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(
f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*
cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) - ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (
```

```
2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) -
 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))
*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*
x + e)), 1/2*(2*((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*
a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arc
tan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) -
((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*
c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) + ((2*a^3*c^3 - a^3*
c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)
*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2
*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*
cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(-2*a^3*c+3*a^3*d)*1/2/d^3*ln(abs(tan((f*x+exp(1))/2)-1))+(-2
*a^3*c+3*a^3*d)*1/2/d^3*ln(abs(tan((f*x+exp(1))/2)+1))+(-4*a^3*c^3+2*a^3*c^
2*d+8*a^3*c*d^2-6*a^3*d^3)*1/2/(c*d^3+d^4)/sqrt(-c^2+d^2)*(atan((c*tan((f*x
+exp(1))/2)-d*tan((f*x+exp(1))/2))/sqrt(-c^2+d^2))+pi*sign(2*c-2*d)*floor((
f*x+exp(1))/2/pi+1/2))+(-2*tan((f*x+exp(1))/2)^3*a^3*c^2+2*tan((f*x+exp(1))
/2)^3*a^3*c*d+2*tan((f*x+exp(1))/2)*a^3*c^2+2*tan((f*x+exp(1))/2)*a^3*d^2)/
(c*d^2+d^3)/(tan((f*x+exp(1))/2)^4*c-tan((f*x+exp(1))/2)^4*d-2*tan((f*x+exp
(1))/2)^2*c+c+d))
```

maple [B] time = 0.70, size = 548, normalized size = 3.40

$$-\frac{2a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2}{f\,d^2\,(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}+\frac{4a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c}{f\,d\,(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x)
```

```
[Out] -2/f*a^3/d^2/(c+d)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2
*f*x)^2*d-c-d)*c^2+4/f*a^3/d/(c+d)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2
```

```
*c-tan(1/2*e+1/2*f*x)^2*d-c-d)*c-2/f*a^3/(c+d)*tan(1/2*e+1/2*f*x)/(tan(1/2*
e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)+4/f*a^3/d^3/(c+d)/((c+d)*(c-d))^
(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*c^3-2/f*a^3/d^2
/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(
1/2))*c^2-8/f*a^3/d/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c
-d)/((c+d)*(c-d))^(1/2))*c+6/f*a^3/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/
2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))-1/f*a^3/d^2/(tan(1/2*e+1/2*f*x)-1)+
2/f*a^3/d^3*ln(tan(1/2*e+1/2*f*x)-1)*c-3/f*a^3/d^2*ln(tan(1/2*e+1/2*f*x)-1)
-1/f*a^3/d^2/(tan(1/2*e+1/2*f*x)+1)-2/f*a^3/d^3*ln(tan(1/2*e+1/2*f*x)+1)*c+
3/f*a^3/d^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 5.27, size = 3135, normalized size = 19.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (a^3*atan(((a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d
^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*
c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10
- 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^
8 + c^2*d^6) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 +
 c^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d)*1
i)/d^3 + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6
 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^
5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 -
4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8
+ c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c
^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d)*1i)
/d^3)/((128*(4*a^9*c^7 - 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c^2*d^5 - 50*a
^9*c^3*d^4 + 20*a^9*c^4*d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) +
```

```
(a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6
*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(
2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2
*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6
) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/
(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d))/d^3 - (a^3*
((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*
d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d
^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9
+ 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (
64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/(d^3*
(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d))/d^3))*(2*c - 3*d
)*2i)/(d^3*f) - ((4*tan(e/2 + (f*x)/2)^3*(a^3*c^2 - a^3*c*d))/(d^2*(c + d))
 - (4*a^3*tan(e/2 + (f*x)/2)*(c^2 + d^2))/(d^2*(c + d)))/(f*(c + d + tan(e/
2 + (f*x)/2)^4*(c - d) - 2*c*tan(e/2 + (f*x)/2)^2)) + (a^3*atan(((a^3*((64*
tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 1
6*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 +
d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a
^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (64*a^
3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^
3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c
^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d
^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*1i)/(3*c*d^5 + d^6 +
 3*c^2*d^4 + c^3*d^3) + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7
 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c
^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 -
3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/
(2*c*d^7 + d^8 + c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3
)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d
^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*
c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2
)*(2*c + 3*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((128*(4*a^9*c^7 -
 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c^2*d^5 - 50*a^9*c^3*d^4 + 20*a^9*c^4*
d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((64*tan(e/2 + (f*x
)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5
- 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4)
 + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a
^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (64*a^3*tan(e/2 + (f
*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^
6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c
+ d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*
d^3) - (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 -
 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*
d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*
```

```
a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 +
c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*
d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^
6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^
5 + d^6 + 3*c^2*d^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3
*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*
d)*2i)/(f*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{}{c^2+2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(3*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(3*sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) +
d**2*sec(e + f*x)**2), x) + Integral(sec(e + f*x)**4/(c**2 + 2*c*d*sec(e +
f*x) + d**2*sec(e + f*x)**2), x))
```

3.207 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=188

$$-\frac{a^3\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3f(c+d)^{5/2}}-\frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2f(c+d)^2(c+d\sec(e+fx))}-\frac{(c-d)\tan(e+fx)\left(a^3\right.}{2df(c+d)(c+d\,\mathrm{s}}$$

```
[Out] a^3*arctanh(sin(f*x+e))/d^3/f-a^3*(2*c^2+6*c*d+7*d^2)*arctanh((c-d)^(1/2)*t
an(1/2*e+1/2*f*x)/(c+d)^(1/2))*(c-d)^(1/2)/d^3/(c+d)^(5/2)/f-1/2*(c-d)*(a^3
+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*sec(f*x+e))^2-1/2*a^3*(c-d)*(2*c
+5*d)*tan(f*x+e)/d^2/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.39, antiderivative size = 301, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3987, 98, 149, 157, 63, 217, 203, 93, 205}

$$\frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{d^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2f(c+d)^2(c+d\sec(e+fx))}-\frac{(c-d)\tan(}{2df(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^4*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^4*Sqrt
[c - d]*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(5/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3
*Sec[e + f*x])*Tan[e + f*x])/(2*d*(c + d)*f*(c + d*Sec[e + f*x])^2) - (a^3*
(c - d)*(2*c + 5*d)*Tan[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.}{2d(c+d)f\sqrt{a-a\,\mathrm{s}}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+}{2d^2(c+d)^2f(c+d\sec(e+}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+}{2d^2(c+d)^2f(c+d\sec(e+}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+}{2d^2(c+d)^2f(c+d\sec(e+}$$

$$= \frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{(c-}{}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right.}{d^3(c+d)^{5/2}f\sqrt{a-a}}$$

Mathematica [C] time = 3.67, size = 393, normalized size = 2.09

$$a^3 \sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(\frac{4\left(2c^3+4c^2d+cd^2-7d^3\right)(\sin(e)+i\cos(e))(c\cos(e+fx)+d)^2\tan^{-1}\left(\frac{(\sin(e)+i}{}\right.}{(c+d)^2\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3*(-4*(d +
c*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*(d + c*Cos[e
 + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (4*(2*c^3 + 4*c^2*d +
 c*d^2 - 7*d^3)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan
[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f
*x])^2*(I*Cos[e] + Sin[e]))/((c + d)^2*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin
[e])^2]) + ((c - d)*d*Sec[e]*((2*c^4 + 6*c^3*d + 5*c^2*d^2 + 12*c*d^3 + 2*d
^4)*Sin[e] - c*(d*(7*c^2 + 18*c*d + 2*d^2)*Sin[f*x] - d*(c^2 + 6*c*d + 2*d^
2)*Sin[2*e + f*x] + c*(2*c^2 + 6*c*d + d^2)*Sin[e + 2*f*x])))/(c^2*(c + d)^
2)))/(32*d^3*f*(c + d*Sec[e + f*x])^3)
```

fricas [B] time = 0.95, size = 1176, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (2*a^3*c^4 + 6*a^3*c^3*d +
 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 + 7*a^3*c*d
^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)
/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x +
 e) + d^2)) + 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3
*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3
)*cos(f*x + e))*log(sin(f*x + e) + 1) - 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*
d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d +
 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(3*a^3
*c^2*d^2 + 3*a^3*c*d^3 - 6*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c
*d^3 - a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5
)*f*cos(f*x + e)^2 + 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*
```

```
d^5 + 2*c*d^6 + d^7)*f), -1/2*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (
2*a^3*c^4 + 6*a^3*c^3*d + 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d +
6*a^3*c^2*d^2 + 7*a^3*c*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arctan(-(
d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) - (a^3*c
^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(
f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) + (a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c
^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + (3*a^3*c^2*d^2 + 3*a^3*c*d^3 - 6
*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x +
e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^
3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*d^5 + 2*c*d^6 + d^7)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-a^3*1/2/d^3*ln(abs(tan((f*x+exp(1))/2)-1))+a^3*1/2/d^3*ln(abs
(tan((f*x+exp(1))/2)+1))+(-2*a^3*c^3-4*a^3*c^2*d-a^3*c*d^2+7*a^3*d^3)*1/2/(
-c^2*d^3-2*c*d^4-d^5)/sqrt(-c^2+d^2)*(atan((c*tan((f*x+exp(1))/2)-d*tan((f*
x+exp(1))/2))/sqrt(-c^2+d^2))+pi*sign(2*c-2*d)*floor((f*x+exp(1))/2/pi+1/2)
)+(2*tan((f*x+exp(1))/2)^3*a^3*c^3+tan((f*x+exp(1))/2)^3*a^3*c^2*d-8*tan((f
*x+exp(1))/2)^3*a^3*c*d^2+5*tan((f*x+exp(1))/2)^3*a^3*d^3-2*tan((f*x+exp(1)
)/2)*a^3*c^3-7*tan((f*x+exp(1))/2)*a^3*c^2*d+2*tan((f*x+exp(1))/2)*a^3*c*d^
2+7*tan((f*x+exp(1))/2)*a^3*d^3)/(2*c^2*d^2+4*c*d^3+2*d^4)/(tan((f*x+exp(1)
)/2)^2*c-tan((f*x+exp(1))/2)^2*d-c-d)^2)
```

maple [B] time = 0.87, size = 768, normalized size = 4.09

$$\frac{2a^3\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^3}{f\,d^2\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^2\left(c^2+2cd+d^2\right)}+\frac{a^3\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c}{f\,d\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 2/f*a^3/d^2/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c^2+2*c*
d+d^2)*tan(1/2*e+1/2*f*x)^3*c^3+1/f*a^3/d/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e
+1/2*f*x)^2*d-c-d)^2/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3*c^2-8/f*a^3*c/(ta
n(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c^2+2*c*d+d^2)*tan(1/2*
e+1/2*f*x)^3-2/f*a^3/d^2/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d
)^2/(c+d)*tan(1/2*e+1/2*f*x)*c^2-5/f*a^3/d*c/(tan(1/2*e+1/2*f*x)^2*c-tan(1/
2*e+1/2*f*x)^2*d-c-d)^2/(c+d)*tan(1/2*e+1/2*f*x)-2/f*a^3/d^3/(c^2+2*c*d+d^2
)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))
*c^3-4/f*a^3/d^2/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*
f*x)*(c-d)/((c+d)*(c-d))^(1/2))*c^2-1/f*a^3/d*c/(c^2+2*c*d+d^2)/((c+d)*(c-d
))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))+5/f*a^3*d/(t
an(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c^2+2*c*d+d^2)*tan(1/2
*e+1/2*f*x)^3+7/f*a^3/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2
/(c+d)*tan(1/2*e+1/2*f*x)+7/f*a^3/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arcta
nh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))-1/f*a^3/d^3*ln(tan(1/2*e+1
/2*f*x)-1)+1/f*a^3/d^3*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 8.50, size = 4131, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] - ((a^3*tan(e/2 + (f*x)/2)*(5*c*d + 2*c^2 - 7*d^2))/(d^2*(c + d)) - (a^3*ta
n(e/2 + (f*x)/2)^3*(c^2*d - 8*c*d^2 + 2*c^3 + 5*d^3))/(d^2*(c + d)^2))/(f*(
2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 -
2*c*d + d^2) + c^2 + d^2)) - (a^3*atan(((a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*
c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^
3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3
*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 -
 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^
```

9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*x)/2)*(8*
c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8
*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1
i)/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6
 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*
c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*
a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^
8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^
7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^
10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8
+ 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1i)/d^3)/((16*(4*a^9*c^6 - 35*a^
9*d^6 + 61*a^9*c*d^5 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 - 35*a^9*c^3*d^3 - 10*a
^9*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^3*((8*
tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d +
53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7
 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*
c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7
 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*
a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 -
8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^
3*d^5 + c^4*d^4))))/d^3))/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53
*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 -
52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c
^4*d^4) - (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*
c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10
 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 +
 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6
))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3))/d^3))*2i
)/(d^3*f) - (a^3*atan(((a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2
)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 -
23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^
6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c
^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6)
)/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*
x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*(8*c*d^12 + 16*c^
2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/((4
*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c*d^7 + d^8 + 10*c^2*d^6
 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)*(3*c*d + c
^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5
d^3))(3*c*d + c^2 + (7*d^2)/2)*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d
^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f
x)/2)(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d
^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c
^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*
a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6

```
*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2
+ (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*(8*c*d^12 +
16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6)
)/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c*d^7 + d^8 + 10*c^
2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)*(3*c*
d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4
+ c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2)*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*
c^3*d^5 + 5*c^4*d^4 + c^5*d^3))/((16*(4*a^9*c^6 - 35*a^9*d^6 + 61*a^9*c*d^5
 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 - 35*a^9*c^3*d^3 - 10*a^9*c^4*d^2))/(4*c*d^
9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^3*((c + d)^5*(c - d))^(1/2
)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^
6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4
*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 1
0*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c
^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6)
 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^
2)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^
6*d^7 + 8*c^7*d^6))/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c
*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c
 - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^
3*d^5 + 5*c^4*d^4 + c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 1
0*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*((c + d)^5*(c - d))^(1
/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*
c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/
(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*a^3*d^12 +
 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3
*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^
6) + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*
d^2)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*
c^6*d^7 + 8*c^7*d^6))/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5
*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*
(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*
c^3*d^5 + 5*c^4*d^4 + c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 +
 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)
*(3*c*d + c^2 + (7*d^2)/2)*2i)/(f*(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5
+ 5*c^4*d^4 + c^5*d^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx+\int \frac{3\sec^2\left(e+\right.}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(3*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(3*sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec
(e + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**4/(c**3 +
 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3),
x))
```

3.208 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=178

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,(c+d)^{7/2}}+\frac{5a^3(c+4d)\tan(e+fx)}{6df(c+d)^3(c+d\sec(e+fx))}-\frac{5a^3(c-d)\tan(e+fx)}{6df(c+d)^2(c+d\sec(e+fx))^2}+\frac{a\tan(e+fx)(}{3f(c+d)(c}$$

```
[Out] 5*a^3*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c+d)^(7/2)/f/(c-
d)^(1/2)+1/3*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3-5/6
*a^3*(c-d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^2+5/6*a^3*(c+4*d)*tan(f*
x+e)/d/(c+d)^3/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.22, antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 94, 93, 205}

$$-\frac{5a^4\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}\,(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{5a^3\tan(e+fx)}{2f(c+d)^3(c+d\sec(e+fx))}+\frac{5\tan(e+fx)\left(a^3\sec\right.}{6f(c+d)^2(c+d\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (-5*a^4*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(7/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^2*Tan[e + f
*x])/(3*(c + d)*f*(c + d*Sec[e + f*x])^3) + (5*(a^3 + a^3*Sec[e + f*x])*Tan
[e + f*x])/(6*(c + d)^2*f*(c + d*Sec[e + f*x])^2) + (5*a^3*Tan[e + f*x])/(2
*(c + d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^4}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{\left(5a^3\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/}}{\sqrt{a-ax}\,(c+}\right.}{3(c+d)f\sqrt{a-a\sec(e+fx)}\,\sqrt{a}}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}$$

$$= -\frac{5a^4\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}\,(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a(a+a\sec}{3(c+d)f}$$

Mathematica [C] time = 3.58, size = 398, normalized size = 2.24

$$\frac{a^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(\frac{c\sec(e)\left(c\left(c\left(22c^2+9cd+2d^2\right)\sin(2e+3fx)+3\left(3c^3+38c^2d\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*Sec[e + f*x]*(1 + Sec[e + f*x]
)^3*(((-120*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[
(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*
x])^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
(c*Sec[e]*(6*(8*c^4 + 6*c^3*d + 30*c^2*d^2 + 9*c*d^3 + 2*d^4)*Sin[f*x] - 3*
(6*c^4 - 3*c^3*d + 30*c^2*d^2 + 18*c*d^3 + 4*d^4)*Sin[2*e + f*x] + c*(3*(3*
c^3 + 38*c^2*d + 12*c*d^2 + 2*d^3)*Sin[e + 2*f*x] + 3*(3*c^3 - 6*c^2*d - 6*
c*d^2 - 2*d^3)*Sin[3*e + 2*f*x] + c*(22*c^2 + 9*c*d + 2*d^2)*Sin[2*e + 3*f*
x])) - 2*d*(66*c^4 + 27*c^3*d + 50*c^2*d^2 + 18*c*d^3 + 4*d^4)*Tan[e])/c^3)
)/(192*(c + d)^3*f*(c + d*Sec[e + f*x])^4)
```

fricas [B] time = 0.56, size = 1012, normalized size = 5.69

$$\left[\frac{15\left(a^3c^3\cos\left(fx+e\right)^3+3\,a^3c^2d\cos\left(fx+e\right)^2+3\,a^3cd^2\cos\left(fx+e\right)+a^3d^3\right)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right.}{}\right.}{12\left(\left(c^8+3\,c^7d+2\,c^6d^2-2\,c^5d^3-3\,c^4d^4-c^3d^5\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*d*cos(f*x + e)^2 + 3*a^3*c*d^
2*cos(f*x + e) + a^3*d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3
*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^3*c^
4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e)^2
+ 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))*sin
(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^5)*f
*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*d^5
```

```
- c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^3*d^
5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 -
2*c^2*d^6 - 3*c*d^7 - d^8)*f), 1/6*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*
d*cos(f*x + e)^2 + 3*a^3*c*d^2*cos(f*x + e) + a^3*d^3)*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2
*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^
3*c^4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e
)^2 + 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))
*sin(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^
5)*f*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*
d^5 - c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^
3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^
5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f)]
```

giac [A] time = 0.52, size = 320, normalized size = 1.80

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^3}{\left(c^3+3c^2d+3cd^2+d^3\right)\sqrt{-c^2+d^2}}+\frac{15a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-30a^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+15a^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}{\left(c^3+3\right.}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^3/((c^3 + 3*
c^2*d + 3*c*d^2 + d^3)*sqrt(-c^2 + d^2)) + (15*a^3*c^2*tan(1/2*f*x + 1/2*e)
^5 - 30*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 + 15*a^3*d^2*tan(1/2*f*x + 1/2*e)^5
- 40*a^3*c^2*tan(1/2*f*x + 1/2*e)^3 + 40*a^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3
3*a^3*c^2*tan(1/2*f*x + 1/2*e) + 66*a^3*c*d*tan(1/2*f*x + 1/2*e) + 33*a^3*d
^2*tan(1/2*f*x + 1/2*e))/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*(c*tan(1/2*f*x +
1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

maple [A] time = 0.96, size = 227, normalized size = 1.28

$$\frac{16a^{3}\left(-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6(c+d)\left(\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^{3}}-\frac{5\left(-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4(c+d)\left(\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^{2}}-\frac{3\left(-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2(c+d)\left(\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right.\right.}\right.}{4(c+d}\right.}{6(c+d)}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 16/f*a^3*(-1/6*tan(1/2*e+1/2*f*x)/(c+d)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1
/2*f*x)^2*d-c-d)^3-5/6/(c+d)*(-1/4*tan(1/2*e+1/2*f*x)/(c+d)/(tan(1/2*e+1/2*
f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2-3/4/(c+d)*(-1/2*tan(1/2*e+1/2*f*x)/(
c+d)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)+1/2/(c+d)/((c+d)*(
c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 4.98, size = 264, normalized size = 1.48

$$\frac{\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(a^3c^2-2a^3cd+a^3d^2\right)}{(c+d)^3}+\frac{11a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+d}-\frac{40\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(a^3c-\right.}{3(c+d)^2}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-3c^3-3c^2d+3cd^2+3d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(-3c^3+3c^2d+3cd^2-3d^3\right)+3cd^2+3c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((5*tan(e/2 + (f*x)/2)^5*(a^3*c^2 + a^3*d^2 - 2*a^3*c*d))/(c + d)^3 + (11*a
^3*tan(e/2 + (f*x)/2))/(c + d) - (40*tan(e/2 + (f*x)/2)^3*(a^3*c - a^3*d))/
(3*(c + d)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3)
 - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c
^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) +
 (5*a^3*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)
^(7/2)*(c - d)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{\sec\left(e+fx\right)}{c^4+4c^3d\sec\left(e+fx\right)+6c^2d^2\sec^2\left(e+fx\right)+4cd^3\sec^3\left(e+fx\right)+d^4\sec^4\left(e+fx\right)}\,dx+\int\frac{}{c^4+4c^3d\sec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(3*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(3*
sec(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2
 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(sec(e +
f*x)**4/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 + 4*c*d
**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

3.209 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=266

$$\frac{5a^3(4c-3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4f(c-d)^{3/2}(c+d)^{9/2}}+\frac{5a^3(4c-3d)(c+4d)\tan(e+fx)}{24df(c-d)(c+d)^4(c+d\sec(e+fx))}-\frac{5a^3(4c-3d)\tan(e+fx)}{24df(c+d)^3(c+d\sec(e+fx)}$$

```
[Out] 5/4*a^3*(4*c-3*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c-d)
^(3/2)/(c+d)^(9/2)/f-1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/(c^2-d^2)/f/(c+d*s
ec(f*x+e))^4+1/12*a*(4*c-3*d)*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c-d)/(c+d)^2/f
/(c+d*sec(f*x+e))^3-5/24*a^3*(4*c-3*d)*tan(f*x+e)/d/(c+d)^3/f/(c+d*sec(f*x+
e))^2+5/24*a^3*(4*c-3*d)*(c+4*d)*tan(f*x+e)/(c-d)/d/(c+d)^4/f/(c+d*sec(f*x+
e))
```

Rubi [A] time = 0.34, antiderivative size = 327, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3987, 96, 94, 93, 205}

$$-\frac{5a^4(4c-3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{3/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{5a^3(4c-3d)\tan(e+fx)}{8f(c-d)(c+d)^4(c+d\sec(e+fx))}+\frac{5(4c-3d)}{24f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (-5*a^4*(4*c - 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(4*(c - d)^(3/2)*(c + d)^(9/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*(a + a*Sec[e + f
*x])^3*Tan[e + f*x])/(4*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^4) + (a*(4*c - 3
*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*(c - d)*(c + d)^2*f*(c + d*Sec
[e + f*x])^3) + (5*(4*c - 3*d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*(
c - d)*(c + d)^3*f*(c + d*Sec[e + f*x])^2) + (5*a^3*(4*c - 3*d)*Tan[e + f*x
])/(8*(c - d)*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^5}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} - \frac{\left(a^2(4c-3d)\tan(e+fx)\right)\text{Subst}\left(\right.}{4\left(c^2-d^2\right)f\sqrt{a-a\sec(e}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan}{12(c-d)(c+d)^2f(c+d\sec(e}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan}{12(c-d)(c+d)^2f(c+d\sec(e}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan}{12(c-d)(c+d)^2f(c+d\sec(e}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan}{12(c-d)(c+d)^2f(c+d\sec(e}$$

$$= -\frac{5a^4(4c-3d)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{4(c-d)^{3/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{d(a+a}{4\left(c^2-\right.}$$

Mathematica [A] time = 9.19, size = 274, normalized size = 1.03

$$\frac{a^3\left(-\frac{120(4c-3d)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{\sin(e+fx)\left(-88c^4\cos(3(e+fx))-72c^4+36c^3d\cos(3(e+fx))-478c^3d+37c^2d^2\cos(3(e+fx))+336c^2d^2+\right.}{}\right.}{96}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*((-120*(4*c - 3*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/Sqrt[c^2 - d^2] - ((-72*c^4 - 478*c^3*d + 336*c^2*d^2 + 28*c*d^3 + 336*d
^4 + (-296*c^4 - 84*c^3*d - 577*c^2*d^2 + 984*c*d^3 + 198*d^4)*Cos[e + f*x]
 + (-72*c^4 - 470*c^3*d + 384*c^2*d^2 + 200*c*d^3 + 48*d^4)*Cos[2*(e + f*x)
```

```
] - 88*c^4*Cos[3*(e + f*x)] + 36*c^3*d*Cos[3*(e + f*x)] + 37*c^2*d^2*Cos[3*
(e + f*x)] + 24*c*d^3*Cos[3*(e + f*x)] + 6*d^4*Cos[3*(e + f*x)])*Sin[e + f*
x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)*(c + d)^4*f)
```

fricas [B] time = 0.58, size = 1714, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)
^4 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*
a^3*c^2*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3*c^5*d + 12*a^3*c^4*d^2 +
 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6
- 36*a^3*c^5*d - 125*a^3*c^4*d^2 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3
*c*d^5 + 6*a^3*d^6)*cos(f*x + e)^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*
c^4*d^2 - 335*a^3*c^3*d^3 + 168*a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*c
os(f*x + e)^2 + (8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d
^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x +
e))/((c^11 + 3*c^10*d + c^9*d^2 - 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d
^6 + c^4*d^7)*f*cos(f*x + e)^4 + 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^
4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^
2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2
*d^9)*f*cos(f*x + e)^2 + 4*(c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c
^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^
5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f), 1/24*(
15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)^4 + 4*
(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*a^3*c^2
*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4
 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6 - 36*a^3*c^5*d - 125*a^3*c^4*d^2
 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3*c*d^5 + 6*a^3*d^6)*cos(f*x + e)
^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*c^4*d^2 - 335*a^3*c^3*d^3 + 168*
a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*cos(f*x + e)^2 + (8*a^3*c^6 + 48*
a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c
*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 + 3*c^10*d + c^9*d^2
- 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d^6 + c^4*d^7)*f*cos(f*x + e)^4 +
 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*
d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*
```

```
d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2*d^9)*f*cos(f*x + e)^2 + 4*(c^8*
d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*
d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f)]
```

giac [B] time = 1.70, size = 626, normalized size = 2.35

$$\frac{15\left(4a^3c-3a^3d\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^5+3c^4d+2c^3d^2-2c^2d^3-3cd^4-d^5\right)\sqrt{-c^2+d^2}}-\frac{60a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-225a^3c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+315a^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(-c^2
+ d^2)) - (60*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 225*a^3*c^3*d*tan(1/2*f*x +
1/2*e)^7 + 315*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 195*a^3*c*d^3*tan(1/2*f
*x + 1/2*e)^7 + 45*a^3*d^4*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c^4*tan(1/2*f*x
 + 1/2*e)^5 + 385*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 55*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 - 385*a^3*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 165*a^3*d^4*tan(1/
2*f*x + 1/2*e)^5 + 292*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 + 73*a^3*c^3*d*tan(1/
2*f*x + 1/2*e)^3 - 511*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 73*a^3*c*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 219*a^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c^4*tan
(1/2*f*x + 1/2*e) - 249*a^3*c^3*d*tan(1/2*f*x + 1/2*e) + 45*a^3*c^2*d^2*tan
(1/2*f*x + 1/2*e) + 309*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + 147*a^3*d^4*tan(1/
2*f*x + 1/2*e))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*(c
*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

maple [A] time = 1.00, size = 303, normalized size = 1.14

$$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\left(\tan^7\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{64\left(c^4+4c^3d+6c^2d^2+4cd^3+d^4\right)}+\frac{55(c-d)(4c-3d)\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{192\left(c^3+3c^2d+3cd^2+d^3\right)}-\frac{73(4c-3d)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{64(c+d)(c-d)}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^4}+\frac{5(4c-3d)\,\mathrm{arctanh}\left(\cdots\right)}{64\left(c^5+3c^4d+2c^3d^2-2c^2d^3-3\cdots\right)}\cdots\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x)
```

```
[Out] 16/f*a^3*((-5/64*(4*c-3*d)*(c^2-2*c*d+d^2)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d
^4)*tan(1/2*e+1/2*f*x)^7+55/192*(c-d)*(4*c-3*d)/(c^3+3*c^2*d+3*c*d^2+d^3)*t
an(1/2*e+1/2*f*x)^5-73/192*(4*c-3*d)/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3+1
/64*(44*c-49*d)/(c+d)/(c-d)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan
(1/2*e+1/2*f*x)^2*d-c-d)^4+5/64*(4*c-3*d)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-
3*c*d^4-d^5)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c
-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 5.08, size = 385, normalized size = 1.45

$$\frac{\frac{55\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(4\,a^3\,c^2-7\,a^3\,c\,d+3\,a^3\,d^2\right)}{12\,(c+d)^3}-\frac{73\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(4\,a^3\,c-3\,a^3\,d\right)}{12\,(c+d)^2}-\frac{5\,\mathrm{ta}}{}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(6\,c^4-12\,c^2\,d^2+6\,d^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(-4\,c^4-8\,c^3\,d+8\,c\,d^3+4\,d^4\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(4\,c^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

```
[Out] ((55*tan(e/2 + (f*x)/2)^5*(4*a^3*c^2 + 3*a^3*d^2 - 7*a^3*c*d))/(12*(c + d)^
3) - (73*tan(e/2 + (f*x)/2)^3*(4*a^3*c - 3*a^3*d))/(12*(c + d)^2) - (5*tan(
e/2 + (f*x)/2)^7*(4*a^3*c^3 - 3*a^3*d^3 + 10*a^3*c*d^2 - 11*a^3*c^2*d))/(4*
(c + d)^4) + (a^3*tan(e/2 + (f*x)/2)*(44*c - 49*d))/(4*(c + d)*(c - d)))/(f
*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*
(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3
*d + 4*c^4 - 4*d^4) + tan(e/2 + (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 +
 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d + c^4 + d^4 + 6*c^2*d^2)) + (5*a^3*atanh((t
an(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(4*c - 3*d))/(4*f*(c + d)^(
9/2)*(c - d)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^5+5c^4d\sec\left(e+fx\right)+10c^3d^2\sec^2\left(e+fx\right)+10c^2d^3\sec^3\left(e+fx\right)+5cd^4\sec^4\left(e+fx\right)+d^5\sec^5\left(e+fx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x
)**3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2
*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x
) + Integral(sec(e + f*x)**4/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*s
ec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 +
d**5*sec(e + f*x)**5), x))
```

3.210 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=183

$$\frac{d\left(8c^3-12c^2d+12cd^2-3d^3\right)\tanh^{-1}(\sin(e+fx))}{2af}-\frac{d\tan(e+fx)\left(d\left(6c^2-20cd+9d^2\right)\sec(e+fx)+4\left(3c^3-16\right.\right.}{6af}$$

```
[Out] 1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*arctanh(sin(f*x+e))/a/f-1/3*(3*c-4*d)
*d*(c+d*sec(f*x+e))^2*tan(f*x+e)/a/f+(c-d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f/
(a+a*sec(f*x+e))-1/6*d*(12*c^3-64*c^2*d+48*c*d^2-16*d^3+d*(6*c^2-20*c*d+9*d
^2)*sec(f*x+e))*tan(f*x+e)/a/f
```

Rubi [A] time = 0.34, antiderivative size = 236, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 153, 147, 63, 217, 203}

$$\frac{d\left(-12c^2d+8c^3+12cd^2-3d^3\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)\left(d\left(6c^2-20cd+9d^2\right)\sec(e+f\right.\right.}{6af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
+ a*Sec[e + f*x]]) - ((3*c - 4*d)*d*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
*a*f) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x
])) - (d*(4*(3*c^3 - 16*c^2*d + 12*c*d^2 - 4*d^3) + d*(6*c^2 - 20*c*d + 9*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
```

```
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^2\left(-a^2\right.}{\sqrt{}}\right.}{af\sqrt{a-a\sec(e+}}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3}{f(a+a\sec(e+f}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3}{f(a+a\sec(e+f}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3}{f(a+a\sec(e+f}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3}{f(a+a\sec(e+f}$$

$$= \frac{d\left(8c^3-12c^2d+12cd^2-3d^3\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{(3c-}{}$$

Mathematica [B] time = 6.46, size = 1243, normalized size = 6.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((-8*c^3*d + 12*c^2*d^2 - 12*c*d^3 + 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*
x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(
```

```
f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x])) + ((8*c^3*d - 12*c^2*d^2 + 1
2*c*d^3 - 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]^3*Log[Cos[e/2 + (f*x)/2]
 + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(f*(d + c*Cos[e + f*x])^4*(a
 + a*Sec[e + f*x])) + (Cos[e/2 + (f*x)/2]*Sec[e/2]*Sec[e]*(c + d*Sec[e + f*
x])^4*(-18*c^4*Sin[(f*x)/2] + 72*c^3*d*Sin[(f*x)/2] - 36*c^2*d^2*Sin[(f*x)/
2] + 24*c*d^3*Sin[(f*x)/2] + 6*d^4*Sin[(f*x)/2] + 18*c^4*Sin[(3*f*x)/2] - 7
2*c^3*d*Sin[(3*f*x)/2] + 180*c^2*d^2*Sin[(3*f*x)/2] - 108*c*d^3*Sin[(3*f*x)
/2] + 39*d^4*Sin[(3*f*x)/2] - 72*c^2*d^2*Sin[e - (f*x)/2] + 48*c*d^3*Sin[e
- (f*x)/2] - 24*d^4*Sin[e - (f*x)/2] - 36*c^2*d^2*Sin[e + (f*x)/2] + 24*c*d
^3*Sin[e + (f*x)/2] - 6*d^4*Sin[e + (f*x)/2] - 18*c^4*Sin[2*e + (f*x)/2] +
72*c^3*d*Sin[2*e + (f*x)/2] - 144*c^2*d^2*Sin[2*e + (f*x)/2] + 96*c*d^3*Sin
[2*e + (f*x)/2] - 24*d^4*Sin[2*e + (f*x)/2] + 72*c^2*d^2*Sin[e + (3*f*x)/2]
 - 36*c*d^3*Sin[e + (3*f*x)/2] + 21*d^4*Sin[e + (3*f*x)/2] + 18*c^4*Sin[2*e
 + (3*f*x)/2] - 72*c^3*d*Sin[2*e + (3*f*x)/2] + 72*c^2*d^2*Sin[2*e + (3*f*x
)/2] - 36*c*d^3*Sin[2*e + (3*f*x)/2] + 9*d^4*Sin[2*e + (3*f*x)/2] - 36*c^2*
d^2*Sin[3*e + (3*f*x)/2] + 36*c*d^3*Sin[3*e + (3*f*x)/2] - 9*d^4*Sin[3*e +
(3*f*x)/2] + 36*c^2*d^2*Sin[e + (5*f*x)/2] - 12*c*d^3*Sin[e + (5*f*x)/2] +
7*d^4*Sin[e + (5*f*x)/2] - 6*c^4*Sin[2*e + (5*f*x)/2] + 24*c^3*d*Sin[2*e +
(5*f*x)/2] + 12*c*d^3*Sin[2*e + (5*f*x)/2] + d^4*Sin[2*e + (5*f*x)/2] + 12*
c*d^3*Sin[3*e + (5*f*x)/2] - 3*d^4*Sin[3*e + (5*f*x)/2] - 6*c^4*Sin[4*e + (
5*f*x)/2] + 24*c^3*d*Sin[4*e + (5*f*x)/2] - 36*c^2*d^2*Sin[4*e + (5*f*x)/2]
 + 36*c*d^3*Sin[4*e + (5*f*x)/2] - 9*d^4*Sin[4*e + (5*f*x)/2] + 6*c^4*Sin[2
*e + (7*f*x)/2] - 24*c^3*d*Sin[2*e + (7*f*x)/2] + 72*c^2*d^2*Sin[2*e + (7*f
*x)/2] - 48*c*d^3*Sin[2*e + (7*f*x)/2] + 16*d^4*Sin[2*e + (7*f*x)/2] + 36*c
^2*d^2*Sin[3*e + (7*f*x)/2] - 24*c*d^3*Sin[3*e + (7*f*x)/2] + 10*d^4*Sin[3*
e + (7*f*x)/2] + 6*c^4*Sin[4*e + (7*f*x)/2] - 24*c^3*d*Sin[4*e + (7*f*x)/2]
 + 36*c^2*d^2*Sin[4*e + (7*f*x)/2] - 24*c*d^3*Sin[4*e + (7*f*x)/2] + 6*d^4*
Sin[4*e + (7*f*x)/2]))/(48*f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x]))
```

fricas [A] time = 0.48, size = 297, normalized size = 1.62

$$\frac{3\left(\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*
d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) -
3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*d - 12
*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*
d^4 + 2*(3*c^4 - 12*c^3*d + 36*c^2*d^2 - 24*c*d^3 + 8*d^4)*cos(f*x + e)^3 +
```

```
(36*c^2*d^2 - 12*c*d^3 + 7*d^4)*cos(f*x + e)^2 + (12*c*d^3 - d^4)*cos(f*x
+ e))*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((tan((f*x+exp(1))/2)*c^4-4*tan((f*x+exp(1))/2)*c^3*d+6*tan((f*
x+exp(1))/2)*c^2*d^2-4*tan((f*x+exp(1))/2)*c*d^3+tan((f*x+exp(1))/2)*d^4)*1
/2/a+(-36*tan((f*x+exp(1))/2)^5*c^2*d^2+36*tan((f*x+exp(1))/2)^5*c*d^3-15*t
an((f*x+exp(1))/2)^5*d^4+72*tan((f*x+exp(1))/2)^3*c^2*d^2-48*tan((f*x+exp(1
))/2)^3*c*d^3+16*tan((f*x+exp(1))/2)^3*d^4-36*tan((f*x+exp(1))/2)*c^2*d^2+1
2*tan((f*x+exp(1))/2)*c*d^3-9*tan((f*x+exp(1))/2)*d^4)*1/6/a/(tan((f*x+exp(
1))/2)^2-1)^3-(8*c^3*d-12*c^2*d^2+12*c*d^3-3*d^4)*1/4/a*ln(abs(tan((f*x+exp
(1))/2)-1))+(8*c^3*d-12*c^2*d^2+12*c*d^3-3*d^4)*1/4/a*ln(abs(tan((f*x+exp(1
))/2)+1)))
```

maple [B] time = 0.67, size = 596, normalized size = 3.26

$$-\frac{d^4}{af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^2}-\frac{d^4}{3af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)^3}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^4}{af}-\frac{d^4}{3af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3}-\frac{3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x)
```

```
[Out] -1/a/f*d^4/(tan(1/2*e+1/2*f*x)-1)^2-1/3/a/f*d^4/(tan(1/2*e+1/2*f*x)+1)^3+1/
a/f*tan(1/2*e+1/2*f*x)*d^4-1/3/a/f*d^4/(tan(1/2*e+1/2*f*x)-1)^3-3/2/a/f*ln(
tan(1/2*e+1/2*f*x)+1)*d^4-5/2/a/f*d^4/(tan(1/2*e+1/2*f*x)+1)+1/a/f*d^4/(tan
(1/2*e+1/2*f*x)+1)^2+3/2/a/f*ln(tan(1/2*e+1/2*f*x)-1)*d^4-5/2/a/f*d^4/(tan(
1/2*e+1/2*f*x)-1)+1/f*c^4/a*tan(1/2*e+1/2*f*x)+6/a/f*ln(tan(1/2*e+1/2*f*x)+
1)*c*d^3-2/a/f*d^3/(tan(1/2*e+1/2*f*x)+1)^2*c+6/a/f*d^3/(tan(1/2*e+1/2*f*x)
+1)*c-4/a/f*tan(1/2*e+1/2*f*x)*c^3*d-4/a/f*ln(tan(1/2*e+1/2*f*x)-1)*c^3*d+6
/a/f*ln(tan(1/2*e+1/2*f*x)-1)*c^2*d^2-6/a/f*ln(tan(1/2*e+1/2*f*x)-1)*c*d^3-
6/a/f*d^2/(tan(1/2*e+1/2*f*x)+1)*c^2+4/a/f*ln(tan(1/2*e+1/2*f*x)+1)*c^3*d-6
/a/f*ln(tan(1/2*e+1/2*f*x)+1)*c^2*d^2+6/a/f*tan(1/2*e+1/2*f*x)*c^2*d^2-4/a/
f*tan(1/2*e+1/2*f*x)*c*d^3-6/a/f*d^2/(tan(1/2*e+1/2*f*x)-1)*c^2+6/a/f*d^3/(
tan(1/2*e+1/2*f*x)-1)*c+2/a/f*d^3/(tan(1/2*e+1/2*f*x)-1)^2*c
```

maxima [B] time = 0.46, size = 596, normalized size = 3.26

$$\frac{d^4\left(\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{16\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a-\frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{6\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-12\,cd^3\left(\frac{2\left(\frac{\mathrm{si}}{\mathrm{co}}\right.}{a-\frac{2a}{(\mathrm{cos}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) - 12*c*d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x +
e)/(a*(cos(f*x + e) + 1))) - 36*c^2*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a
- a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)
/(a*(cos(f*x + e) + 1))) + 24*c^3*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*
x + e) + 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 2.45, size = 211, normalized size = 1.15

$$\frac{\left(12\,c^2\,d^2-12\,c\,d^3+5\,d^4\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-24\,c^2\,d^2+16\,c\,d^3-\frac{16\,d^4}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(12\,c^2\,d^2-4\,c\,d^3+3\,d^4\right.}{f\,\left(-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3*d^4 - 4*c*d^3 + 12*c^2*d^2) + tan(e/2 + (f*x)/2)^5*(
5*d^4 - 12*c*d^3 + 12*c^2*d^2) - tan(e/2 + (f*x)/2)^3*((16*d^4)/3 - 16*c*d^
3 + 24*c^2*d^2))/(f*(a - 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)/2)^
4 - a*tan(e/2 + (f*x)/2)^6)) + (tan(e/2 + (f*x)/2)*(c - d)^4)/(a*f) + (d*at
anh(tan(e/2 + (f*x)/2))*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^4\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d^4\sec^5\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{4cd^3\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{6c^2d^2\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{4c^3d\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**4*sec(e +
f*x)**5/(sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(sec(e +
 f*x) + 1), x) + Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x) + 1), x
) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/a
```

3.211 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{3d(2c^2-2cd+d^2)\tanh^{-1}(\sin(e+fx))}{2af}-\frac{d\tan(e+fx)\left(4(c^2-3cd+d^2)+d(2c-3d)\sec(e+fx)\right)}{2af}+\frac{(c-d)\tan \ldots}{f \ldots}$$

```
[Out] 3/2*d*(2*c^2-2*c*d+d^2)*arctanh(sin(f*x+e))/a/f+(c-d)*(c+d*sec(f*x+e))^2*ta
n(f*x+e)/f/(a+a*sec(f*x+e))-1/2*d*(4*c^2-12*c*d+4*d^2+(2*c-3*d)*d*sec(f*x+e
))*tan(f*x+e)/a/f
```

Rubi [A] time = 0.25, antiderivative size = 171, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 147, 63, 217, 203}

$$\frac{3d(2c^2-2cd+d^2)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)\left(4(c^2-3cd+d^2)+d(2c-3d)\sec(e+f\ldots\right.}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[
e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]
)) - (d*(4*(c^2 - 3*c*d + d^2) + (2*c - 3*d)*d*Sec[e + f*x])*Tan[e + f*x])/
(2*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{(c+dx)(-\cdots}{\cdots}\right.}{af\sqrt{a-a\sec(e\cdots}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d\left(4\left(c^2-3cd+d^2\right)+(2c-3\cdots\right.}{2a\cdots}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d\left(4\left(c^2-3cd+d^2\right)+(2c-3\cdots\right.}{2a\cdots}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d\left(4\left(c^2-3cd+d^2\right)+(2c-3\cdots\right.}{2a\cdots}$$

$$= \frac{3d\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec\cdots}{f(a+\cdots}$$

Mathematica [B] time = 2.66, size = 275, normalized size = 2.35

$$\frac{\cos^6\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)\left(3d\left(2c^2-2cd+d^2\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\cdots\right.\right.\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^6*Sec[e + f*x]^2*(16*d^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^
4 + (-1 + Tan[(e + f*x)/2]^2)*(3*d*(2*c^2 - 2*c*d + d^2)*(Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(c^
3 - 3*c^2*d + 9*c*d^2 - 3*d^3)*Tan[(e + f*x)/2] - 3*d*(2*c^2 - 2*c*d + d^2)
*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]])*Tan[(e + f*x)/2]^2 + 2*(c - d)^3*Tan[(e + f*x)/2]^3)))/(a*f*(1
+ Cos[e + f*x]))
```

fricas [A] time = 0.45, size = 216, normalized size = 1.85

$$\frac{3\left(\left(2c^2d-2cd^2+d^3\right)\cos\left(fx+e\right)^3+\left(2c^2d-2cd^2+d^3\right)\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(\left(2c^2d-2\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/4*(3*((2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3
)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 3*((2*c^2*d - 2*c*d^2 + d^3)*cos(
f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^2)*log(-sin(f*x + e) +
1) + 2*(d^3 + 2*(c^3 - 3*c^2*d + 6*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (6*c*d^2
 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^
2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((tan((f*x+exp(1))/2)*c^3-3*tan((f*x+exp(1))/2)*c^2*d+3*tan((f*
x+exp(1))/2)*c*d^2-tan((f*x+exp(1))/2)*d^3)*1/2/a-(6*tan((f*x+exp(1))/2)^3*
c*d^2-3*tan((f*x+exp(1))/2)^3*d^3-6*tan((f*x+exp(1))/2)*c*d^2+tan((f*x+exp(
1))/2)*d^3)*1/2/a/(tan((f*x+exp(1))/2)^2-1)^2+(-6*c^2*d+6*c*d^2-3*d^3)*1/4/
a*ln(abs(tan((f*x+exp(1))/2)-1))-(-6*c^2*d+6*c*d^2-3*d^3)*1/4/a*ln(abs(tan(
(f*x+exp(1))/2)+1)))
```

maple [B] time = 0.68, size = 371, normalized size = 3.17

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^{3}}{af}-\frac{3c^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{af}+\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d^{2}}{af}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^{3}}{af}+\frac{d^{3}}{2af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{e}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x)
```

```
[Out] 1/a/f*tan(1/2*e+1/2*f*x)*c^3-3/a/f*c^2*tan(1/2*e+1/2*f*x)*d+3/a/f*tan(1/2*e
+1/2*f*x)*c*d^2-1/a/f*tan(1/2*e+1/2*f*x)*d^3+1/2/a/f*d^3/(tan(1/2*e+1/2*f*x
)-1)^2-3/a/f*ln(tan(1/2*e+1/2*f*x)-1)*c^2*d+3/a/f*ln(tan(1/2*e+1/2*f*x)-1)*
c*d^2-3/2/a/f*ln(tan(1/2*e+1/2*f*x)-1)*d^3-3/a/f*d^2/(tan(1/2*e+1/2*f*x)-1)
*c+3/2/a/f*d^3/(tan(1/2*e+1/2*f*x)-1)-1/2/a/f*d^3/(tan(1/2*e+1/2*f*x)+1)^2+
3/a/f*ln(tan(1/2*e+1/2*f*x)+1)*c^2*d-3/a/f*ln(tan(1/2*e+1/2*f*x)+1)*c*d^2+3
```

```
/2/a/f*ln(tan(1/2*e+1/2*f*x)+1)*d^3-3/a/f*d^2/(tan(1/2*e+1/2*f*x)+1)*c+3/2/
a/f*d^3/(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.53, size = 388, normalized size = 3.32

$$-\frac{d^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+6cd^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
 + 1))) + 6*c*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)
)) - 6*c^2*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c^3*s
in(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.94, size = 139, normalized size = 1.19

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(6\,c\,d^2-d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(6\,c\,d^2-3\,d^3\right)}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a\right)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(c-d)^3}{a\,f}+\frac{3\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\,(2}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(6*c*d^2 - d^3) - tan(e/2 + (f*x)/2)^3*(6*c*d^2 - 3*d^3
))/(f*(a - 2*a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4)) + (tan(e/2 +
 (f*x)/2)*(c - d)^3)/(a*f) + (3*d*atanh(tan(e/2 + (f*x)/2))*(2*c^2 - 2*c*d
+ d^2))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^3\sec(e+fx)}{\sec(e+fx)+1}\,dx+\int\frac{d^3\sec^4(e+fx)}{\sec(e+fx)+1}\,dx+\int\frac{3cd^2\sec^3(e+fx)}{\sec(e+fx)+1}\,dx+\int\frac{3c^2d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**3*sec(e +
f*x)**4/(sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(sec(e +
 f*x) + 1), x) + Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/
a
```

3.212 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=68

$$\frac{d(2c-d)\tanh^{-1}(\sin(e+fx))}{af}+\frac{(c-d)^2\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d^2\tan(e+fx)}{af}$$

```
[Out] (2*c-d)*d*arctanh(sin(f*x+e))/a/f+d^2*tan(f*x+e)/a/f+(c-d)^2*tan(f*x+e)/f/(
a+a*sec(f*x+e))
```

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.84, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 89, 80, 63, 217, 203}

$$\frac{2d(2c-d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)^2\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d^2\tan(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d^2*Tan[e + f*x])/(a*f) + ((c - d)^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])
) + (2*(2*c - d)*d*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{a^3(2c-d)d+a^3d^2x}{\sqrt{a-ax}\,\sqrt{a+ax}}\,dx, x, \sec(e\right.}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(a(2c-d)d\tan(e+fx))\,\text{Subst}}{f\sqrt{a-a\sec(e+f}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\,\text{Subs}}{f\sqrt{a-a\sec(e+}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\,\text{Subs}}{f\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{2(2c-d)d\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx}}{\sqrt{a+a\sec(e+fx}}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a}}$$

Mathematica [B] time = 1.73, size = 237, normalized size = 3.49

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos(e+fx)(c+d\sec(e+fx))^2\left((c-d)^2\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\sin\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]*(c + d*Sec[e + f*x])^2*((c - d)^2*Sec[e/2]
*Sin[(f*x)/2] + d*Cos[(e + f*x)/2]*(-((2*c - d)*(Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + (d*Sin[f*x])/
((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(d + c*Cos[e + f*x])
^2*(1 + Sec[e + f*x]))
```

fricas [B] time = 0.44, size = 155, normalized size = 2.28

$$\frac{\left(\left(2\,cd-d^2\right)\cos\left(fx+e\right)^2+\left(2\,cd-d^2\right)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(\left(2\,cd-d^2\right)\cos\left(fx+e\right)^2+\left(2\,cd\right.\right.}{2\left(af\cos\left(fx+e\right)^2+af\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/2*(((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*log(sin(f*
x + e) + 1) - ((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*l
og(-sin(f*x + e) + 1) + 2*(d^2 + (c^2 - 2*c*d + 2*d^2)*cos(f*x + e))*sin(f*
x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((tan((f*x+exp(1))/2)*c^2-2*tan((f*x+exp(1))/2)*c*d+tan((f*x+ex
p(1))/2)*d^2)*1/2/a-tan((f*x+exp(1))/2)*d^2/a/(tan((f*x+exp(1))/2)^2-1)-(2*
c*d-d^2)*1/2/a*ln(abs(tan((f*x+exp(1))/2)-1))+(2*c*d-d^2)*1/2/a*ln(abs(tan(
(f*x+exp(1))/2)+1)))
```

maple [B] time = 0.64, size = 196, normalized size = 2.88

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2}{af}-\frac{2cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^2}{af}-\frac{d^2}{af\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}-\frac{2d\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)c}{af}+\frac{d^2\ln\left(\text{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x)
```

```
[Out] 1/a/f*tan(1/2*e+1/2*f*x)*c^2-2/a/f*c*d*tan(1/2*e+1/2*f*x)+1/a/f*tan(1/2*e+1
/2*f*x)*d^2-1/a/f*d^2/(tan(1/2*e+1/2*f*x)-1)-2/a/f*d*ln(tan(1/2*e+1/2*f*x)-
1)*c+1/a/f*d^2*ln(tan(1/2*e+1/2*f*x)-1)-1/a/f*d^2/(tan(1/2*e+1/2*f*x)+1)+2/
a/f*d*ln(tan(1/2*e+1/2*f*x)+1)*c-1/a/f*d^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.44, size = 223, normalized size = 3.28

$$-\frac{d^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-2\,cd\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c*d*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.82, size = 85, normalized size = 1.25

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c-d\right)}^2}{a\,f}+\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\right)}+\frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c-d\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c - d)^2)/(a*f) + (2*d^2*tan(e/2 + (f*x)/2))/(f*(a - a
*tan(e/2 + (f*x)/2)^2)) + (2*d*atanh(tan(e/2 + (f*x)/2))*(2*c - d))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d^2\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**2*sec(e +
f*x)**3/(sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e + f*
x) + 1), x))/a
```

3.213 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d\tanh^{-1}(\sin(e+fx))}{af}$$

[Out] d*arctanh(sin(f*x+e))/a/f+(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3998, 3770, 3794}

$$\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d\tanh^{-1}(\sin(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(a*f) + ((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e +
f*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = (c-d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx + \frac{d\int \sec(e+fx)\,dx}{a}$$
$$= \frac{d\tanh^{-1}(\sin(e+fx))}{af} + \frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [B] time = 0.27, size = 109, normalized size = 2.53

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left((c-d)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(c\right.\right.\right.}{af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d*Cos[(e + f*x)/2]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Sec[e/2]*Sin[
(f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

fricas [A] time = 0.48, size = 74, normalized size = 1.72

$$\frac{\left(d\cos\left(fx+e\right)+d\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(d\cos\left(fx+e\right)+d\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\,(c-d)\sin\left(fx+e\right.}{2\left(af\cos\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/2*((d*cos(f*x + e) + d)*log(sin(f*x + e) + 1) - (d*cos(f*x + e) + d)*log(
-sin(f*x + e) + 1) + 2*(c - d)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)2/f*(-d*1/2/a*ln(abs(tan((f*x+exp(1))/2)-1))+d*1/2/a*ln(abs(tan((f*
x+exp(1))/2)+1))+(tan((f*x+exp(1))/2)*c-tan((f*x+exp(1))/2)*d)*1/2/a)
```

maple [A] time = 0.74, size = 78, normalized size = 1.81

$$-\frac{d\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{af}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c}{af}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{af}+\frac{d\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -1/a/f*d*ln(tan(1/2*e+1/2*f*x)-1)+1/a/f*tan(1/2*e+1/2*f*x)*c-1/a/f*tan(1/2*
e+1/2*f*x)*d+1/a/f*d*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.38, size = 99, normalized size = 2.30

$$\frac{d\left(\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a}-\frac{\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}\right)+\frac{c\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] (d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*sin(f*x + e)/(a
*(cos(f*x + e) + 1)))/f
```

mupad [B] time = 1.73, size = 41, normalized size = 0.95

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c-d\right)}{a\,f}+\frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c - d))/(a*f) + (2*d*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d*sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

3.214 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)}{f(c-d)(a\sec(e+fx)+a)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{af(c-d)^{3/2}\sqrt{c+d}}$$

```
[Out] -2*d*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/a/(c-d)^(3/2)/f/(c
+d)^(1/2)+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.61, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 96, 93, 205}

$$\frac{2d\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{\tan(e+fx)}{f(c-d)(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/((c - d)*f*(a + a*Sec[e + f*x])) + (2*d*ArcTan[(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x
])/((c - d)^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{(ad\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,\sqrt{a}\cdots}\right.}{(c-d)f\sqrt{a-a\sec(e+fx)}\cdots}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{(2ad\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(\cdots}\right.}{(c-d)f\sqrt{a-a\sec(e+fx)}\cdots}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{2d\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\cdots}{(c-d)^{3/2}\sqrt{c+d}\,f\sqrt{a-a\sec(e+f\cdots}}$$

Mathematica [C] time = 0.67, size = 160, normalized size = 1.93

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+\frac{2d(\sin(e)+i\cos(e))\cos\left(\frac{1}{2}(e+fx)\right)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\cos(e)-d)+c\sin(e)\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{af(c-d)(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((2*d*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*C
os[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e
 + f*x)/2]*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + Sec[e/2]*Sin[(f*x)/2]))/(a*(c - d)*f*(1 + Cos[e + f*x]))
```

fricas [A] time = 0.48, size = 353, normalized size = 4.25

$$\left[-\frac{\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+d\right)\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)-2\left(c^2-\right.}{2\left(\left(ac^3-ac^2d-acd^2+ad^3\right)f\cos\left(fx+e\right)+\left(ac^3-ac^2d-acd^2+ad^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(sqrt(c^2 - d^2)*(d*cos(f*x + e) + d)*log((2*c*d*cos(f*x + e) - (c^2
- 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x +
e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^2
 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) +
 (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), -(sqrt(-c^2 + d^2)*(d*cos(f*x + e)
 + d)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
e))) - (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos
(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

giac [A] time = 0.52, size = 114, normalized size = 1.37

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)d}{(ac-ad)\sqrt{-c^2+d^2}}+\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac-ad}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d/((a*c - a*d)*sqrt(
-c^2 + d^2)) + tan(1/2*f*x + 1/2*e)/(a*c - a*d))/f
```

maple [A] time = 0.69, size = 74, normalized size = 0.89

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c-d}-\frac{2d\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)\sqrt{(c+d)(c-d)}}}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f/a*(1/(c-d)*tan(1/2*e+1/2*f*x)-2*d/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan
(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 1.95, size = 110, normalized size = 1.33

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\ (c-d)}-\frac{2\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)c^2-2\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)d^2}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c+d}\ (c-d)^{3/2}}\right)}{af\sqrt{c+d}\ (c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)) - (2*d*atanh((c^2*sin(e/2 + (f*x)/2) + d^2
*sin(e/2 + (f*x)/2) - 2*c*d*sin(e/2 + (f*x)/2))/(cos(e/2 + (f*x)/2)*(c + d)
^(1/2)*(c - d)^(3/2))))/(a*f*(c + d)^(1/2)*(c - d)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(e+fx\right)}{c\sec\left(e+fx\right)+c+d\sec^2\left(e+fx\right)+d\sec\left(e+fx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x) + c + d*sec(e + f*x)**2 + d*sec(e + f
*x)), x)/a
```

3.215 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=145

$$-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)(c+d\sec(e+fx))}-\frac{2d(2c+d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{af(c-d)^{5/2}(c+d)^{3/2}}+\frac{(c+2d)\tan(e}{f(c-d)^2(c+d)(a\sec}$$

```
[Out] -2*d*(2*c+d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/a/(c-d)^(5
/2)/(c+d)^(3/2)/f+(c+2*d)*tan(f*x+e)/(c-d)^2/(c+d)/f/(a+a*sec(f*x+e))-d*tan
(f*x+e)/(c^2-d^2)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 103, 152, 12, 93, 205}

$$-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)(c+d\sec(e+fx))}+\frac{2d(2c+d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{5/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 2*d)*Tan[e + f*x])/((c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])) + (2*d*
(2*c + d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(5/2)*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Tan[e + f*x])/((c^2 - d^2)*f
*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{3/2}(c+dx)^2}\,dx, x, \sec(e+fx\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))} - \frac{\tan(e+fx}{\left(c^2-\right.}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+}{\left(c^2-d^2\right)f(a+a\sec(e+f}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+}{\left(c^2-d^2\right)f(a+a\sec(e+f}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+}{\left(c^2-d^2\right)f(a+a\sec(e+f}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} + \frac{2d(2c+d)\tan^{-1}\left(\frac{\sqrt{c+}}{\sqrt{c-}}\right.}{(c-d)^{5/2}(c+d)^{3/2}f\sqrt{a-}}$$

Mathematica [C] time = 3.25, size = 286, normalized size = 1.97

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)(c\cos(e+fx)+d)\left(\frac{2d(2c+d)(\sin(e)+i\cos(e))\cos\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\right.}{\sqrt{c^2-d^2}\sqrt{(}}\right.}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{af(c-d)^2(\sec(e+fx)+1)(c+d\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^3*((2*d*(2*c + d)*Arc
Tan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c
^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x
])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + (d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (d^2*Cos[(e + f*x)/2]*(-(
d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e
/2]))))/(a*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x])^2)
```

fricas [B] time = 0.48, size = 691, normalized size = 4.77

$$\left[\frac{\left(2\,cd^2+d^3+\left(2\,c^2d+cd^2\right)\cos\left(fx+e\right)^2+\left(2\,c^2d+3\,cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cdots}{c\cdots}\right.}{2\left(\left(ac^6-ac^5d-2\,ac^4d^2+2\,ac^3d^3+ac^2d^4-acd^5\right)f\cos\left(fx+e\right)^2+\left(ac^6-3\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d^
2 + d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d
^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(c^3*d +
2*c^2*d^2 - c*d^3 - 2*d^4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f
*x + e))/((a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e)^2 + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x
 + e) + (a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)
*f), -((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d
^2 + d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*
x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^
4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f*x + e))/((a*c^6 - a*c^5
*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e)^2 + (a
*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x + e) + (a*c^5*d - a*c^4
*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f)]
```

giac [A] time = 0.30, size = 228, normalized size = 1.57

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(ac^3-ac^2d-acd^2+ad^3\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,c-2\,d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)\left(2\,cd+d^2\right)\cdots}{\left(ac^3-ac^2d-acd^2+ad^3\right)\sqrt{-c^2+d^2}}\cdots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*(c*tan(1/
2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) - 2*(pi*floor(1/2*(f*
x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/
2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*c*d^
```

```
2 + a*d^3)*sqrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^2 - 2*a*c*d + a*d^
2))/f
```

maple [A] time = 0.79, size = 146, normalized size = 1.01

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c^2-2cd+d^2}+\frac{4d\left(-\frac{d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}-\frac{(2c+d)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2}}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f/a*(1/(c^2-2*c*d+d^2)*tan(1/2*e+1/2*f*x)+4*d/(c-d)^2*(-1/2*d/(c+d)*tan(1
/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)-1/2*(2*c+
d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))
^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.04, size = 187, normalized size = 1.29

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af(c-d)^2}-\frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f(c+d)\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(ac^3-3ac^2d+3acd^2-ad^3\right)-ad^3-ac^3+acd^2+ac^2d\right)}-\frac{2d\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)^2) - (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*
(tan(e/2 + (f*x)/2)^2*(a*c^3 - a*d^3 + 3*a*c*d^2 - 3*a*c^2*d) - a*d^3 - a*c
^3 + a*c*d^2 + a*c^2*d)) - (2*d*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a*c^
2 + a*d^2 - 2*a*c*d))/(2*a*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c + d))/(a*f*(c
 + d)^(3/2)*(c - d)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^2\sec\left(e+fx\right)+c^2+2cd\sec^2\left(e+fx\right)+2cd\sec\left(e+fx\right)+d^2\sec^3\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**2 + 2
*c*d*sec(e + f*x) + d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a
```

3.216 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{3d\left(2c^2+2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{af(c-d)^{7/2}(c+d)^{5/2}}+\frac{d(2c+d)(c+4d)\tan(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sec(e+fx))}+\frac{d(2c+3d)\tan(e+fx)}{2af(c-d)^2(c+d)(c+d\sec(e+fx))^2}$$

```
[Out] -3*d*(2*c^2+2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/
a/(c-d)^(7/2)/(c+d)^(5/2)/f+1/2*d*(2*c+3*d)*tan(f*x+e)/a/(c-d)^2/(c+d)/f/(c
+d*sec(f*x+e))^2+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/2
*d*(2*c+d)*(c+4*d)*tan(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 103, 151, 152, 12, 93, 205}

$$\frac{3d\left(2c^2+2cd+d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{7/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{d(4c+d)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(a\sec(e+fx)+a)(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c + d)*(c + 4*d)*Tan[e + f*x])/(2*(c - d)^3*(c + d)^2*f*(a + a*Sec[e +
f*x])) + (3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(7/2)
*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Ta
n[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2) -
 (d*(4*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])*(c + d*
Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
```

```
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{3/2}(c+dx)^3}\,dx,x,\sec(e+fx\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{\tan(e+}{2\left(c\right.}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{}{2\left(c^2-d\right.}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{d\tan}{2\left(c^2-d^2\right)f(a+a\sec(e}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{d\tan}{2\left(c^2-d^2\right)f(a+a\sec(e}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{d\tan}{2\left(c^2-d^2\right)f(a+a\sec(e}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} + \frac{3d\left(2c^2+2cd+d^2\right)\tan}{(c-d)^{7/2}(c+d)^{5/2}f\sqrt{a}}$$

Mathematica [C] time = 6.82, size = 1422, normalized size = 6.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 2*c*d + d^2)*Cos[e/2 + (f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e +
f*x]^4*(((-6*I)*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e
```

```
] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]])
)*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - (6*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 -
 d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2
*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(
Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^3*(c + d)^2*(a +
 a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e
 + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^4*(8*c^5*d*Sin[(f*x)/2] + 10*c^4*d^2*
Sin[(f*x)/2] - 11*c^3*d^3*Sin[(f*x)/2] - 17*c^2*d^4*Sin[(f*x)/2] - 2*c*d^5*
Sin[(f*x)/2] + 2*d^6*Sin[(f*x)/2] - 8*c^5*d*Sin[(3*f*x)/2] - 22*c^4*d^2*Sin
[(3*f*x)/2] - 27*c^3*d^3*Sin[(3*f*x)/2] - 5*c^2*d^4*Sin[(3*f*x)/2] + 2*c*d^
5*Sin[(3*f*x)/2] + 4*c^6*Sin[e - (f*x)/2] + 8*c^5*d*Sin[e - (f*x)/2] + 18*c
^4*d^2*Sin[e - (f*x)/2] + 35*c^3*d^3*Sin[e - (f*x)/2] + 25*c^2*d^4*Sin[e -
(f*x)/2] + 2*c*d^5*Sin[e - (f*x)/2] - 2*d^6*Sin[e - (f*x)/2] - 4*c^6*Sin[e
+ (f*x)/2] - 8*c^5*d*Sin[e + (f*x)/2] - 6*c^4*d^2*Sin[e + (f*x)/2] - 7*c^3*
d^3*Sin[e + (f*x)/2] + 5*c^2*d^4*Sin[e + (f*x)/2] + 2*c*d^5*Sin[e + (f*x)/2
] - 2*d^6*Sin[e + (f*x)/2] + 8*c^5*d*Sin[2*e + (f*x)/2] + 22*c^4*d^2*Sin[2*
e + (f*x)/2] + 17*c^3*d^3*Sin[2*e + (f*x)/2] + 13*c^2*d^4*Sin[2*e + (f*x)/2
] + 2*c*d^5*Sin[2*e + (f*x)/2] - 2*d^6*Sin[2*e + (f*x)/2] + 2*c^6*Sin[e + (
3*f*x)/2] + 4*c^5*d*Sin[e + (3*f*x)/2] - 4*c^4*d^2*Sin[e + (3*f*x)/2] - 19*
c^3*d^3*Sin[e + (3*f*x)/2] - 5*c^2*d^4*Sin[e + (3*f*x)/2] + 2*c*d^5*Sin[e +
 (3*f*x)/2] - 8*c^5*d*Sin[2*e + (3*f*x)/2] - 16*c^4*d^2*Sin[2*e + (3*f*x)/2
] - c^3*d^3*Sin[2*e + (3*f*x)/2] + 2*c^2*d^4*Sin[2*e + (3*f*x)/2] - 2*c*d^5
*Sin[2*e + (3*f*x)/2] + 2*c^6*Sin[3*e + (3*f*x)/2] + 4*c^5*d*Sin[3*e + (3*f
*x)/2] + 2*c^4*d^2*Sin[3*e + (3*f*x)/2] + 7*c^3*d^3*Sin[3*e + (3*f*x)/2] +
2*c^2*d^4*Sin[3*e + (3*f*x)/2] - 2*c*d^5*Sin[3*e + (3*f*x)/2] - 2*c^6*Sin[e
 + (5*f*x)/2] - 4*c^5*d*Sin[e + (5*f*x)/2] - 8*c^4*d^2*Sin[e + (5*f*x)/2] -
 2*c^3*d^3*Sin[e + (5*f*x)/2] + c^2*d^4*Sin[e + (5*f*x)/2] - 6*c^4*d^2*Sin[
2*e + (5*f*x)/2] - 2*c^3*d^3*Sin[2*e + (5*f*x)/2] + c^2*d^4*Sin[2*e + (5*f*
x)/2] - 2*c^6*Sin[3*e + (5*f*x)/2] - 4*c^5*d*Sin[3*e + (5*f*x)/2] - 2*c^4*d
^2*Sin[3*e + (5*f*x)/2]))/(8*c^2*(-c + d)^3*(c + d)^2*f*(a + a*Sec[e + f*x]
)*(c + d*Sec[e + f*x])^3)
```

fricas [B] time = 0.77, size = 1331, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*(3*(2*c^2*d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f
*x + e)^3 + (2*c^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4
*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*
c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(
```

```
f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f
*x + e) + d^2)) - 2*(2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 +
(2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3 - 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f
*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6
)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3
 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6 + a*c^2*d^7)*f*cos(f*x + e)^3 + (a
*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*
a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(f*x + e)^2 + (2*a*c^8*d - a*c^7*d^
2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a
*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a
*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f), -1/2*(3*(2*c^2*
d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f*x + e)^3 + (2*c
^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4*c^3*d^2 + 6*c^2
*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^
2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4*d^2 + 9*c^3*d^
3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 + (2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3
- 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3
*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a
*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6
+ a*c^2*d^7)*f*cos(f*x + e)^3 + (a*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^
3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(
f*x + e)^2 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d
^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d
^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*
c*d^8 + a*d^9)*f)]
```

giac [A] time = 0.72, size = 374, normalized size = 1.81

$$-\frac{\frac{3\left(2c^2d+2cd^2+d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5\right)\sqrt{-c^2+d^2}}-\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^3-3ac^2d+3acd^2-ad^3}+\frac{6c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3\cdots}{(a\cdots}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
+ d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*s
qrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*
d^3) + (6*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^3*tan(1/2*f*x + 1/2*e)^3 -
 3*d^4*tan(1/2*f*x + 1/2*e)^3 - 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 7*c*d^3*ta
```

```
n(1/2*f*x + 1/2*e) - d^4*tan(1/2*f*x + 1/2*e))/((a*c^5 - a*c^4*d - 2*a*c^3*
d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*
f*x + 1/2*e)^2 - c - d)^2))/f
```

maple [A] time = 1.01, size = 221, normalized size = 1.07

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c^3-3c^2d+3c\,d^2-d^3}+\frac{2d\left(\frac{-\frac{3d\left(2c^2-cd-d^2\right)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)}+\frac{d(6c+d)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2c+2d}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^2}-\frac{3\left(2c^2+2cd+d^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^3}}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f/a*(1/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*e+1/2*f*x)+2*d/(c-d)^3*((-3/2*d*
(2*c^2-c*d-d^2)/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3+1/2*d*(6*c+d)/(c+d)*ta
n(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2-3/2
*(2*c^2+2*c*d+d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/
2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.97, size = 379, normalized size = 1.83

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,(c-d)^3}-\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(d^3+6\,c\,d^2\right)}{c+d}+\frac{3}{}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(2\,a\,c^5-6\,a\,c^4\,d+4\,a\,c^3\,d^2+4\,a\,c^2\,d^3-6\,a\,c\,d^4+2\,a\,d^5\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(a\,c^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)^3) - ((tan(e/2 + (f*x)/2)*(6*c*d^2 + d^3))/
(c + d) + (3*tan(e/2 + (f*x)/2)^3*(c*d^3 + d^4 - 2*c^2*d^2))/(c + d)^2)/(f*
(tan(e/2 + (f*x)/2)^2*(2*a*c^5 + 2*a*d^5 + 4*a*c^2*d^3 + 4*a*c^3*d^2 - 6*a*
c*d^4 - 6*a*c^4*d) - tan(e/2 + (f*x)/2)^4*(a*c^5 - a*d^5 - 10*a*c^2*d^3 + 1
0*a*c^3*d^2 + 5*a*c*d^4 - 5*a*c^4*d) - a*c^5 + a*d^5 - 2*a*c^2*d^3 + 2*a*c^
3*d^2 - a*c*d^4 + a*c^4*d)) + (d*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(
e/2 + (f*x)/2)*1i - c*d^3*tan(e/2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*
4i + c^2*d^2*tan(e/2 + (f*x)/2)*6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(2*c*d +
 2*c^2 + d^2)*3i)/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^3\sec\left(e+fx\right)+c^3+3c^2d\sec^2\left(e+fx\right)+3c^2d\sec\left(e+fx\right)+3cd^2\sec^3\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^4\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**2
+ 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f*x)*
*2 + d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)**3), x)/a
```

3.217 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=258

$$\frac{5d^2(2c-d)\left(2c^2-3cd+2d^2\right)\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{d\left(c^2+10cd-12d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{3a^2f}-\frac{d\tan(e$$

```
[Out] 5/2*(2*c-d)*d^2*(2*c^2-3*c*d+2*d^2)*arctanh(sin(f*x+e))/a^2/f-1/3*d*(c^2+10
*c*d-12*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/a^2/f+1/3*(c-d)*(c+10*d)*(c+d*se
c(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+e))^4*
tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^4+40*c^3*d-176*c^2*d^2+160*c*d^3
-48*d^4+d*(2*c^3+20*c^2*d-57*c*d^2+30*d^3)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A] time = 0.44, antiderivative size = 315, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 98, 150, 153, 147, 63, 217, 203}

$$-\frac{d\left(c^2+10cd-12d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{3a^2f}-\frac{d\tan(e+fx)\left(d\left(20c^2d+2c^3-57cd^2+30d^3\right)\sec(e+fx\right.}{6a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]) - (d*(c^2 + 10*c*d - 12*d^2)*(c + d*Sec[e + f*x])^2*Ta
n[e + f*x])/(3*a^2*f) + ((c - d)*(c + 10*d)*(c + d*Sec[e + f*x])^3*Tan[e +
f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^4*Tan[
e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^4 + 10*c^3*d - 44*c^2*d^2
 + 40*c*d^3 - 12*d^4) + d*(2*c^3 + 20*c^2*d - 57*c*d^2 + 30*d^3)*Sec[e + f*
x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^3(-}{}\right.}{3af\sqrt{a-a\sec}}$$

$$= \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+}{3f(a+a\sec}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10}{3f}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10}{3f}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10}{3f}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10}{3f}$$

$$= \frac{5(2c-d)d^2\left(2c^2-3cd+2d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} - \frac{d\left(c^2\right.}{}$$

Mathematica [A] time = 4.03, size = 446, normalized size = 1.73

$$\frac{240d^2\left(-4c^3+8c^2d-7cd^2+2d^3\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (240*d^2*(-4*c^3 + 8*c^2*d - 7*c*d^2 + 2*d^3)*Cos[(e + f*x)/2]^4*(Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
 + 2*Cos[(e + f*x)/2]*(6*c^5 + 15*c^4*d - 120*c^3*d^2 + 420*c^2*d^3 - 300*c
```

```
*d^4 + 104*d^5 + (6*c^5 + 60*c^4*d - 300*c^3*d^2 + 840*c^2*d^3 - 585*c*d^4
+ 190*d^5)*Cos[e + f*x] + 4*(2*c^5 + 5*c^4*d - 40*c^3*d^2 + 130*c^2*d^3 - 9
5*c*d^4 + 30*d^5)*Cos[2*(e + f*x)] + 2*c^5*Cos[3*(e + f*x)] + 20*c^4*d*Cos[
3*(e + f*x)] - 100*c^3*d^2*Cos[3*(e + f*x)] + 280*c^2*d^3*Cos[3*(e + f*x)]
- 215*c*d^4*Cos[3*(e + f*x)] + 66*d^5*Cos[3*(e + f*x)] + 2*c^5*Cos[4*(e + f
*x)] + 5*c^4*d*Cos[4*(e + f*x)] - 40*c^3*d^2*Cos[4*(e + f*x)] + 100*c^2*d^3
*Cos[4*(e + f*x)] - 80*c*d^4*Cos[4*(e + f*x)] + 24*d^5*Cos[4*(e + f*x)])*Se
c[e + f*x]^3*Sin[(e + f*x)/2])/(24*a^2*f*(1 + Cos[e + f*x])^2)
```

fricas [A] time = 0.50, size = 456, normalized size = 1.77

$$\frac{15\left(\left(4\,c^3d^2-8\,c^2d^3+7\,cd^4-2\,d^5\right)\cos\left(fx+e\right)^5+2\left(4\,c^3d^2-8\,c^2d^3+7\,cd^4-2\,d^5\right)\cos\left(fx+e\right)^4+\left(4\,c^3d^2-8\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(15*((4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c
^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d
^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((4*c^3*d^
2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c^3*d^2 - 8*c^2*d^3
+ 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^
5)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*d^5 + 2*(2*c^5 + 5*c^4*d -
 40*c^3*d^2 + 100*c^2*d^3 - 80*c*d^4 + 24*d^5)*cos(f*x + e)^4 + (2*c^5 + 20
*c^4*d - 100*c^3*d^2 + 280*c^2*d^3 - 215*c*d^4 + 66*d^5)*cos(f*x + e)^3 + 6
*(10*c^2*d^3 - 5*c*d^4 + 2*d^5)*cos(f*x + e)^2 + (15*c*d^4 - 2*d^5)*cos(f*x
 + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f
*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-16/3*tan((f*x+exp(1))/2)^3*c^5*a^4+80/3*tan((f*x+exp(1))/2)^
3*c^4*a^4*d-160/3*tan((f*x+exp(1))/2)^3*c^3*a^4*d^2+160/3*tan((f*x+exp(1))/
2)^3*c^2*a^4*d^3-80/3*tan((f*x+exp(1))/2)^3*c*a^4*d^4+16/3*tan((f*x+exp(1))
```

```
/2)^3*a^4*d^5+16*tan((f*x+exp(1))/2)*c^5*a^4+80*tan((f*x+exp(1))/2)*c^4*a^4
*d-480*tan((f*x+exp(1))/2)*c^3*a^4*d^2+800*tan((f*x+exp(1))/2)*c^2*a^4*d^3-
560*tan((f*x+exp(1))/2)*c*a^4*d^4+144*tan((f*x+exp(1))/2)*a^4*d^5)*1/64/a^6
+(-60*tan((f*x+exp(1))/2)^5*c^2*d^3+75*tan((f*x+exp(1))/2)^5*c*d^4-30*tan((
f*x+exp(1))/2)^5*d^5+120*tan((f*x+exp(1))/2)^3*c^2*d^3-120*tan((f*x+exp(1))
/2)^3*c*d^4+40*tan((f*x+exp(1))/2)^3*d^5-60*tan((f*x+exp(1))/2)*c^2*d^3+45*
tan((f*x+exp(1))/2)*c*d^4-18*tan((f*x+exp(1))/2)*d^5)*1/6/a^2/(tan((f*x+exp
(1))/2)^2-1)^3-(20*c^3*d^2-40*c^2*d^3+35*c*d^4-10*d^5)*1/4/a^2*ln(abs(tan((
f*x+exp(1))/2)-1))+(20*c^3*d^2-40*c^2*d^3+35*c*d^4-10*d^5)*1/4/a^2*ln(abs(t
an((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.02, size = 766, normalized size = 2.97

$$\frac{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^5}{2f\,a^2}-\frac{d^5}{3f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)^3}+\frac{5\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)d^5}{f\,a^2}-\frac{5d^5}{f\,a^2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}-\frac{3}{2f\,a^2\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 9/2/f/a^2*tan(1/2*e+1/2*f*x)*d^5-1/3/f/a^2*d^5/(tan(1/2*e+1/2*f*x)-1)^3+5/f
/a^2*ln(tan(1/2*e+1/2*f*x)-1)*d^5-5/f/a^2*d^5/(tan(1/2*e+1/2*f*x)-1)-1/6/f/
a^2*tan(1/2*e+1/2*f*x)^3*c^5-3/2/f/a^2*d^5/(tan(1/2*e+1/2*f*x)-1)^2-1/3/f/a
^2*d^5/(tan(1/2*e+1/2*f*x)+1)^3-5/f/a^2*ln(tan(1/2*e+1/2*f*x)+1)*d^5-5/f/a^
2*d^5/(tan(1/2*e+1/2*f*x)+1)+3/2/f/a^2*d^5/(tan(1/2*e+1/2*f*x)+1)^2+1/6/f/a
^2*tan(1/2*e+1/2*f*x)^3*d^5+1/2/f/a^2*tan(1/2*e+1/2*f*x)*c^5+25/2/f/a^2*d^4
/(tan(1/2*e+1/2*f*x)-1)*c+5/6/f/a^2*tan(1/2*e+1/2*f*x)^3*c^4*d-5/3/f/a^2*ta
n(1/2*e+1/2*f*x)^3*c^3*d^2-5/2/f/a^2*d^4/(tan(1/2*e+1/2*f*x)+1)^2*c-20/f/a^
2*ln(tan(1/2*e+1/2*f*x)+1)*c^2*d^3-15/f/a^2*tan(1/2*e+1/2*f*x)*c^3*d^2+25/f
/a^2*tan(1/2*e+1/2*f*x)*c^2*d^3-35/2/f/a^2*tan(1/2*e+1/2*f*x)*c*d^4-10/f/a^
2*ln(tan(1/2*e+1/2*f*x)-1)*c^3*d^2+20/f/a^2*ln(tan(1/2*e+1/2*f*x)-1)*c^2*d^
3-35/2/f/a^2*ln(tan(1/2*e+1/2*f*x)-1)*c*d^4-10/f/a^2*d^3/(tan(1/2*e+1/2*f*x
)-1)*c^2+5/3/f/a^2*tan(1/2*e+1/2*f*x)^3*c^2*d^3-5/6/f/a^2*tan(1/2*e+1/2*f*x
)^3*c*d^4+5/2/f/a^2*tan(1/2*e+1/2*f*x)*c^4*d+35/2/f/a^2*ln(tan(1/2*e+1/2*f*
x)+1)*c*d^4-10/f/a^2*d^3/(tan(1/2*e+1/2*f*x)+1)*c^2+25/2/f/a^2*d^4/(tan(1/2
*e+1/2*f*x)+1)*c+5/2/f/a^2*d^4/(tan(1/2*e+1/2*f*x)-1)^2*c+10/f/a^2*ln(tan(1
/2*e+1/2*f*x)+1)*c^3*d^2
```

maxima [B] time = 0.36, size = 772, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) -
5*c*d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
 + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^2*d^3*(
(15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/
a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 10*c^3*d^2*((9*sin(f*x + e)/(co
s(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) -
1)/a^2) + 5*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(
f*x + e) + 1)^3)/a^2 + c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.97, size = 268, normalized size = 1.04

$$\frac{5\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2\,{\left(c-d\right)}^5}{a^2}-\frac{5\,\left(c+d\right)\,{\left(c-d\right)}^4}{2\,a^2}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,(c -}{6\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (5*d^2*atanh(tan(e/2 + (f*x)/2))*(2*c - d)*(2*c^2 - 3*c*d + 2*d^2))/(a^2*f)
 - (tan(e/2 + (f*x)/2)*((2*(c - d)^5)/a^2 - (5*(c + d)*(c - d)^4)/(2*a^2)))
/f - (tan(e/2 + (f*x)/2)^3*(c - d)^5)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*(6*d^
5 - 15*c*d^4 + 20*c^2*d^3) + tan(e/2 + (f*x)/2)^5*(10*d^5 - 25*c*d^4 + 20*c
^2*d^3) - tan(e/2 + (f*x)/2)^3*((40*d^5)/3 - 40*c*d^4 + 40*c^2*d^3))/(f*(3*
a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)
/2)^6 - a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^5 \sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^5 \sec^6(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{5cd^4 \sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{10c^2d^3 \sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**5*sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(5*c*d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(10*c**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1), x))/a**2
```

3.218 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{d^2\left(12c^2-16cd+7d^2\right)\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{d\tan(e+fx)\left(d\left(2c^2+16cd-21d^2\right)\sec(e+fx)+4\left(c^3+8c^2d-20\right.\right.}{6a^2f}$$

```
[Out] 1/2*d^2*(12*c^2-16*c*d+7*d^2)*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+8*d)*(
c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+
e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^3+32*c^2*d-80*c*d^2+32*d^3
+d*(2*c^2+16*c*d-21*d^2)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A] time = 0.32, antiderivative size = 249, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 150, 147, 63, 217, 203}

$$-\frac{d\tan(e+fx)\left(d\left(2c^2+16cd-21d^2\right)\sec(e+fx)+4\left(8c^2d+c^3-20cd^2+8d^3\right)\right)}{6a^2f}+\frac{(c-d)(c+8d)\tan(e+fx)(c\ldots}{3f\left(a^2\sec(e+fx)\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]) + ((c - d)*(c + 8*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(
a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(3
*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^3 + 8*c^2*d - 20*c*d^2 + 8*d^3) + d*(
2*c^2 + 16*c*d - 21*d^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
```

```
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^2\left(-a^2\right.}{}\right.}{3af\sqrt{a-a\sec(e}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx}{3f(a+a\sec(e}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx}{3f(a+a\sec(e}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx}{3f(a+a\sec(e}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx}{3f(a+a\sec(e}$$

$$= \frac{d^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+8}{3}$$

Mathematica [A] time = 2.83, size = 310, normalized size = 1.61

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(2c^4\cos(3(e+fx))+2c^4+4c^3d\cos(3(e+fx))+16c^3d-24c^2d^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-24*d^2*(12*c^2 - 16*c*d + 7*d^2)*Cos[(e + f*x)/2]^4*(Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[(e
 + f*x)/2]*(2*c^4 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 37*d^4 + 6*(c^4 + 2
*c^3*d - 12*c^2*d^2 + 28*c*d^3 - 10*d^4)*Cos[e + f*x] + (2*c^4 + 16*c^3*d -
 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*Cos[2*(e + f*x)] + 2*c^4*Cos[3*(e + f*x)]
 + 4*c^3*d*Cos[3*(e + f*x)] - 24*c^2*d^2*Cos[3*(e + f*x)] + 40*c*d^3*Cos[3*
(e + f*x)] - 16*d^4*Cos[3*(e + f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(12*
a^2*f*(1 + Cos[e + f*x])^2)
```

fricas [A] time = 0.46, size = 361, normalized size = 1.87

$$\frac{3\left(\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^4+2\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^3+\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^4 + 2*(12*c^2*d^2 - 1
6*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^2)*log(sin(f*x + e) + 1) - 3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^4 + 2*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16
*c*d^3 + 7*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(3*d^4 + 4*(c^4
+ 2*c^3*d - 12*c^2*d^2 + 20*c*d^3 - 8*d^4)*cos(f*x + e)^3 + (2*c^4 + 16*c^3
*d - 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*cos(f*x + e)^2 + 6*(4*c*d^3 - d^4)*co
s(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 +
a^2*f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-16/3*tan((f*x+exp(1))/2)^3*c^4*a^4+64/3*tan((f*x+exp(1))/2)^
3*c^3*a^4*d-32*tan((f*x+exp(1))/2)^3*c^2*a^4*d^2+64/3*tan((f*x+exp(1))/2)^3
*c*a^4*d^3-16/3*tan((f*x+exp(1))/2)^3*a^4*d^4+16*tan((f*x+exp(1))/2)*c^4*a^
4+64*tan((f*x+exp(1))/2)*c^3*a^4*d-288*tan((f*x+exp(1))/2)*c^2*a^4*d^2+320*
tan((f*x+exp(1))/2)*c*a^4*d^3-112*tan((f*x+exp(1))/2)*a^4*d^4)*1/64/a^6-(8*
```

```
tan((f*x+exp(1))/2)^3*c*d^3-5*tan((f*x+exp(1))/2)^3*d^4-8*tan((f*x+exp(1))/
2)*c*d^3+3*tan((f*x+exp(1))/2)*d^4)*1/2/a^2/(tan((f*x+exp(1))/2)^2-1)^2+(-1
2*c^2*d^2+16*c*d^3-7*d^4)*1/4/a^2*ln(abs(tan((f*x+exp(1))/2)-1))-(-12*c^2*d
^2+16*c*d^3-7*d^4)*1/4/a^2*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 0.84, size = 514, normalized size = 2.66

$$-\frac{c^4\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{6f\,a^2}+\frac{2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^3d}{3f\,a^2}-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2d^2}{f\,a^2}+\frac{2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c\,d^3}{3f\,a^2}-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^4}{6f\,a^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -1/6/f*c^4/a^2*tan(1/2*e+1/2*f*x)^3+2/3/f/a^2*tan(1/2*e+1/2*f*x)^3*c^3*d-1/
f/a^2*tan(1/2*e+1/2*f*x)^3*c^2*d^2+2/3/f/a^2*tan(1/2*e+1/2*f*x)^3*c*d^3-1/6
/f/a^2*tan(1/2*e+1/2*f*x)^3*d^4+1/2/f*c^4/a^2*tan(1/2*e+1/2*f*x)+2/f/a^2*ta
n(1/2*e+1/2*f*x)*c^3*d-9/f/a^2*tan(1/2*e+1/2*f*x)*c^2*d^2+10/f/a^2*tan(1/2*
e+1/2*f*x)*c*d^3-7/2/f/a^2*tan(1/2*e+1/2*f*x)*d^4-4/f/a^2*d^3/(tan(1/2*e+1/
2*f*x)-1)*c+5/2/f/a^2*d^4/(tan(1/2*e+1/2*f*x)-1)+1/2/f/a^2*d^4/(tan(1/2*e+1
/2*f*x)-1)^2-6/f/a^2*ln(tan(1/2*e+1/2*f*x)-1)*c^2*d^2+8/f/a^2*ln(tan(1/2*e+
1/2*f*x)-1)*c*d^3-7/2/f/a^2*ln(tan(1/2*e+1/2*f*x)-1)*d^4+6/f/a^2*ln(tan(1/2
*e+1/2*f*x)+1)*c^2*d^2-8/f/a^2*ln(tan(1/2*e+1/2*f*x)+1)*c*d^3+7/2/f/a^2*ln(
tan(1/2*e+1/2*f*x)+1)*d^4-4/f/a^2*d^3/(tan(1/2*e+1/2*f*x)+1)*c+5/2/f/a^2*d^
4/(tan(1/2*e+1/2*f*x)+1)-1/2/f/a^2*d^4/(tan(1/2*e+1/2*f*x)+1)^2
```

maxima [B] time = 0.37, size = 536, normalized size = 2.78

$$-\frac{d^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-4\,cd^3\left(\frac{15\sin(fx\cdots}{\cos(fx+e)\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 4*c*d^3*((
```

```
15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a
^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^2*d^2*((9*sin(f*x + e)/(cos(
f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/a^2) - 4*c^3*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.91, size = 193, normalized size = 1.00

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c\,d^3-3\,d^4\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(8\,c\,d^3-5\,d^4\right)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,(c-d)^4}{2\,a^2}-\frac{2\,(c+d)\,(c-d)^3}{a^2}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(8*c*d^3 - 3*d^4) - tan(e/2 + (f*x)/2)^3*(8*c*d^3 - 5*d
^4))/(f*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (t
an(e/2 + (f*x)/2)*((3*(c - d)^4)/(2*a^2) - (2*(c + d)*(c - d)^3)/a^2))/f -
(tan(e/2 + (f*x)/2)^3*(c - d)^4)/(6*a^2*f) + (d^2*atanh(tan(e/2 + (f*x)/2))
*(12*c^2 - 16*c*d + 7*d^2))/(a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^4\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\frac{d^4\sec^5\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\frac{4cd^3\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\frac{6c^2d^2\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right.}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(4*c*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) +
1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1), x))/a**2
```

3.219 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{\tan(e+fx)\left(c^3+4c^2d-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{d^2(3c-2d)\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{(c-d)\tan(e}{3f(a}$$

```
[Out] (3*c-2*d)*d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+d*sec(f*x+e))^2*tan(f*
x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c^3+4*c^2*d-12*c*d^2+10*d^3-(c-4*d)*d^2*sec(
f*x+e))*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 143, 63, 217, 203}

$$\frac{\tan(e+fx)\left(4c^2d+c^3-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2d^2(3c-2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(3*c - 2*d)*d^2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) +
 ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
 + ((c^3 + 4*c^2*d - 12*c*d^2 + 10*d^3 - (c - 4*d)*d^2*Sec[e + f*x])*Tan[e
+ f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)\left(-a^2\right.}{}\right.}{3af\sqrt{a-a\sec(e}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c^3+4c^2d-12cd^2+10d^3-(c\right.}{3f\left(a^2+a^2\right.}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c^3+4c^2d-12cd^2+10d^3-(c\right.}{3f\left(a^2+a^2\right.}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c^3+4c^2d-12cd^2+10d^3-(c\right.}{3f\left(a^2+a^2\right.}$$

$$= \frac{2(3c-2d)d^2\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+f}{3f(a+a\sec(e}$$

Mathematica [B] time = 1.70, size = 294, normalized size = 2.21

$$\frac{2\cos^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(2\left(2c^3+3c^2d-12cd^2+13d^3\right)\tan\left(\frac{1}{2}(e+fx)\right)+6d^2(2d-3c)\left(\log\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]^6*Sec[e + f*x]*(6*d^2*(-3*c + 2*d)*(Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 8*(c -
d)^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^4 + 32*(c - d)^3*Csc[e + f*x]^5*Sin[(e
 + f*x)/2]^8 + 2*(2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3)*Tan[(e + f*x)/2] + 6
*(3*c - 2*d)*d^2*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^2 - 2*(c - d)^2*(2*c + 7*d)*Ta
n[(e + f*x)/2]^3))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

fricas [B] time = 0.47, size = 268, normalized size = 2.02

$$\frac{3\left(\left(3\,cd^2-2\,d^3\right)\cos\left(fx+e\right)^3+2\left(3\,cd^2-2\,d^3\right)\cos\left(fx+e\right)^2+\left(3\,cd^2-2\,d^3\right)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((3*c*d^2 - 2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)
^2 + (3*c*d^2 - 2*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*((3*c*d^2 -
2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (3*c*d^2 - 2*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(3*d^3 + (2*c^3 + 3*c^2*d - 12
*c*d^2 + 10*d^3)*cos(f*x + e)^2 + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3)*cos(f
*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2
*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-16/3*tan((f*x+exp(1))/2)^3*c^3*a^4+16*tan((f*x+exp(1))/2)^3*
c^2*a^4*d-16*tan((f*x+exp(1))/2)^3*c*a^4*d^2+16/3*tan((f*x+exp(1))/2)^3*a^4
*d^3+16*tan((f*x+exp(1))/2)*c^3*a^4+48*tan((f*x+exp(1))/2)*c^2*a^4*d-144*ta
n((f*x+exp(1))/2)*c*a^4*d^2+80*tan((f*x+exp(1))/2)*a^4*d^3)*1/64/a^6-tan((f
*x+exp(1))/2)*d^3/a^2/(tan((f*x+exp(1))/2)^2-1)-(3*c*d^2-2*d^3)*1/2/a^2*ln(
abs(tan((f*x+exp(1))/2)-1))+(3*c*d^2-2*d^3)*1/2/a^2*ln(abs(tan((f*x+exp(1))
/2)+1)))
```

maple [B] time = 0.68, size = 316, normalized size = 2.38

$$-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^3}{6f\,a^2}+\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2d}{2f\,a^2}-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c\,d^2}{2f\,a^2}+\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^3}{6f\,a^2}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3}{2f\,a^2}+\frac{3c^2\tan}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -1/6/f/a^2*tan(1/2*e+1/2*f*x)^3*c^3+1/2/f/a^2*tan(1/2*e+1/2*f*x)^3*c^2*d-1/
2/f/a^2*tan(1/2*e+1/2*f*x)^3*c*d^2+1/6/f/a^2*tan(1/2*e+1/2*f*x)^3*d^3+1/2/f
/a^2*tan(1/2*e+1/2*f*x)*c^3+3/2/f/a^2*c^2*tan(1/2*e+1/2*f*x)*d-9/2/f/a^2*ta
n(1/2*e+1/2*f*x)*c*d^2+5/2/f/a^2*tan(1/2*e+1/2*f*x)*d^3-1/f/a^2*d^3/(tan(1/
2*e+1/2*f*x)-1)-3/f/a^2*d^2*ln(tan(1/2*e+1/2*f*x)-1)*c+2/f/a^2*d^3*ln(tan(1
/2*e+1/2*f*x)-1)-1/f/a^2*d^3/(tan(1/2*e+1/2*f*x)+1)+3/f/a^2*d^2*ln(tan(1/2*
e+1/2*f*x)+1)*c-2/f/a^2*d^3*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.35, size = 342, normalized size = 2.57

$$\frac{d^3\left(\frac{\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}{a^2}-\frac{12\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^2}+\frac{12\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^2}+\frac{12\,\sin\left(fx+e\right)}{\left(a^2-\frac{a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)\left(\cos\left(fx+e\right)+1\right)}\right)-3\,cd^2\left(\frac{\frac{9\,\sin\left(fx+\cdots\right.}{\cos\left(fx+e\right)\cdots}\cdots}{\cdots}\right.}{6\,f\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 3*c*d^2*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/a^2) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2 + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.88, size = 136, normalized size = 1.02

$$\frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(3\,c-2\,d\right)}{a^2\,f}-\frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(c-d\right)^3}{6\,a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{\left(c-d\right)^3}{a^2}-\cdots\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*atanh(tan(e/2 + (f*x)/2))*(3*c - 2*d))/(a^2*f) - (2*d^3*tan(e/2 + (f
*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2)) - (tan(e/2 + (f*x)/2)^3*(c - d
```

```
)^3)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^3/a^2 - (3*(c + d)*(c - d)^2)
/(2*a^2)))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^3\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{3cd^2\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{3c^2d\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(3*c*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
 x))/a**2
```

3.220 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{(c+5d)(c-d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{d^2\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{(c-d)^2\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1
/3*(c-d)*(c+5*d)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.67, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 89, 78, 63, 217, 203}

$$\frac{(c+5d)(c-d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)^2\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}+\frac{2d^2\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + (2*d^2*ArcTan[Sqrt[
a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*(c + 5*d)*Tan[e + f*x
])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{a^3\left(c^2+4cd-2d^2\right)+3a^3d^2x}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x,\right.}{3a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} - \frac{\left(d^2\tan(e+fx)\right)}{f\sqrt{a-a}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{\left(2d^2\tan(e+fx)\right)}{af\sqrt{a-}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{\left(2d^2\tan(e+fx)\right)}{af\sqrt{a-a\,\text{s}}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{2d^2\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c}{3}$$

Mathematica [B] time = 0.78, size = 181, normalized size = 2.03

$$-\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-4\left(c^2+cd-2d^2\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+(c-d)^2\tan\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)+(c-\right.}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*Cos[(e + f*x)/2]*(6*d^2*Cos[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)^2*Sec[e
/2]*Sin[(f*x)/2] - 4*(c^2 + c*d - 2*d^2)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f
*x)/2] + (c - d)^2*Cos[(e + f*x)/2]*Tan[e/2]))/(3*a^2*f*(1 + Cos[e + f*x])^
2)
```

fricas [A] time = 0.46, size = 155, normalized size = 1.74

$$\frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right.}{6\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(sin(f*x + e) + 1
) - 3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(-sin(f*x + e) + 1
) + 2*(c^2 + 4*c*d - 5*d^2 + 2*(c^2 + c*d - 2*d^2)*cos(f*x + e))*sin(f*x +
e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-d^2*1/2/a^2*ln(abs(tan((f*x+exp(1))/2)-1))+d^2*1/2/a^2*ln(abs
(tan((f*x+exp(1))/2)+1))+(-16/3*tan((f*x+exp(1))/2)^3*c^2*a^4+32/3*tan((f*x
+exp(1))/2)^3*c*a^4*d-16/3*tan((f*x+exp(1))/2)^3*a^4*d^2+16*tan((f*x+exp(1)
)/2)*c^2*a^4+32*tan((f*x+exp(1))/2)*c*a^4*d-48*tan((f*x+exp(1))/2)*a^4*d^2)
*1/64/a^6)
```

maple [A] time = 0.77, size = 170, normalized size = 1.91

$$\frac{cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,a^{2}}+\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{3f\,a^{2}}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^{2}}{2f\,a^{2}}-\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^{2}}{2f\,a^{2}}-\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^{2}}{6f\,a^{2}}-\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{6f\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/f/a^2*c*d*tan(1/2*e+1/2*f*x)+1/3/f/a^2*tan(1/2*e+1/2*f*x)^3*c*d+1/2/f/a^2
*tan(1/2*e+1/2*f*x)*c^2-3/2/f/a^2*tan(1/2*e+1/2*f*x)*d^2-1/6/f/a^2*tan(1/2*
e+1/2*f*x)^3*c^2-1/6/f/a^2*tan(1/2*e+1/2*f*x)^3*d^2-1/f/a^2*d^2*ln(tan(1/2*
e+1/2*f*x)-1)+1/f/a^2*d^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.34, size = 195, normalized size = 2.19

$$-\frac{d^{2}\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{a^{2}}-\frac{6\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}}+\frac{6\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^{2}}\right)-\frac{2\,cd\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{a^{2}}-\frac{c^{2}\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\cdots\right)}{a^{2}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.77, size = 89, normalized size = 1.00

$$\frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{{\left(c-d\right)}^2}{2\,a^2}-\frac{c^2-d^2}{a^2}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,{\left(c-d\right)}^2}{6\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^2/
(2*a^2) - (c^2 - d^2)/a^2))/f - (tan(e/2 + (f*x)/2)^3*(c - d)^2)/(6*a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^2\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(2*c*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a*
*2
```

3.221 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(c+2d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] 1/3*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c+2*d)*tan(f*x+e)/f/(a^2+a^2
*sec(f*x+e))
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4000, 3794}

$$\frac{(c+2d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c + 2*d)*Tan[e + f*
x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c+2d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}$$
$$= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

Mathematica [A] time = 0.22, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left((2c+d)\sin\left(e+\frac{3fx}{2}\right)+3(c+d)\sin\left(\frac{fx}{2}\right)-3c\sin\left(e+\frac{fx}{2}\right)\right)}{3a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(3*(c + d)*Sin[(f*x)/2] - 3*c*Sin[e + (f*x)/2] +
 (2*c + d)*Sin[e + (3*f*x)/2]))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

fricas [A] time = 0.45, size = 58, normalized size = 0.89

$$\frac{\left((2\,c+d)\cos\left(fx+e\right)+c+2\,d\right)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*((2*c + d)*cos(f*x + e) + c + 2*d)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 +
 2*a^2*f*cos(f*x + e) + a^2*f)
```

giac [A] time = 0.29, size = 64, normalized size = 0.98

$$-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{6\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(c*tan(1/2*f*x + 1/2*e)^3 - d*tan(1/2*f*x + 1/2*e)^3 - 3*c*tan(1/2*f*x
 + 1/2*e) - 3*d*tan(1/2*f*x + 1/2*e))/(a^2*f)
```

maple [A] time = 0.72, size = 60, normalized size = 0.92

$$\frac{-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c}{3}+\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{2f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/2/f/a^2*(-1/3*tan(1/2*e+1/2*f*x)^3*c+1/3*tan(1/2*e+1/2*f*x)^3*d+tan(1/2*e
+1/2*f*x)*c+tan(1/2*e+1/2*f*x)*d)
```

maxima [A] time = 0.35, size = 93, normalized size = 1.43

$$\frac{\frac{d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/a^2)/f
```

mupad [B] time = 1.71, size = 45, normalized size = 0.69

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c+d)}{2\,a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(c-d)}{6\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c + d))/(2*a^2*f) - (tan(e/2 + (f*x)/2)^3*(c - d))/(6*
a^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

3.222 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=129

$$\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2f(c-d)^{5/2}\sqrt{c+d}}+\frac{(c-4d)\tan(e+fx)}{3f(c-d)^2\left(a^2\sec(e+fx)+a^2\right)}+\frac{\tan(e+fx)}{3f(c-d)(a\sec(e+fx)+a)^2}$$

```
[Out] 2*d^2*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/a^2/(c-d)^(5/2)/f
/(c+d)^(1/2)+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2+1/3*(c-4*d)*tan(f*x+
e)/(c-d)^2/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 104, 152, 12, 93, 205}

$$\frac{(c-4d)\tan(e+fx)}{3f(c-d)^2\left(a^2\sec(e+fx)+a^2\right)}-\frac{2d^2\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{5/2}\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{\tan(e+f}{3f(c-d)(a\sec(e}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(3*(c - d)*f*(a + a*Sec[e + f*x])^2) - (2*d^2*ArcTan[(Sqrt[c +
 d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e
 + f*x])/(a*(c - d)^(5/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]) + ((c - 4*d)*Tan[e + f*x])/(3*(c - d)^2*f*(a^2 + a^2*Sec[e
+ f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{5/2}(c+dx)}\,dx, x, \sec(e+fx)\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{-a^2(c-3d}{\sqrt{a-ax}\,(a+a}\right.}{3a(c-d)f\sqrt{a-a\sec(e+}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} - \frac{2d^2\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(}}{\sqrt{c-d}\,\sqrt{a-a\sec(}}\right.}{a(c-d)^{5/2}\sqrt{c+d}\,f\sqrt{a-a\sec(e}}$$

Mathematica [C] time = 1.65, size = 209, normalized size = 1.62

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(\sec\left(\frac{e}{2}\right)\left(-3(c-2d)\sin\left(e+\frac{fx}{2}\right)+(2c-5d)\sin\left(e+\frac{3fx}{2}\right)+3(c-3d)\sin\left(\frac{fx}{2}\right)\right)-\frac{24id^2(\cos(e)-i\sin}{}\right.}{3a^2f(c-d)^2(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*(((-24*I)*d^2*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d
 + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*
Cos[(e + f*x)/2]^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*S
in[e])^2]) + Sec[e/2]*(3*(c - 3*d)*Sin[(f*x)/2] - 3*(c - 2*d)*Sin[e + (f*x)
/2] + (2*c - 5*d)*Sin[e + (3*f*x)/2])))/(3*a^2*(c - d)^2*f*(1 + Cos[e + f*x
])^2)
```

fricas [B] time = 0.48, size = 598, normalized size = 4.64

$$\left[\frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin(}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{6\left(\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)f\cos\left(fx+e\right)^2+2\left(a^2c^4-2\,a^2c^3d+2\,a^2c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d -
 2*c*d^2 + 5*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a
^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d
^3 - a^2*d^4)*f*cos(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d
^4)*f), 1/3*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(-c^2 +
d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e
))) + (c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4
)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*co
s(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)]
```

giac [B] time = 0.59, size = 258, normalized size = 2.00

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{\left(a^2c^2-2\,a^2cd+a^2d^2\right)\sqrt{-c^2+d^2}}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2\,a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3\,a^4}{a^6c^3-3\,a^6c^2d+3\,a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((a^2*c^2
- 2*a^2*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + (a^4*c^2*tan(1/2*f*x + 1/2*e)^3
- 2*a^4*c*d*tan(1/2*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4
*c^2*tan(1/2*f*x + 1/2*e) + 12*a^4*c*d*tan(1/2*f*x + 1/2*e) - 9*a^4*d^2*tan
(1/2*f*x + 1/2*e))/(a^6*c^3 - 3*a^6*c^2*d + 3*a^6*c*d^2 - a^6*d^3))/f
```

maple [A] time = 0.74, size = 122, normalized size = 0.95

$$\frac{-\frac{\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c}{3}-\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d}{3}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c+3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{(c-d)^{2}}+\frac{4d^{2}\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)^{2}\sqrt{(c+d)(c-d)}}}{2fa^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/2/f/a^2*(-1/(c-d)^2*(1/3*tan(1/2*e+1/2*f*x)^3*c-1/3*tan(1/2*e+1/2*f*x)^3*
d-tan(1/2*e+1/2*f*x)*c+3*tan(1/2*e+1/2*f*x)*d)+4*d^2/(c-d)^2/((c+d)*(c-d))^
(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 1.92, size = 168, normalized size = 1.30

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1}{a^{2}(c-d)}-\frac{c+d}{2a^{2}(c-d)^{2}}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}}{6a^{2}f(c-d)}-\frac{d^{2}\operatorname{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^{3}-3\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^{2}d+3\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)cd^{2}-1\mathrm{i}\tan\left(\frac{e}{2}+\right.}{\sqrt{c+d}\,(c-d)^{5/2}}\right.}{a^{2}f\sqrt{c+d}\,(c-d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(1/(a^2*(c - d)) - (c + d)/(2*a^2*(c - d)^2)))/f - tan(
e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)) - (d^2*atan((c^3*tan(e/2 + (f*x)/2)*1i -
 d^3*tan(e/2 + (f*x)/2)*1i + c*d^2*tan(e/2 + (f*x)/2)*3i - c^2*d*tan(e/2 +
(f*x)/2)*3i)/((c + d)^(1/2)*(c - d)^(5/2)))*2i)/(a^2*f*(c + d)^(1/2)*(c - d
)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c\sec^2\left(e+fx\right)+2c\sec\left(e+fx\right)+c+d\sec^3\left(e+fx\right)+2d\sec^2\left(e+fx\right)+d\sec\left(e+fx\right)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**2 + 2*c*sec(e + f*x) + c + d*sec(e +
 f*x)**3 + 2*d*sec(e + f*x)**2 + d*sec(e + f*x)), x)/a**2
```

3.223 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{d\left(c^2-6cd-10d^2\right)\tan(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sec(e+fx))}+\frac{2d^2(3c+2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2f(c-d)^{7/2}(c+d)^{3/2}}+\frac{(c-6d)\tan(e+}{3a^2f(c-d)^2(\sec(e+fx)+1)(}$$

[Out] 2*d^2*(3*c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/a^2/(c-d)^(7/2)/(c+d)^(3/2)/f+1/3*d*(c^2-6*c*d-10*d^2)*tan(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))+1/3*(c-6*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(c+d*sec(f*x+e))+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))

Rubi [A] time = 0.37, antiderivative size = 260, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 103, 152, 12, 93, 205}

$$\frac{\left(c^2-6cd-10d^2\right)\tan(e+fx)}{3f(c-d)^3(c+d)\left(a^2\sec(e+fx)+a^2\right)}-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}-\frac{2d^2(3c+2d)\,\mathrm{ta}}{af(c-d)^{7/2}(c+d)^{3}}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]

[Out] ((c + 4*d)*Tan[e + f*x])/(3*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^2) - (2*d^2*(3*c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a*(c - d)^(7/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c^2 - 6*c*d - 10*d^2)*Tan[e + f*x])/(3*(c - d)^3*(c + d)*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{5/2}(c+dx)^2}\,dx, x, \sec(e+f\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(e+f}{\left(c^2-\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{d\tan}{\left(c^2-d^2\right)f(a+a\sec(e}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{\left(c^2-6cd-10d^2\right)}{3(c-d)^3(c+d)f\left(a^2+\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{\left(c^2-6cd-10d^2\right)}{3(c-d)^3(c+d)f\left(a^2+\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{\left(c^2-6cd-10d^2\right)}{3(c-d)^3(c+d)f\left(a^2+\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{2d^2(3c+2d)\tan^{-1}}{a(c-d)^{7/2}(c+d)^{3/2}f\sqrt{}}$$

Mathematica [C] time = 3.96, size = 376, normalized size = 1.78

$$2\cos\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)(c\cos(e+fx)+d)\left(\frac{12d^2(3c+2d)(\sin(e)+i\cos(e))\cos^3\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))}{\sqrt{c^2-}}\right.}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^4*((12*d^2*(3*c + 2*d
)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]^3*(d + c*Cos[
```

```
e + f*x])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Si
n[e])^2]) + (c - d)*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] - 4*(c - 4*d
)*Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (6*d^3*Co
s[(e + f*x)/2]^3*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2
])*(Cos[e/2] + Sin[e/2])) + (c - d)*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*T
an[e/2]))/(3*a^2*(-c + d)^3*f*(1 + Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2)
```

fricas [B] time = 0.53, size = 1242, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/6*(3*(3*c*d^3 + 2*d^4 + (3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d
^2 + 7*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*
x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e
)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c
^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^4*d - 6*c^3*d^2 - 11*
c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^3*d^2 + 3*c^2*d^3 + 8*
c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 +
13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^7 - 2*a^2*c^6*d - a^
2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d^5 + a^2*c*d^6)*f*cos(
f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2 + 7*a^2*c^4*d^3 + 2*a
^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^2 + (a^2*c^7 - 5*a^2*c
^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^
2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^
3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f), 1/3*(3*(3*c*d^3 + 2*d^4 +
(3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d^2 + 7*c*d^3 + 2*d^4)*cos(f*
x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e))*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (c
^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^
3*d^2 + 3*c^2*d^3 + 8*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c
^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^7 - 2*a^2*c^6*d - a^2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d
^5 + a^2*c*d^6)*f*cos(f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2
 + 7*a^2*c^4*d^3 + 2*a^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^
2 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^
5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 -
a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f)]
```

giac [B] time = 0.39, size = 490, normalized size = 2.32

$$\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}+\frac{12\left(3\,cd^2+2\,d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\,t}{\sqrt{-c^2+d}}\right.\right.}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^
2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) + 12*
(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arcta
n(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a
^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)) - (a^4*c^4*
tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^2*d^2
*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 + a^4*d^4*tan(
1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1/2*e) + 24*a^4*c^3*d*tan(1/2*
f*x + 1/2*e) - 54*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e) + 48*a^4*c*d^3*tan(1/2*f
*x + 1/2*e) - 15*a^4*d^4*tan(1/2*f*x + 1/2*e))/(a^6*c^6 - 6*a^6*c^5*d + 15*
a^6*c^4*d^2 - 20*a^6*c^3*d^3 + 15*a^6*c^2*d^4 - 6*a^6*c*d^5 + a^6*d^6))/f
```

maple [A] time = 0.80, size = 203, normalized size = 0.96

$$\frac{-\frac{\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c}{3}-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d}{3}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{(c+d)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}-\frac{(3c+2d)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^3}}{2f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/2/f/a^2*(-1/(c^2-2*c*d+d^2)/(c-d)*(1/3*tan(1/2*e+1/2*f*x)^3*c-1/3*tan(1/2
*e+1/2*f*x)^3*d-tan(1/2*e+1/2*f*x)*c+5*tan(1/2*e+1/2*f*x)*d)-4*d^2/(c-d)^3*
(-d/(c+d)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d
-c-d)-(3*c+2*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/
((c+d)*(c-d))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.18, size = 314, normalized size = 1.49

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{3}{2\,a^2\,(c-d)^2}-\frac{c^2-d^2}{a^2\,(c-d)^4}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6\,a^2\,f\,(c-d)^2}+\frac{2\,d^3\,\tan\left(\frac{e}{2}+\right.}{f\,(c+d)\left(a^2\,d^4-a^2\,c^4+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(a^2\,c^4-4\,a^2\,c^3\,d+6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3/(2*a^2*(c - d)^2) - (c^2 - d^2)/(a^2*(c - d)^4)))/f
- tan(e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^2) + (2*d^3*tan(e/2 + (f*x)/2))/(f*
(c + d)*(a^2*d^4 - a^2*c^4 + tan(e/2 + (f*x)/2)^2*(a^2*c^4 + a^2*d^4 - 4*a^
2*c*d^3 - 4*a^2*c^3*d + 6*a^2*c^2*d^2) - 2*a^2*c*d^3 + 2*a^2*c^3*d)) - (d^2
*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(e/2 + (f*x)/2)*1i - c*d^3*tan(e/
2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*4i + c^2*d^2*tan(e/2 + (f*x)/2)*
6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(3*c + 2*d)*2i)/(a^2*f*(c + d)^(3/2)*(c
- d)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^2\sec^2\left(e+fx\right)+2c^2\sec\left(e+fx\right)+c^2+2cd\sec^3\left(e+fx\right)+4cd\sec^2\left(e+fx\right)+2cd\sec\left(e+fx\right)+d^2\sec^4\left(e+fx\right)+2d^2\sec^3\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**2 + 2*c**2*sec(e + f*x) + c**2 +
2*c*d*sec(e + f*x)**3 + 4*c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*s
ec(e + f*x)**4 + 2*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**2
```

3.224 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{d^2\left(12c^2+16cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2f(c-d)^{9/2}(c+d)^{5/2}}+\frac{d\left(2c^2-16cd-21d^2\right)\tan(e+fx)}{6a^2f(c-d)^3(c+d)(c+d\sec(e+fx))^2}+\frac{d\left(2c^3-16c^2d-59\right.}{6a^2f(c-d)^4(c+}$$

```
[Out] d^2*(12*c^2+16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2
))/a^2/(c-d)^(9/2)/(c+d)^(5/2)/f+1/6*d*(2*c^2-16*c*d-21*d^2)*tan(f*x+e)/a^2
/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2+1/3*(c-8*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1
+sec(f*x+e))/(c+d*sec(f*x+e))^2+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(
c+d*sec(f*x+e))^2+1/6*d*(2*c^3-16*c^2*d-59*c*d^2-32*d^3)*tan(f*x+e)/a^2/(c-
d)^4/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.56, antiderivative size = 346, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 103, 151, 152, 12, 93, 205}

$$\frac{\left(-16c^2d+2c^3-59cd^2-32d^3\right)\tan(e+fx)}{6f(c-d)^4(c+d)^2\left(a^2\sec(e+fx)+a^2\right)}-\frac{d^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{9/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 22*c*d + 11*d^2)*Tan[e + f*x])/(6*(c - d)^3*(c + d)^2*f*(a + a*Se
c[e + f*x])^2) - (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a
+ a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a
*(c - d)^(9/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) + ((2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Tan[e + f*x])/(6*(c - d)^4
*(c + d)^2*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2) - (d*(5*c + 2*d)*Tan[e + f*x
])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{5/2}(c+dx)^3}\,dx, x, \sec(e+f\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{\tan(e}{2}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{}{2\left(c^2-\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d\,t}{2\left(c^2-d^2\right)f(a+a\sec}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^3-16c^2d-59cd^2\right.}{6(c-d)^4(c+d)^2f\left(a\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^3-16c^2d-59cd^2\right.}{6(c-d)^4(c+d)^2f\left(a\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^3-16c^2d-59cd^2\right.}{6(c-d)^4(c+d)^2f\left(a\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d^2\left(12c^2+16cd+7d^2\right.}{a(c-d)^{9/2}(c+d)^{5/2}f}$$

Mathematica [C] time = 7.35, size = 2220, normalized size = 7.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((12*c^2 + 16*c*d + 7*d^2)*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[
e + f*x]^5*(((-4*I)*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[C
os[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[
2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d
^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (4*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sq
rt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sq
rt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*S
in[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d
)^2*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d
 + c*Cos[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-16*c^7*Sin[(f*x)/2] + 1
4*c^6*d*Sin[(f*x)/2] + 220*c^5*d^2*Sin[(f*x)/2] + 334*c^4*d^3*Sin[(f*x)/2]
+ 54*c^3*d^4*Sin[(f*x)/2] - 156*c^2*d^5*Sin[(f*x)/2] - 48*c*d^6*Sin[(f*x)/2
] + 18*d^7*Sin[(f*x)/2] + 14*c^7*Sin[(3*f*x)/2] - 16*c^6*d*Sin[(3*f*x)/2] -
 226*c^5*d^2*Sin[(3*f*x)/2] - 532*c^4*d^3*Sin[(3*f*x)/2] - 583*c^3*d^4*Sin[
(3*f*x)/2] - 232*c^2*d^5*Sin[(3*f*x)/2] - 6*c*d^6*Sin[(3*f*x)/2] + 6*d^7*Si
n[(3*f*x)/2] - 12*c^7*Sin[e - (f*x)/2] + 20*c^6*d*Sin[e - (f*x)/2] + 236*c^
5*d^2*Sin[e - (f*x)/2] + 628*c^4*d^3*Sin[e - (f*x)/2] + 778*c^3*d^4*Sin[e -
 (f*x)/2] + 420*c^2*d^5*Sin[e - (f*x)/2] + 48*c*d^6*Sin[e - (f*x)/2] - 18*d
^7*Sin[e - (f*x)/2] + 12*c^7*Sin[e + (f*x)/2] - 20*c^6*d*Sin[e + (f*x)/2] -
 236*c^5*d^2*Sin[e + (f*x)/2] - 460*c^4*d^3*Sin[e + (f*x)/2] - 310*c^3*d^4*
Sin[e + (f*x)/2] + 39*c^2*d^5*Sin[e + (f*x)/2] + 48*c*d^6*Sin[e + (f*x)/2]
- 18*d^7*Sin[e + (f*x)/2] - 16*c^7*Sin[2*e + (f*x)/2] + 14*c^6*d*Sin[2*e +
(f*x)/2] + 220*c^5*d^2*Sin[2*e + (f*x)/2] + 502*c^4*d^3*Sin[2*e + (f*x)/2]
+ 522*c^3*d^4*Sin[2*e + (f*x)/2] + 303*c^2*d^5*Sin[2*e + (f*x)/2] + 48*c*d^
6*Sin[2*e + (f*x)/2] - 18*d^7*Sin[2*e + (f*x)/2] - 6*c^7*Sin[e + (3*f*x)/2]
 + 6*c^6*d*Sin[e + (3*f*x)/2] + 126*c^5*d^2*Sin[e + (3*f*x)/2] + 114*c^4*d^
3*Sin[e + (3*f*x)/2] - 159*c^3*d^4*Sin[e + (3*f*x)/2] - 144*c^2*d^5*Sin[e +
 (3*f*x)/2] - 6*c*d^6*Sin[e + (3*f*x)/2] + 6*d^7*Sin[e + (3*f*x)/2] + 14*c^
7*Sin[2*e + (3*f*x)/2] - 16*c^6*d*Sin[2*e + (3*f*x)/2] - 226*c^5*d^2*Sin[2*
e + (3*f*x)/2] - 412*c^4*d^3*Sin[2*e + (3*f*x)/2] - 235*c^3*d^4*Sin[2*e + (
3*f*x)/2] - 7*c^2*d^5*Sin[2*e + (3*f*x)/2] + 6*c*d^6*Sin[2*e + (3*f*x)/2] -
 6*d^7*Sin[2*e + (3*f*x)/2] - 6*c^7*Sin[3*e + (3*f*x)/2] + 6*c^6*d*Sin[3*e
+ (3*f*x)/2] + 126*c^5*d^2*Sin[3*e + (3*f*x)/2] + 234*c^4*d^3*Sin[3*e + (3*
f*x)/2] + 189*c^3*d^4*Sin[3*e + (3*f*x)/2] + 81*c^2*d^5*Sin[3*e + (3*f*x)/2
] + 6*c*d^6*Sin[3*e + (3*f*x)/2] - 6*d^7*Sin[3*e + (3*f*x)/2] + 6*c^7*Sin[e
 + (5*f*x)/2] - 14*c^6*d*Sin[e + (5*f*x)/2] - 134*c^5*d^2*Sin[e + (5*f*x)/2
] - 274*c^4*d^3*Sin[e + (5*f*x)/2] - 193*c^3*d^4*Sin[e + (5*f*x)/2] - 27*c^
```

```
2*d^5*Sin[e + (5*f*x)/2] + 6*c*d^6*Sin[e + (5*f*x)/2] - 6*c^7*Sin[2*e + (5*
f*x)/2] + 12*c^6*d*Sin[2*e + (5*f*x)/2] + 42*c^5*d^2*Sin[2*e + (5*f*x)/2] -
 48*c^4*d^3*Sin[2*e + (5*f*x)/2] - 105*c^3*d^4*Sin[2*e + (5*f*x)/2] - 27*c^
2*d^5*Sin[2*e + (5*f*x)/2] + 6*c*d^6*Sin[2*e + (5*f*x)/2] + 6*c^7*Sin[3*e +
 (5*f*x)/2] - 14*c^6*d*Sin[3*e + (5*f*x)/2] - 134*c^5*d^2*Sin[3*e + (5*f*x)
/2] - 202*c^4*d^3*Sin[3*e + (5*f*x)/2] - 61*c^3*d^4*Sin[3*e + (5*f*x)/2] +
12*c^2*d^5*Sin[3*e + (5*f*x)/2] - 6*c*d^6*Sin[3*e + (5*f*x)/2] - 6*c^7*Sin[
4*e + (5*f*x)/2] + 12*c^6*d*Sin[4*e + (5*f*x)/2] + 42*c^5*d^2*Sin[4*e + (5*
f*x)/2] + 24*c^4*d^3*Sin[4*e + (5*f*x)/2] + 27*c^3*d^4*Sin[4*e + (5*f*x)/2]
 + 12*c^2*d^5*Sin[4*e + (5*f*x)/2] - 6*c*d^6*Sin[4*e + (5*f*x)/2] + 4*c^7*S
in[2*e + (7*f*x)/2] - 14*c^6*d*Sin[2*e + (7*f*x)/2] - 40*c^5*d^2*Sin[2*e +
(7*f*x)/2] - 46*c^4*d^3*Sin[2*e + (7*f*x)/2] - 12*c^3*d^4*Sin[2*e + (7*f*x)
/2] + 3*c^2*d^5*Sin[2*e + (7*f*x)/2] - 24*c^4*d^3*Sin[3*e + (7*f*x)/2] - 12
*c^3*d^4*Sin[3*e + (7*f*x)/2] + 3*c^2*d^5*Sin[3*e + (7*f*x)/2] + 4*c^7*Sin[
4*e + (7*f*x)/2] - 14*c^6*d*Sin[4*e + (7*f*x)/2] - 40*c^5*d^2*Sin[4*e + (7*
f*x)/2] - 22*c^4*d^3*Sin[4*e + (7*f*x)/2]))/(48*c^2*(-c + d)^4*(c + d)^2*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3)
```

fricas [B] time = 0.61, size = 2030, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2*
d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*co
s(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*co
s(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^
3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 3
2*c^4*d^3 + 28*c^3*d^4 + 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2
*c^7 - 8*c^6*d - 68*c^5*d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c
*d^6 + 6*d^7)*cos(f*x + e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^
3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^10 - 2*a^2*c^9*d - 2*a^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*
c^4*d^6 + 2*a^2*c^3*d^7 - a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2
*c^9*d - 4*a^2*c^8*d^2 + 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*
a^2*c^4*d^6 + 4*a^2*c^3*d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (
a^2*c^10 + 2*a^2*c^9*d - 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 2
2*a^2*c^4*d^6 + 4*a^2*c^3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*c
os(f*x + e)^2 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4
```

```
+ 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9
 - a^2*d^10)*f*cos(f*x + e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f
), 1/6*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2
*d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*c
os(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*c
os(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)
)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^
2)*sin(f*x + e))) + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*
c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 32*c^4*d^3 + 28*c^3*d^4 +
 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2*c^7 - 8*c^6*d - 68*c^5*
d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c*d^6 + 6*d^7)*cos(f*x +
e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134
*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^10 - 2*a^2*c^9*d - 2*a
^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*c^4*d^6 + 2*a^2*c^3*d^7
- a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2*c^9*d - 4*a^2*c^8*d^2 +
 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*a^2*c^4*d^6 + 4*a^2*c^3*
d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (a^2*c^10 + 2*a^2*c^9*d -
 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 22*a^2*c^4*d^6 + 4*a^2*c^
3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 + 2*(a^2*c
^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x +
 e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f)]
```

giac [B] time = 0.53, size = 776, normalized size = 2.73

$$\frac{\frac{6\left(12c^2d^2+16cd^3+7d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(a^2c^6-2a^2c^5d-a^2c^4d^2+4a^2c^3d^3-a^2c^2d^4-2a^2cd^5+a^2d^6\right)\sqrt{-c^2+d^2}}-\frac{a^4c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6a^4c^5d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+15a^4\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sg
n(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/s
qrt(-c^2 + d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a
^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(-c^2 + d^2)) - (a^4*c^6*tan(1/2*f*
x + 1/2*e)^3 - 6*a^4*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^4*d^2*tan(1/2*
f*x + 1/2*e)^3 - 20*a^4*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^2*d^4*tan
(1/2*f*x + 1/2*e)^3 - 6*a^4*c*d^5*tan(1/2*f*x + 1/2*e)^3 + a^4*d^6*tan(1/2*
```

```
f*x + 1/2*e)^3 - 3*a^4*c^6*tan(1/2*f*x + 1/2*e) + 36*a^4*c^5*d*tan(1/2*f*x
+ 1/2*e) - 135*a^4*c^4*d^2*tan(1/2*f*x + 1/2*e) + 240*a^4*c^3*d^3*tan(1/2*f
*x + 1/2*e) - 225*a^4*c^2*d^4*tan(1/2*f*x + 1/2*e) + 108*a^4*c*d^5*tan(1/2*
f*x + 1/2*e) - 21*a^4*d^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9 - 9*a^6*c^8*d + 36
*a^6*c^7*d^2 - 84*a^6*c^6*d^3 + 126*a^6*c^5*d^4 - 126*a^6*c^4*d^5 + 84*a^6*
c^3*d^6 - 36*a^6*c^2*d^7 + 9*a^6*c*d^8 - a^6*d^9) + 6*(8*c^2*d^3*tan(1/2*f*
x + 1/2*e)^3 - 3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*d^5*tan(1/2*f*x + 1/2*e)^
3 - 8*c^2*d^3*tan(1/2*f*x + 1/2*e) - 11*c*d^4*tan(1/2*f*x + 1/2*e) - 3*d^5*
tan(1/2*f*x + 1/2*e))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3
 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1
/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

maple [A] time = 0.95, size = 280, normalized size = 0.99

$$\frac{-\frac{\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c}{3}-\frac{\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d}{3}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c+7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{\left(c^3-3c^2d+3c\,d^2-d^3\right)(c-d)}-\frac{8d^2\left(\frac{-\frac{d\left(8c^2-3cd-5d^2\right)\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{4\left(c^2+2cd+d^2\right)}+\frac{d(8c+3d)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4c+4d}}{\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^2}-\frac{\left(12c^2+16cd+7d^2\right)\operatorname{arctanh}\left(\cdots\right)}{4\left(c^2+2cd+d^2\right)\sqrt{(c+\cdots}}\cdots\right)}{(c-d)^4}}{2f\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f/a^2*(-1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c-d)*(1/3*tan(1/2*e+1/2*f*x)^3*c-1
/3*tan(1/2*e+1/2*f*x)^3*d-tan(1/2*e+1/2*f*x)*c+7*tan(1/2*e+1/2*f*x)*d)-8*d^
2/(c-d)^4*((-1/4*d*(8*c^2-3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3
+1/4*d*(8*c+3*d)/(c+d)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*
e+1/2*f*x)^2*d-c-d)^2-1/4*(12*c^2+16*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d
))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.28, size = 505, normalized size = 1.78

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(-8c^2 \cdots\right.}{(c+d)^2 \cdots}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(2a^2c^6-8a^2c^5d+10a^2c^4d^2-10a^2c^2d^4+8a^2cd^5-2a^2d^6\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(a^2c^6-6a^2 \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^3*(3*c*d^4 + 5*d^5 - 8*c^2*d^3))/(c + d)^2 + (tan(e/2
+ (f*x)/2)*(8*c*d^3 + 3*d^4))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2*c^6
- 2*a^2*d^6 + 8*a^2*c*d^5 - 8*a^2*c^5*d - 10*a^2*c^2*d^4 + 10*a^2*c^4*d^2)
- tan(e/2 + (f*x)/2)^4*(a^2*c^6 + a^2*d^6 - 6*a^2*c*d^5 - 6*a^2*c^5*d + 15*
a^2*c^2*d^4 - 20*a^2*c^3*d^3 + 15*a^2*c^4*d^2) - a^2*c^6 - a^2*d^6 + 2*a^2*
c*d^5 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)) + (tan(e/
2 + (f*x)/2)*(2/(a^2*(c - d)^3) - (3*(c + d))/(2*a^2*(c - d)^4)))/f - tan(e
/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^3) - (d^2*atan((c^5*tan(e/2 + (f*x)/2)*1i
- d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*5i - c^4*d*tan(e/2 +
 (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2*tan(e/2 + (f*x)/2)*
10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(16*c*d + 12*c^2 + 7*d^2)*1i)/(a^2*f*(c
 + d)^(5/2)*(c - d)^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^3\sec^2(e+fx)+2c^3\sec(e+fx)+c^3+3c^2d\sec^3(e+fx)+6c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^4(e+fx)+6cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3 \cdots}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x)**2 + 2*c**3*sec(e + f*x) + c**3 +
3*c**2*d*sec(e + f*x)**3 + 6*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x)
 + 3*c*d**2*sec(e + f*x)**4 + 6*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**5 + 2*d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)*
*3), x)/a**2
```

3.225 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=363

$$\frac{(c-d)\left(2c^2+18cd+115d^2\right)\tan(e+fx)(c+d\sec(e+fx))^3}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{d^3\left(40c^3-90c^2d+78cd^2-23d^3\right)\tanh^{-1}(\sin(e+$$

```
[Out] 1/2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*arctanh(sin(f*x+e))/a^3/f-2/15*d*
(2*c^5+18*c^4*d+107*c^3*d^2-472*c^2*d^3+456*c*d^4-136*d^5)*tan(f*x+e)/a^3/f
-1/30*d^2*(4*c^4+36*c^3*d+216*c^2*d^2-626*c*d^3+345*d^4)*sec(f*x+e)*tan(f*x
+e)/a^3/f-1/15*d*(2*c^3+18*c^2*d+111*c*d^2-136*d^3)*(c+d*sec(f*x+e))^2*tan(
f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+18*c*d+115*d^2)*(c+d*sec(f*x+e))^3*tan(f*x+e
)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+13*d)*(c+d*sec(f*x+e))^4*tan(f*x+e
)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec
(f*x+e))^3
```

Rubi [A] time = 0.54, antiderivative size = 405, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 98, 150, 153, 147, 63, 217, 203}

$$\frac{d^3\left(-90c^2d+40c^3+78cd^2-23d^3\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\left(2c^2+18cd+115d^2\right)\tan(e+fx}{15f\left(a^3\sec(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]
]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^3)*(c
 + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a^3*f) + ((c - d)*(2*c^2 + 18*c*d +
115*d^2)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]
)) + ((c - d)*(2*c + 13*d)*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(15*a*f*(a
+ a*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^5*Tan[e + f*x])/(5*f*(
a + a*Sec[e + f*x])^3) - (d*(4*(2*c^5 + 18*c^4*d + 107*c^3*d^2 - 472*c^2*d^
3 + 456*c*d^4 - 136*d^5) + d*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 +
345*d^4)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^6}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)^4(-a^2}{}\right.}{5af\sqrt{a-a\sec(}}$$

$$= \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+}{5f(a+a\sec}$$

$$= \frac{(c-d)\left(2c^2+18cd+115d^2\right)(c+d\sec(e+fx))^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c}{}$$

$$= \frac{d^3\left(40c^3-90c^2d+78cd^2-23d^3\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d}{}$$

Mathematica [B] time = 6.72, size = 1338, normalized size = 3.69

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (4*(-40*c^3*d^3 + 90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[
e + f*x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x]
```

```
)^6)/(f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*(-40*c^3*d^3 +
90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[e + f*x]^3*Log[Cos
[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^6)/(f*(d + c*Cos
[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]
^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[e/2] - 6*c^5*d*Sin[e/2] + 15*c^
4*d^2*Sin[e/2] - 20*c^3*d^3*Sin[e/2] + 15*c^2*d^4*Sin[e/2] - 6*c*d^5*Sin[e/
2] + d^6*Sin[e/2]))/(5*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (
8*Cos[e/2 + (f*x)/2]^4*Cos[e + f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(-4*c
^6*Sin[e/2] + 9*c^5*d*Sin[e/2] + 15*c^4*d^2*Sin[e/2] - 70*c^3*d^3*Sin[e/2]
+ 90*c^2*d^4*Sin[e/2] - 51*c*d^5*Sin[e/2] + 11*d^6*Sin[e/2]))/(15*f*(d + c*
Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]*Cos[e + f*x
]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[(f*x)/2] - 6*c^5*d*Sin[(f*x)/2
] + 15*c^4*d^2*Sin[(f*x)/2] - 20*c^3*d^3*Sin[(f*x)/2] + 15*c^2*d^4*Sin[(f*x
)/2] - 6*c*d^5*Sin[(f*x)/2] + d^6*Sin[(f*x)/2]))/(5*f*(d + c*Cos[e + f*x])^
6*(a + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^3*Cos[e + f*x]^3*Sec[e/2]
*(c + d*Sec[e + f*x])^6*(-4*c^6*Sin[(f*x)/2] + 9*c^5*d*Sin[(f*x)/2] + 15*c^
4*d^2*Sin[(f*x)/2] - 70*c^3*d^3*Sin[(f*x)/2] + 90*c^2*d^4*Sin[(f*x)/2] - 51
*c*d^5*Sin[(f*x)/2] + 11*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^5*Cos[e + f*x]^3*Sec[e/2]*(c
+ d*Sec[e + f*x])^6*(7*c^6*Sin[(f*x)/2] + 18*c^5*d*Sin[(f*x)/2] + 30*c^4*d^
2*Sin[(f*x)/2] - 440*c^3*d^3*Sin[(f*x)/2] + 855*c^2*d^4*Sin[(f*x)/2] - 642*
c*d^5*Sin[(f*x)/2] + 172*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*d^6*Cos[e/2 + (f*x)/2]^6*Sec[e]*(c + d*Sec[e + f
*x])^6*Sin[f*x])/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*C
os[e/2 + (f*x)/2]^6*Cos[e + f*x]^2*Sec[e]*(c + d*Sec[e + f*x])^6*(-18*c*d^5
*Sin[e] + 9*d^6*Sin[e] - 90*c^2*d^4*Sin[f*x] + 108*c*d^5*Sin[f*x] - 40*d^6*
Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (4*Cos[e/2
 + (f*x)/2]^6*Cos[e + f*x]*Sec[e]*(c + d*Sec[e + f*x])^6*(2*d^6*Sin[e] + 18
*c*d^5*Sin[f*x] - 9*d^6*Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e
 + f*x])^3)
```

fricas [A] time = 0.50, size = 620, normalized size = 1.71

$$\frac{15\left(\left(40\,c^3d^3-90\,c^2d^4+78\,cd^5-23\,d^6\right)\cos\left(fx+e\right)^6+3\left(40\,c^3d^3-90\,c^2d^4+78\,cd^5-23\,d^6\right)\cos\left(fx+e\right)^5+3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*
(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^
3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d
```

```
^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((40*c^3
*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*(40*c^3*d^3 - 90*
c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^3 - 90*c^2*d^4 +
78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 2
3*d^6)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(10*d^6 + 2*(7*c^6 + 18*c
^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 1080*c^2*d^4 - 912*c*d^5 + 272*d^6)*cos(f
*x + e)^5 + 3*(4*c^6 + 36*c^5*d + 60*c^4*d^2 - 680*c^3*d^3 + 1710*c^2*d^4 -
 1434*c*d^5 + 429*d^6)*cos(f*x + e)^4 + (4*c^6 + 36*c^5*d + 210*c^4*d^2 - 1
280*c^3*d^3 + 3510*c^2*d^4 - 2874*c*d^5 + 869*d^6)*cos(f*x + e)^3 + 5*(90*c
^2*d^4 - 54*c*d^5 + 19*d^6)*cos(f*x + e)^2 + 15*(6*c*d^5 - d^6)*cos(f*x + e
))*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((4096/5*tan((f*x+exp(1))/2)^5*c^6*a^12-24576/5*tan((f*x+exp(1)
)/2)^5*c^5*a^12*d+12288*tan((f*x+exp(1))/2)^5*c^4*a^12*d^2-16384*tan((f*x+e
xp(1))/2)^5*c^3*a^12*d^3+12288*tan((f*x+exp(1))/2)^5*c^2*a^12*d^4-24576/5*t
an((f*x+exp(1))/2)^5*c*a^12*d^5+4096/5*tan((f*x+exp(1))/2)^5*a^12*d^6-8192/
3*tan((f*x+exp(1))/2)^3*c^6*a^12+40960*tan((f*x+exp(1))/2)^3*c^4*a^12*d^2-3
27680/3*tan((f*x+exp(1))/2)^3*c^3*a^12*d^3+122880*tan((f*x+exp(1))/2)^3*c^2
*a^12*d^4-65536*tan((f*x+exp(1))/2)^3*c*a^12*d^5+40960/3*tan((f*x+exp(1))/2
)^3*a^12*d^6+4096*tan((f*x+exp(1))/2)*c^6*a^12+24576*tan((f*x+exp(1))/2)*c^
5*a^12*d+61440*tan((f*x+exp(1))/2)*c^4*a^12*d^2-573440*tan((f*x+exp(1))/2)*
c^3*a^12*d^3+1044480*tan((f*x+exp(1))/2)*c^2*a^12*d^4-761856*tan((f*x+exp(1
))/2)*c*a^12*d^5+200704*tan((f*x+exp(1))/2)*a^12*d^6)*1/32768/a^15+(-90*tan
((f*x+exp(1))/2)^5*c^2*d^4+126*tan((f*x+exp(1))/2)^5*c*d^5-51*tan((f*x+exp(
1))/2)^5*d^6+180*tan((f*x+exp(1))/2)^3*c^2*d^4-216*tan((f*x+exp(1))/2)^3*c*
d^5+76*tan((f*x+exp(1))/2)^3*d^6-90*tan((f*x+exp(1))/2)*c^2*d^4+90*tan((f*x
+exp(1))/2)*c*d^5-33*tan((f*x+exp(1))/2)*d^6)*1/6/a^3/(tan((f*x+exp(1))/2)^
2-1)^3-(40*c^3*d^3-90*c^2*d^4+78*c*d^5-23*d^6)*1/4/a^3*ln(abs(tan((f*x+exp(
1))/2)-1))+(40*c^3*d^3-90*c^2*d^4+78*c*d^5-23*d^6)*1/4/a^3*ln(abs(tan((f*x+
exp(1))/2)+1)))
```

maple [B] time = 0.84, size = 956, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 5/6/f/a^3*tan(1/2*e+1/2*f*x)^3*d^6+49/4/f/a^3*tan(1/2*e+1/2*f*x)*d^6+5/2/f/
a^3*tan(1/2*e+1/2*f*x)^3*c^4*d^2+15/2/f/a^3*tan(1/2*e+1/2*f*x)^3*c^2*d^4-20
/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)*c^3*d^3+45/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)*c
^2*d^4-39/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)*c*d^5-15/f/a^3*d^4/(tan(1/2*e+1/2*
f*x)-1)*c^2-4/f/a^3*tan(1/2*e+1/2*f*x)^3*c*d^5+1/20/f*c^6/a^3*tan(1/2*e+1/2
*f*x)^5-1/6/f*c^6/a^3*tan(1/2*e+1/2*f*x)^3+1/4/f*c^6/a^3*tan(1/2*e+1/2*f*x)
+23/2/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)*d^6-17/2/f/a^3*d^6/(tan(1/2*e+1/2*f*x)
-1)-2/f/a^3*d^6/(tan(1/2*e+1/2*f*x)-1)^2-17/2/f/a^3*d^6/(tan(1/2*e+1/2*f*x)
+1)+2/f/a^3*d^6/(tan(1/2*e+1/2*f*x)+1)^2-23/2/f/a^3*ln(tan(1/2*e+1/2*f*x)+1
)*d^6-1/3/f/a^3*d^6/(tan(1/2*e+1/2*f*x)-1)^3-1/3/f/a^3*d^6/(tan(1/2*e+1/2*f
*x)+1)^3+1/20/f/a^3*tan(1/2*e+1/2*f*x)^5*d^6-35/f/a^3*c^3*d^3*tan(1/2*e+1/2
*f*x)+21/f/a^3*d^5/(tan(1/2*e+1/2*f*x)+1)*c-3/f/a^3*d^5/(tan(1/2*e+1/2*f*x)
+1)^2*c+20/f/a^3*ln(tan(1/2*e+1/2*f*x)+1)*c^3*d^3-45/f/a^3*ln(tan(1/2*e+1/2
*f*x)+1)*c^2*d^4+39/f/a^3*ln(tan(1/2*e+1/2*f*x)+1)*c*d^5+255/4/f/a^3*tan(1/
2*e+1/2*f*x)*c^2*d^4+3/2/f/a^3*tan(1/2*e+1/2*f*x)*c^5*d+15/4/f/a^3*tan(1/2*
e+1/2*f*x)*c^4*d^2-93/2/f/a^3*tan(1/2*e+1/2*f*x)*c*d^5-3/10/f/a^3*tan(1/2*e
+1/2*f*x)^5*c^5*d+3/4/f/a^3*tan(1/2*e+1/2*f*x)^5*c^4*d^2+3/4/f/a^3*tan(1/2*
e+1/2*f*x)^5*c^2*d^4-3/10/f/a^3*tan(1/2*e+1/2*f*x)^5*c*d^5+21/f/a^3*d^5/(ta
n(1/2*e+1/2*f*x)-1)*c+3/f/a^3*d^5/(tan(1/2*e+1/2*f*x)-1)^2*c-15/f/a^3*d^4/(
tan(1/2*e+1/2*f*x)+1)*c^2-1/f/a^3*tan(1/2*e+1/2*f*x)^5*c^3*d^3-20/3/f/a^3*t
an(1/2*e+1/2*f*x)^3*c^3*d^3
```

maxima [B] time = 0.39, size = 946, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 6*c*d^5*(60*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e
```

```
)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/a^3) + 45*c^2*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) - 20*c^3*d^3*((105*sin(f*x + e)/(cos(f*x + e)
+ 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^4*d^2*(15*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 + 18*c^5*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.91, size = 327, normalized size = 0.90

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{5\,(c-d)^6}{2\,a^3}-\frac{6\,(c+d)\,(c-d)^5}{a^3}+\frac{15\,(c+d)^2\,(c-d)^4}{4\,a^3}\right)}{f}-\frac{\left(30\,c^2\,d^4-42\,c\,d^5+17\,d^6\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-60\,c^2\,d^4+72\cdots\right.}{f\,\left(a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^3\,\mathrm{ta}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((5*(c - d)^6)/(2*a^3) - (6*(c + d)*(c - d)^5)/a^3 + (1
5*(c + d)^2*(c - d)^4)/(4*a^3)))/f - (tan(e/2 + (f*x)/2)*(11*d^6 - 30*c*d^5
 + 30*c^2*d^4) + tan(e/2 + (f*x)/2)^5*(17*d^6 - 42*c*d^5 + 30*c^2*d^4) - ta
n(e/2 + (f*x)/2)^3*((76*d^6)/3 - 72*c*d^5 + 60*c^2*d^4))/(f*(3*a^3*tan(e/2
+ (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a^3)
) + (tan(e/2 + (f*x)/2)^3*((c - d)^6/(3*a^3) - ((c + d)*(c - d)^5)/(2*a^3))
)/f + (tan(e/2 + (f*x)/2)^5*(c - d)^6)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f
*x)/2))*(78*c*d^2 - 90*c^2*d + 40*c^3 - 23*d^3))/(a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^6\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d^6\sec^7\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{6cd^5\sec^6\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**6*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**6*sec(e + f*x)**7/(sec(e + f*x)**3 + 3*sec(e
```

```
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*d**5*sec(e + f*x)**6/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c
**2*d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(20*c**3*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c**4*d**2*sec(e + f*
x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(6*c**5*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x))/a**3
```

3.226 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=287

$$\frac{(c-d)\left(2c^2+15cd+76d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{d^3\left(20c^2-30cd+13d^2\right)\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{d^2\left(4c\right.}{}$$

```
[Out] 1/2*d^3*(20*c^2-30*c*d+13*d^2)*arctanh(sin(f*x+e))/a^3/f-2/15*d*(2*c^4+15*c
^3*d+72*c^2*d^2-180*c*d^3+76*d^4)*tan(f*x+e)/a^3/f-1/30*d^2*(4*c^3+30*c^2*d
+146*c*d^2-195*d^3)*sec(f*x+e)*tan(f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+15*c*d+76
*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+
11*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*s
ec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A] time = 0.42, antiderivative size = 329, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 150, 147, 63, 217, 203}

$$\frac{d^3\left(20c^2-30cd+13d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\left(2c^2+15cd+76d^2\right)\tan(e+fx)(c+d\sec(e\right.}{15f\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
 Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((c - d)*(2*c^2 + 15*c*d + 76*d^2)*(c + d*Sec[e + f*x])^2*T
an[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])) + ((c - d)*(2*c + 11*d)*(c + d
*Sec[e + f*x])^3*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)*(
c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (d*(4*(2
*c^4 + 15*c^3*d + 72*c^2*d^2 - 180*c*d^3 + 76*d^4) + d*(4*c^3 + 30*c^2*d +
146*c*d^2 - 195*d^3)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^3\left(-a^{\cdots}\right.}{\cdots}\right.}{5af\sqrt{a-a\sec(e\cdots}}$$

$$= \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+\cdots}{5f(a+a\sec\cdots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c\cdots}{\cdots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c\cdots}{\cdots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c\cdots}{\cdots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c\cdots}{\cdots}$$

$$= \frac{d^3\left(20c^2-30cd+13d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)\left(2c^{2}\cdots\right.}{\cdots}$$

Mathematica [A] time = 2.23, size = 439, normalized size = 1.53

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(12c^5\cos(3(e+fx))+7c^5\cos(4(e+fx))+29c^5+90c^4d\cos(3(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-480*d^3*(20*c^2 - 30*c*d + 13*d^2)*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[
(e + f*x)/2]*(29*c^5 + 105*c^4*d + 340*c^3*d^2 - 1940*c^2*d^3 + 3420*c*d^4
- 1354*d^5 + 3*(12*c^5 + 90*c^4*d + 120*c^3*d^2 - 1020*c^2*d^3 + 1910*c*d^4
 - 777*d^5)*Cos[e + f*x] + 6*(6*c^5 + 20*c^4*d + 60*c^3*d^2 - 360*c^2*d^3 +
 630*c*d^4 - 261*d^5)*Cos[2*(e + f*x)] + 12*c^5*Cos[3*(e + f*x)] + 90*c^4*d
*Cos[3*(e + f*x)] + 120*c^3*d^2*Cos[3*(e + f*x)] - 1020*c^2*d^3*Cos[3*(e +
f*x)] + 1710*c*d^4*Cos[3*(e + f*x)] - 717*d^5*Cos[3*(e + f*x)] + 7*c^5*Cos[
4*(e + f*x)] + 15*c^4*d*Cos[4*(e + f*x)] + 20*c^3*d^2*Cos[4*(e + f*x)] - 22
0*c^2*d^3*Cos[4*(e + f*x)] + 360*c*d^4*Cos[4*(e + f*x)] - 152*d^5*Cos[4*(e
+ f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(120*a^3*f*(1 + Cos[e + f*x])^3)
```

fricas [A] time = 0.48, size = 502, normalized size = 1.75

$$\frac{15\left(\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^5+3\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^4+3\left(20\,c^2d^3-30\,cd^4+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2*d^3 -
 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos
(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(sin(f*x
+ e) + 1) - 15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2
*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^
5)*cos(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(-s
in(f*x + e) + 1) + 2*(15*d^5 + 2*(7*c^5 + 15*c^4*d + 20*c^3*d^2 - 220*c^2*d
^3 + 360*c*d^4 - 152*d^5)*cos(f*x + e)^4 + 3*(4*c^5 + 30*c^4*d + 40*c^3*d^2
 - 340*c^2*d^3 + 570*c*d^4 - 239*d^5)*cos(f*x + e)^3 + (4*c^5 + 30*c^4*d +
140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 479*d^5)*cos(f*x + e)^2 + 15*(10*c
*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((4096/5*tan((f*x+exp(1))/2)^5*c^5*a^12-4096*tan((f*x+exp(1))/2
)^5*c^4*a^12*d+8192*tan((f*x+exp(1))/2)^5*c^3*a^12*d^2-8192*tan((f*x+exp(1)
)/2)^5*c^2*a^12*d^3+4096*tan((f*x+exp(1))/2)^5*c*a^12*d^4-4096/5*tan((f*x+e
xp(1))/2)^5*a^12*d^5-8192/3*tan((f*x+exp(1))/2)^3*c^5*a^12+81920/3*tan((f*x
+exp(1))/2)^3*c^3*a^12*d^2-163840/3*tan((f*x+exp(1))/2)^3*c^2*a^12*d^3+4096
0*tan((f*x+exp(1))/2)^3*c*a^12*d^4-32768/3*tan((f*x+exp(1))/2)^3*a^12*d^5+4
096*tan((f*x+exp(1))/2)*c^5*a^12+20480*tan((f*x+exp(1))/2)*c^4*a^12*d+40960
*tan((f*x+exp(1))/2)*c^3*a^12*d^2-286720*tan((f*x+exp(1))/2)*c^2*a^12*d^3+3
48160*tan((f*x+exp(1))/2)*c*a^12*d^4-126976*tan((f*x+exp(1))/2)*a^12*d^5)*1
/32768/a^15-(10*tan((f*x+exp(1))/2)^3*c*d^4-7*tan((f*x+exp(1))/2)^3*d^5-10*
tan((f*x+exp(1))/2)*c*d^4+5*tan((f*x+exp(1))/2)*d^5)*1/2/a^3/(tan((f*x+exp(
1))/2)^2-1)^2+(-20*c^2*d^3+30*c*d^4-13*d^5)*1/4/a^3*ln(abs(tan((f*x+exp(1))
/2)-1))-(-20*c^2*d^3+30*c*d^4-13*d^5)*1/4/a^3*ln(abs(tan((f*x+exp(1))/2)+1)
))
```

maple [B] time = 0.79, size = 679, normalized size = 2.37

$$-\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^4d}{4f\,a^3}+\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^3d^2}{2f\,a^3}-\frac{c^5\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{6f\,a^3}+\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4d}{4f\,a^3}-\frac{10\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)c^2}{f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -1/4/f/a^3*tan(1/2*e+1/2*f*x)^5*c^4*d+1/2/f/a^3*tan(1/2*e+1/2*f*x)^5*c^3*d^
2-1/2/f/a^3*tan(1/2*e+1/2*f*x)^5*c^2*d^3+5/4/f/a^3*tan(1/2*e+1/2*f*x)*c^4*d
+5/2/f/a^3*tan(1/2*e+1/2*f*x)^3*c*d^4-10/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)*c^2
*d^3+15/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)*c*d^4-5/f/a^3*d^4/(tan(1/2*e+1/2*f*x
)-1)*c+7/2/f/a^3*d^5/(tan(1/2*e+1/2*f*x)-1)-1/6/f/a^3*tan(1/2*e+1/2*f*x)^3*
c^5-1/20/f/a^3*tan(1/2*e+1/2*f*x)^5*d^5-2/3/f/a^3*tan(1/2*e+1/2*f*x)^3*d^5+
1/4/f/a^3*tan(1/2*e+1/2*f*x)*c^5-31/4/f/a^3*tan(1/2*e+1/2*f*x)*d^5-35/2/f/a
^3*tan(1/2*e+1/2*f*x)*c^2*d^3+10/f/a^3*ln(tan(1/2*e+1/2*f*x)+1)*c^2*d^3-15/
f/a^3*ln(tan(1/2*e+1/2*f*x)+1)*c*d^4-1/2/f/a^3*d^5/(tan(1/2*e+1/2*f*x)+1)^2
```

```
+1/2/f/a^3*d^5/(tan(1/2*e+1/2*f*x)-1)^2-13/2/f/a^3*ln(tan(1/2*e+1/2*f*x)-1)
*d^5+13/2/f/a^3*ln(tan(1/2*e+1/2*f*x)+1)*d^5+7/2/f/a^3*d^5/(tan(1/2*e+1/2*f
*x)+1)+1/20/f/a^3*tan(1/2*e+1/2*f*x)^5*c^5+85/4/f/a^3*tan(1/2*e+1/2*f*x)*c*
d^4+1/4/f/a^3*tan(1/2*e+1/2*f*x)^5*c*d^4-5/f/a^3*d^4/(tan(1/2*e+1/2*f*x)+1)
*c+5/3/f/a^3*tan(1/2*e+1/2*f*x)^3*c^3*d^2-10/3/f/a^3*tan(1/2*e+1/2*f*x)^3*c
^2*d^3+5/2/f/a^3*tan(1/2*e+1/2*f*x)*c^3*d^2
```

maxima [B] time = 0.37, size = 689, normalized size = 2.40

$$-\frac{d^5\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^3-\frac{2a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 4
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 15*c*d^4*(40*sin(f*x + e)/((a^3 - a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e
)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e
)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^2*d^3*((105*
sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 10*c
^3*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^5*(15*sin(f*x + e
)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5)/a^3 - 15*c^4*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.87, size = 252, normalized size = 0.88

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,(c-d)^5}{2\,a^3}-\frac{15\,(c+d)\,(c-d)^4}{4\,a^3}+\frac{5\,(c+d)^2\,(c-d)^3}{2\,a^3}\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(10\,c\,d^4-5\,d^5\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(10\,c\,d^4-7\right.}{f\,\left(a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*(c - d)^5)/(2*a^3) - (15*(c + d)*(c - d)^4)/(4*a^3)
 + (5*(c + d)^2*(c - d)^3)/(2*a^3)))/f + (tan(e/2 + (f*x)/2)*(10*c*d^4 - 5*
d^5) - tan(e/2 + (f*x)/2)^3*(10*c*d^4 - 7*d^5))/(f*(a^3*tan(e/2 + (f*x)/2)^
4 - 2*a^3*tan(e/2 + (f*x)/2)^2 + a^3)) + (tan(e/2 + (f*x)/2)^3*((c - d)^5/(
4*a^3) - (5*(c + d)*(c - d)^4)/(12*a^3)))/f + (tan(e/2 + (f*x)/2)^5*(c - d)
^5)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f*x)/2))*(20*c^2 - 30*c*d + 13*d^2))
/(a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^5 \sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int \frac{d^5 \sec^6\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int \frac{5cd^4 \sec^5\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**5*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c*d**4*sec(e + f*x)**5/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*c
**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**
2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.227 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=205

$$\frac{\tan(e+fx)\left(2c^4+8c^3d-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d^2-88cd^3+72d^4\right)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{d^3(4c-3d)\tanh^{-1}(\mathrm{si}}{a^3f}$$

```
[Out] (4*c-3*d)*d^3*arctanh(sin(f*x+e))/a^3/f+1/15*(c-d)*(2*c+9*d)*(c+d*sec(f*x+e
))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^3*tan(f*x
+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c^4+8*c^3*d+21*c^2*d^2-88*c*d^3+72*d^4-d^2
*(2*c^2+10*c*d-27*d^2)*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] time = 0.28, antiderivative size = 265, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 150, 143, 63, 217, 203}

$$\frac{\tan(e+fx)\left(-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d^2+8c^3d+2c^4-88cd^3+72d^4\right)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3(4c-3d)\tan(e+}{a^2f\sqrt{a-a\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*(4*c - 3*d)*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + ((c - d)*(2*c + 9*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a
*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a +
 a*Sec[e + f*x])^3) + ((2*c^4 + 8*c^3*d + 21*c^2*d^2 - 88*c*d^3 + 72*d^4 -
d^2*(2*c^2 + 10*c*d - 27*d^2)*Sec[e + f*x])*Tan[e + f*x])/(15*f*(a^3 + a^3*
Sec[e + f*x]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
```

```
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^2(-\cdots}{\cdots}\right.}{5af\sqrt{a-a\sec\cdots}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+\cdots}{5f(a+a\sec\cdots}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+\cdots}{5f(a+a\sec\cdots}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+\cdots}{5f(a+a\sec\cdots}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+\cdots}{5f(a+a\sec\cdots}$$

$$= \frac{2(4c-3d)d^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(2c+9d)(c+\cdots}{15af(a+\cdots}$$

Mathematica [A] time = 2.39, size = 292, normalized size = 1.42

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(4(c-d)^2\left(7c^2+26cd+57d^2\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)-60d^3\cos^5\left(\frac{1}{2}(e+fx)\right)\left((4c\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(3*(c - d)^4*Sec[e/2]*Sin[(f*x)/2] - 8*(c - d)^3*(2*c +
 3*d)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f*x)/2] + 4*(c - d)^2*(7*c^2 + 26*c*
```

```
d + 57*d^2)*Cos[(e + f*x)/2]^4*Sec[e/2]*Sin[(f*x)/2] - 60*d^3*Cos[(e + f*x)
/2]^5*((4*c - 3*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) - d*Sec[e]*Sec[e + f*x]*Sin[f*x]) + 3*(c - d)
^4*Cos[(e + f*x)/2]*Tan[e/2] - 8*(c - d)^3*(2*c + 3*d)*Cos[(e + f*x)/2]^3*T
an[e/2]))/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

fricas [A] time = 0.46, size = 385, normalized size = 1.88

$$\frac{15\left(\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^2+\left(4\,cd^3-3\,d^4\right)c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3 - 3*d^4)*cos(f*x +
e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3*d^4)*cos(f*x + e))
*log(sin(f*x + e) + 1) - 15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3
- 3*d^4)*cos(f*x + e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3
*d^4)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(15*d^4 + (7*c^4 + 12*c^3*d
+ 12*c^2*d^2 - 88*c*d^3 + 72*d^4)*cos(f*x + e)^3 + 3*(2*c^4 + 12*c^3*d + 12
*c^2*d^2 - 68*c*d^3 + 57*d^4)*cos(f*x + e)^2 + (2*c^4 + 12*c^3*d + 42*c^2*d
^2 - 128*c*d^3 + 117*d^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^4
 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((4096/5*tan((f*x+exp(1))/2)^5*c^4*a^12-16384/5*tan((f*x+exp(1)
)/2)^5*c^3*a^12*d+24576/5*tan((f*x+exp(1))/2)^5*c^2*a^12*d^2-16384/5*tan((f
*x+exp(1))/2)^5*c*a^12*d^3+4096/5*tan((f*x+exp(1))/2)^5*a^12*d^4-8192/3*tan
((f*x+exp(1))/2)^3*c^4*a^12+16384*tan((f*x+exp(1))/2)^3*c^2*a^12*d^2-65536/
3*tan((f*x+exp(1))/2)^3*c*a^12*d^3+8192*tan((f*x+exp(1))/2)^3*a^12*d^4+4096
*tan((f*x+exp(1))/2)*c^4*a^12+16384*tan((f*x+exp(1))/2)*c^3*a^12*d+24576*ta
n((f*x+exp(1))/2)*c^2*a^12*d^2-114688*tan((f*x+exp(1))/2)*c*a^12*d^3+69632*
tan((f*x+exp(1))/2)*a^12*d^4)*1/32768/a^15-tan((f*x+exp(1))/2)*d^4/a^3/(tan
```

```
((f*x+exp(1))/2)^2-1)-(4*c*d^3-3*d^4)*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)-1)
)+(4*c*d^3-3*d^4)*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 0.71, size = 454, normalized size = 2.21

$$\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^4}{20f\,a^3}-\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^3d}{5f\,a^3}+\frac{3\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2d^2}{10f\,a^3}-\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c\,d^3}{5f\,a^3}+\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^4}{20f\,a^3}-\frac{(\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/20/f/a^3*tan(1/2*e+1/2*f*x)^5*c^4-1/5/f/a^3*tan(1/2*e+1/2*f*x)^5*c^3*d+3/
10/f/a^3*tan(1/2*e+1/2*f*x)^5*c^2*d^2-1/5/f/a^3*tan(1/2*e+1/2*f*x)^5*c*d^3+
1/20/f/a^3*tan(1/2*e+1/2*f*x)^5*d^4-1/6/f/a^3*tan(1/2*e+1/2*f*x)^3*c^4+1/f/
a^3*tan(1/2*e+1/2*f*x)^3*c^2*d^2-4/3/f/a^3*tan(1/2*e+1/2*f*x)^3*c*d^3+1/2/f
/a^3*tan(1/2*e+1/2*f*x)^3*d^4+1/4/f/a^3*tan(1/2*e+1/2*f*x)*c^4+1/f/a^3*tan(
1/2*e+1/2*f*x)*c^3*d+3/2/f/a^3*tan(1/2*e+1/2*f*x)*c^2*d^2-7/f/a^3*tan(1/2*e
+1/2*f*x)*c*d^3+17/4/f/a^3*tan(1/2*e+1/2*f*x)*d^4-1/f/a^3*d^4/(tan(1/2*e+1/
2*f*x)-1)-4/f/a^3*d^3*ln(tan(1/2*e+1/2*f*x)-1)*c+3/f/a^3*d^4*ln(tan(1/2*e+1
/2*f*x)-1)-1/f/a^3*d^4/(tan(1/2*e+1/2*f*x)+1)+4/f/a^3*d^3*ln(tan(1/2*e+1/2*
f*x)+1)*c-3/f/a^3*d^4*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [B] time = 0.36, size = 475, normalized size = 2.32

$$\frac{3\,d^4\left(\frac{40\,\sin\left(fx+e\right)}{\left(a^3-\frac{a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)\left(\cos\left(fx+e\right)+1\right)}+\frac{\frac{85\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) - 4*c*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^2*d^2*(15*sin(f*x + e)/(cos(f*x + e) +
```

```
 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 12*c^
3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3)/f
```

mupad [B] time = 1.82, size = 195, normalized size = 0.95

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3(c-d)^4}{4a^3}+\frac{3\left(c^2-d^2\right)^2}{2a^3}-\frac{2(c+d)(c-d)^3}{a^3}\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{(c-d)^4}{6a^3}-\frac{(c+d)(c-d)^3}{3a^3}\right)}{f}-\frac{2d^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-a^3\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*(c - d)^4)/(4*a^3) + (3*(c^2 - d^2)^2)/(2*a^3) - (2
*(c + d)*(c - d)^3)/a^3))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^4/(6*a^3) - ((
c + d)*(c - d)^3)/(3*a^3)))/f - (2*d^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2
+ (f*x)/2)^2 - a^3)) + (tan(e/2 + (f*x)/2)^5*(c - d)^4)/(20*a^3*f) + (2*d^3
*atanh(tan(e/2 + (f*x)/2))*(4*c - 3*d))/(a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^4\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d^4\sec^5\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{4cd^3\sec^4\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*
*2*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*
x) + 1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.228 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{d^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{(c-d)\tan(e+fx)\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd+11d^2\right)\right)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\,\mathrm{t}}{}$$

```
[Out] d^3*arctanh(sin(f*x+e))/a^3/f+1/5*(c-d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+
a*sec(f*x+e))^3+1/15*(c-d)*(4*c^2+16*c*d+22*d^2+(2*c^2+11*c*d+29*d^2)*sec(f
*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 145, 63, 217, 203}

$$\frac{2d^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd\right.\right.}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - d
)*(2*(2*c^2 + 8*c*d + 11*d^2) + (2*c^2 + 11*c*d + 29*d^2)*Sec[e + f*x])*Tan
[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)(-}{}\right.}{5af\sqrt{a-a\sec(e}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)\right.}{15a}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)\right.}{15a}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)\right.}{15a}$$

$$= \frac{2d^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+fx))}{5f(a+a\sec(e+}$$

Mathematica [B] time = 1.48, size = 295, normalized size = 2.22

$$\frac{(c-d)\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(-15\left(2c^2+5cd+5d^2\right)\sin\left(e+\frac{fx}{2}\right)+5\left(8c^2+17cd+29d^2\right)\sin\left(\frac{fx}{2}\right)+20c^2\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-240*d^3*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Cos[(e + f*x)/2]*Sec[e/2]
*(5*(8*c^2 + 17*c*d + 29*d^2)*Sin[(f*x)/2] - 15*(2*c^2 + 5*c*d + 5*d^2)*Sin
[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 65*c*d*Sin[e + (3*f*x)/2] + 95*
d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] - 15*c*d*Sin[2*e + (3*
f*x)/2] - 15*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (5*f*x)/2] + 16*c*d
*Sin[2*e + (5*f*x)/2] + 22*d^2*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(1 + Cos[e
+ f*x])^3)
```

fricas [A] time = 0.50, size = 248, normalized size = 1.86

$$\frac{15\left(d^{3}\cos\left(fx+e\right)^{3}+3\,d^{3}\cos\left(fx+e\right)^{2}+3\,d^{3}\cos\left(fx+e\right)+d^{3}\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(d^{3}\cos\left(fx+e\right)^{3}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e) +
d^3)*log(sin(f*x + e) + 1) - 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2
+ 3*d^3*cos(f*x + e) + d^3)*log(-sin(f*x + e) + 1) + 2*(2*c^3 + 9*c^2*d + 2
1*c*d^2 - 32*d^3 + (7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)*cos(f*x + e)^2 + 3*
(2*c^3 + 9*c^2*d + 6*c*d^2 - 17*d^3)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-d^3*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)-1))+d^3*1/2/a^3*ln(abs
(tan((f*x+exp(1))/2)+1))+(4096/5*tan((f*x+exp(1))/2)^5*c^3*a^12-12288/5*tan
((f*x+exp(1))/2)^5*c^2*a^12*d+12288/5*tan((f*x+exp(1))/2)^5*c*a^12*d^2-4096
/5*tan((f*x+exp(1))/2)^5*a^12*d^3-8192/3*tan((f*x+exp(1))/2)^3*c^3*a^12+819
2*tan((f*x+exp(1))/2)^3*c*a^12*d^2-16384/3*tan((f*x+exp(1))/2)^3*a^12*d^3+4
096*tan((f*x+exp(1))/2)*c^3*a^12+12288*tan((f*x+exp(1))/2)*c^2*a^12*d+12288
*tan((f*x+exp(1))/2)*c*a^12*d^2-28672*tan((f*x+exp(1))/2)*a^12*d^3)*1/32768
/a^15)
```

maple [B] time = 0.84, size = 286, normalized size = 2.15

$$\frac{3c^{2}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{4fa^{3}}+\frac{\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd^{2}}{2fa^{3}}+\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)cd^{2}}{4fa^{3}}-\frac{3\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^{2}d}{20fa^{3}}+\frac{3\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd^{2}}{20fa^{3}}-\frac{7\,\mathrm{t}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 3/4/f/a^3*c^2*tan(1/2*e+1/2*f*x)*d+1/2/f/a^3*tan(1/2*e+1/2*f*x)^3*c*d^2+3/4
/f/a^3*tan(1/2*e+1/2*f*x)*c*d^2-3/20/f/a^3*tan(1/2*e+1/2*f*x)^5*c^2*d+3/20/
f/a^3*tan(1/2*e+1/2*f*x)^5*c*d^2-7/4/f/a^3*tan(1/2*e+1/2*f*x)*d^3-1/f/a^3*d
^3*ln(tan(1/2*e+1/2*f*x)-1)-1/6/f/a^3*tan(1/2*e+1/2*f*x)^3*c^3-1/3/f/a^3*ta
n(1/2*e+1/2*f*x)^3*d^3+1/4/f/a^3*tan(1/2*e+1/2*f*x)*c^3+1/f/a^3*d^3*ln(tan(
1/2*e+1/2*f*x)+1)+1/20/f/a^3*tan(1/2*e+1/2*f*x)^5*c^3-1/20/f/a^3*tan(1/2*e+
1/2*f*x)^5*d^3
```

maxima [B] time = 0.34, size = 307, normalized size = 2.31

$$-\frac{d^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-\frac{3cd^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\cdots\right.}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) - 3*c*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^3*(
15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^2*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.80, size = 147, normalized size = 1.11

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{(c-d)^3}{4a^3}-\frac{3(c+d)(c-d)^2}{4a^3}+\frac{3(c+d)^2(c-d)}{4a^3}\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{(c-d)^3}{12a^3}-\frac{(c+d)(c-d)^2}{4a^3}\right)}{f}+\frac{2d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}{a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((c - d)^3/(4*a^3) - (3*(c + d)*(c - d)^2)/(4*a^3) + (3
*(c + d)^2*(c - d))/(4*a^3)))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^3/(12*a^3)
 - ((c + d)*(c - d)^2)/(4*a^3)))/f + (2*d^3*atanh(tan(e/2 + (f*x)/2)))/(a^3
*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^3)/(20*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d^3\sec^4\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{3cd^2\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+\right.}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*
*2*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

3.229 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=115

$$\frac{\left(2c^2+6cd+7d^2\right)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(c-d)^2\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}+\frac{2(c+4d)(c-d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}$$

```
[Out] 1/5*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+2/15*(c-d)*(c+4*d)*tan(f*x+e)/a
/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2+6*c*d+7*d^2)*tan(f*x+e)/f/(a^3+a^3*sec(f*
x+e))
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 89, 78, 37}

$$\frac{\left(2c^2+6cd+7d^2\right)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(c-d)^2\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}+\frac{2(c+4d)(c-d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (2*(c - d)*(c + 4*d
)*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c^2 + 6*c*d + 7*d^2)*
Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{a^3\left(2c^2+6cd-3d^2\right)+5a^3d^2x}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x\right.}{5a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{\left(\left(2c^2+6cd+7\right.\right.}{}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{\left(2c^2+6cd+7d\right.}{15f\left(a^3+a^3\right.}$$

Mathematica [A] time = 0.48, size = 180, normalized size = 1.57

$$\frac{\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(10\left(4c^2+3cd+2d^2\right)\sin\left(\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right)-15c^2\sin\left(2e+\frac{3fx}{2}\right)+7c^2\sin\left(2e+\right.\right.}{30a^3f(\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(10*(4*c^2 + 3*c*d + 2*d^2)*Sin[(f*x)/2] - 30*c*
(c + d)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 30*c*d*Sin[e + (3*f*
x)/2] + 10*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin
[2*e + (5*f*x)/2] + 6*c*d*Sin[2*e + (5*f*x)/2] + 2*d^2*Sin[2*e + (5*f*x)/2]
))/(30*a^3*f*(1 + Cos[e + f*x])^3)
```

fricas [A] time = 0.42, size = 113, normalized size = 0.98

$$\frac{\left(\left(7\,c^{2}+6\,cd+2\,d^{2}\right)\cos\left(fx+e\right)^{2}+2\,c^{2}+6\,cd+7\,d^{2}+6\left(c^{2}+3\,cd+d^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((7*c^2 + 6*c*d + 2*d^2)*cos(f*x + e)^2 + 2*c^2 + 6*c*d + 7*d^2 + 6*(c
^2 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

giac [A] time = 0.35, size = 137, normalized size = 1.19

$$\frac{3\,c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3\,d^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-10\,c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+10\,d^{2}\tan\left(\frac{1}{2}\,f\right.}{60\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(3*c^2*tan(1/2*f*x + 1/2*e)^5 - 6*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*d^2*t
an(1/2*f*x + 1/2*e)^5 - 10*c^2*tan(1/2*f*x + 1/2*e)^3 + 10*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 15*c^2*tan(1/2*f*x + 1/2*e) + 30*c*d*tan(1/2*f*x + 1/2*e) + 15
*d^2*tan(1/2*f*x + 1/2*e))/(a^3*f)
```

maple [A] time = 0.83, size = 128, normalized size = 1.11

$$\frac{\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^{2}}{5}-\frac{2\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{5}+\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^{2}}{5}-\frac{2\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^{2}}{3}+\frac{2\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^{2}}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^{2}+2cd\tan\left(\frac{e}{2}\right.}{4f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/4/f/a^3*(1/5*tan(1/2*e+1/2*f*x)^5*c^2-2/5*tan(1/2*e+1/2*f*x)^5*c*d+1/5*ta
n(1/2*e+1/2*f*x)^5*d^2-2/3*tan(1/2*e+1/2*f*x)^3*c^2+2/3*tan(1/2*e+1/2*f*x)^
3*d^2+tan(1/2*e+1/2*f*x)*c^2+2*c*d*tan(1/2*e+1/2*f*x)+tan(1/2*e+1/2*f*x)*d^
2)
```

maxima [A] time = 0.35, size = 184, normalized size = 1.60

$$\frac{\frac{d^{2}\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}+\frac{c^{2}\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\,\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}+\frac{6\,cd\left(\frac{5\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.83, size = 79, normalized size = 0.69

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c+d\right)}^2}{4\,a^3\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,c^2-2\,d^2\right)}{12\,a^3\,f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^2}{20\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c + d)^2)/(4*a^3*f) - (tan(e/2 + (f*x)/2)^3*(2*c^2 - 2
*d^2))/(12*a^3*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^2)/(20*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^{2}\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d^{2}\sec^{3}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+\right.}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.230 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(2c+3d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] 1/5*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c+3*d)*tan(f*x+e)/a/f/(a+
a*sec(f*x+e))^2+1/15*(2*c+3*d)*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4000, 3796, 3794}

$$\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(2c+3d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c + 3*d)*Tan[e +
f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c + 3*d)*Tan[e + f*x])/(15*f*(a
^3 + a^3*Sec[e + f*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
```

```
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\int \frac{\sec(e+}{a+a\sec(}}{15a^2}$$

$$= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\tan(e-}{15f\left(a^3+a^3\sec(e\right.}$$

Mathematica [A] time = 0.34, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(-15(2c+d)\sin\left(e+\frac{fx}{2}\right)+5(8c+3d)\sin\left(\frac{fx}{2}\right)+20c\sin\left(e+\frac{3fx}{2}\right)-15c\sin\left(2e+\frac{3fx}{2}\right)\right.}{30a^3f(\cos(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(5*(8*c + 3*d)*Sin[(f*x)/2] - 15*(2*c + d)*Sin[e
 + (f*x)/2] + 20*c*Sin[e + (3*f*x)/2] + 15*d*Sin[e + (3*f*x)/2] - 15*c*Sin[
2*e + (3*f*x)/2] + 7*c*Sin[2*e + (5*f*x)/2] + 3*d*Sin[2*e + (5*f*x)/2]))/(3
0*a^3*f*(1 + Cos[e + f*x])^3)
```

fricas [A] time = 0.45, size = 93, normalized size = 0.91

$$\frac{\left((7\,c+3\,d)\cos\left(fx+e\right)^2+3\,(2\,c+3\,d)\cos\left(fx+e\right)+2\,c+3\,d\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((7*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x + e) + 2*c + 3*d)*
sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f
*x + e) + a^3*f)
```

giac [A] time = 1.39, size = 80, normalized size = 0.78

$$\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{60\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(3*c*tan(1/2*f*x + 1/2*e)^5 - 3*d*tan(1/2*f*x + 1/2*e)^5 - 10*c*tan(1/
2*f*x + 1/2*e)^3 + 15*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e))/(
a^3*f)
```

maple [A] time = 0.73, size = 64, normalized size = 0.63

$$\frac{\frac{(c-d)\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{4f\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/4/f/a^3*(1/5*(c-d)*tan(1/2*e+1/2*f*x)^5-2/3*tan(1/2*e+1/2*f*x)^3*c+tan(1/
2*e+1/2*f*x)*c+tan(1/2*e+1/2*f*x)*d)
```

maxima [A] time = 0.33, size = 115, normalized size = 1.13

$$\frac{\frac{c\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}+\frac{3\,d\left(\frac{5\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 3*d*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

mupad [B] time = 1.74, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(15\,c+15\,d-10\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right)}{60\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(15*c + 15*d - 10*c*tan(e/2 + (f*x)/2)^2 + 3*c*tan(e/2
+ (f*x)/2)^4 - 3*d*tan(e/2 + (f*x)/2)^4))/(60*a^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

3.231 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=181

$$\frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15f(c-d)^3(a^3\sec(e+fx)+a^3)}-\frac{2d^3\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3f(c-d)^{7/2}\sqrt{c+d}}+\frac{(2c-7d)\tan(e+fx)}{15af(c-d)^2(a\sec(e+fx)+a)^2}+\frac{\tan}{5f(c-d)(a}$$

```
[Out] -2*d^3*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/a^3/(c-d)^(7/2)/
f/(c+d)^(1/2)+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^3+1/15*(2*c-7*d)*tan(
f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2-9*c*d+22*d^2)*tan(f*x+e)/
(c-d)^3/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] time = 0.32, antiderivative size = 235, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 104, 152, 12, 93, 205}

$$\frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15f(c-d)^3(a^3\sec(e+fx)+a^3)}+\frac{2d^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{7/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(2c-7d)\,\mathrm{ta}}{15af(c-d)^2(a\,\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3) + ((2*c - 7*d)*Tan[e + f*
x])/(15*a*(c - d)^2*f*(a + a*Sec[e + f*x])^2) + (2*d^3*ArcTan[(Sqrt[c + d]*
Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f
*x])/(a^2*(c - d)^(7/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((2*c^2 - 9*c*d + 22*d^2)*Tan[e + f*x])/(15*(c - d)^3*f*(a^
3 + a^3*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{7/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{-a^2(2c-5d)-}{\sqrt{a-ax}\,(a+ax)}\right.}{5a(c-d)f\sqrt{a-a\sec(e+fx}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2}$$

Mathematica [C] time = 3.03, size = 345, normalized size = 1.91

$$\cos\left(\frac{1}{2}(e+fx)\right)\left(\sec\left(\frac{e}{2}\right)\left(-15\left(2c^2-7cd+9d^2\right)\sin\left(e+\frac{fx}{2}\right)+5\left(8c^2-27cd+37d^2\right)\sin\left(\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*((480*d^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c
*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[
(e + f*x)/2]^5*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
```

```
])^2]) + Sec[e/2]*(5*(8*c^2 - 27*c*d + 37*d^2)*Sin[(f*x)/2] - 15*(2*c^2 - 7
*c*d + 9*d^2)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] - 75*c*d*Sin[e +
 (3*f*x)/2] + 115*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 45
*c*d*Sin[2*e + (3*f*x)/2] - 45*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (
5*f*x)/2] - 24*c*d*Sin[2*e + (5*f*x)/2] + 32*d^2*Sin[2*e + (5*f*x)/2])))/(3
0*a^3*(c - d)^3*f*(1 + Cos[e + f*x])^3)
```

fricas [B] time = 0.51, size = 1001, normalized size = 5.53

$$\Bigg[-\frac{15\big(d^3\cos(fx+e)^3+3d^3\cos(fx+e)^2+3d^3\cos(fx+e)+d^3\big)\sqrt{c^2-d^2}\,\log\Big(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2}{c^2\cos(fx+}}{30\Big(\big(a^3c^5-3a^3c^4d+2a^3c^3d^2+2a^3c^2d^3-3a^3cd^4+a^3d^5\big)f\cos(fx+e)^3+3\big(a^3c^5-3a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e)
+ d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)
^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^
2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(2*c^4 - 9*c^3*d + 20*c^2
*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3 - 32*d^
4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^
2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d
+ 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 +
 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a
^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f), -1/15*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos
(f*x + e)^2 + 3*d^3*cos(f*x + e) + d^3)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2
+ d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4 - 9*c^3*d
+ 20*c^2*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3
 - 32*d^4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*
d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^
3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x
+ e)^2 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2
*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)]
```

giac [B] time = 1.07, size = 489, normalized size = 2.70

$$-\frac{\frac{120\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^3}{\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)\sqrt{-c^2+d^2}}-\frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+18a^{12}c^2d^2\tan\left(\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/60*(120*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*t
an(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^3/((a^3*
c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)) - (3*a^12*c^4*
tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2
*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12
*d^4*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*
c^3*d*tan(1/2*f*x + 1/2*e)^3 - 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 70*
a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^4*tan(1/2*f*x + 1/2*e)^3 + 15
*a^12*c^4*tan(1/2*f*x + 1/2*e) - 90*a^12*c^3*d*tan(1/2*f*x + 1/2*e) + 240*a
^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 270*a^12*c*d^3*tan(1/2*f*x + 1/2*e) + 10
5*a^12*d^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5 - 5*a^15*c^4*d + 10*a^15*c^3*d^2
 - 10*a^15*c^2*d^3 + 5*a^15*c*d^4 - a^15*d^5))/f
```

maple [A] time = 0.87, size = 203, normalized size = 1.12

$$\frac{\frac{\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2}{5}-\frac{2\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{5}+\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^2}{5}-\frac{2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2}{3}+2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd-\frac{4\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^2}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2-4cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+7\tan\left(\frac{e}{2}\right.}{(c-d)^3}}{4fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/4/f/a^3*(1/(c-d)^3*(1/5*tan(1/2*e+1/2*f*x)^5*c^2-2/5*tan(1/2*e+1/2*f*x)^5
*c*d+1/5*tan(1/2*e+1/2*f*x)^5*d^2-2/3*tan(1/2*e+1/2*f*x)^3*c^2+2*tan(1/2*e+
1/2*f*x)^3*c*d-4/3*tan(1/2*e+1/2*f*x)^3*d^2+tan(1/2*e+1/2*f*x)*c^2-4*c*d*ta
n(1/2*e+1/2*f*x)+7*tan(1/2*e+1/2*f*x)*d^2)-8*d^3/(c-d)^3/((c+d)*(c-d))^(1/2
)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 1.95, size = 228, normalized size = 1.26

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{3}{4\,a^3\,(c-d)}-\frac{(c+d)\left(\frac{3}{4\,a^3\,(c-d)}-\frac{c+d}{4\,a^3\,(c-d)^2}\right)}{c-d}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{1}{4\,a^3\,(c-d)}-\frac{c+d}{12\,a^3\,(c-d)^2}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{20\,a^3\,f\,\left(c-d\right)}-\frac{2\,d^3\,\mathrm{at}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3/(4*a^3*(c - d)) - ((c + d)*(3/(4*a^3*(c - d)) - (c +
 d)/(4*a^3*(c - d)^2)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(1/(4*a^3*(c -
d)) - (c + d)/(12*a^3*(c - d)^2)))/f + tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c -
d)) - (2*d^3*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a^3*c^3 - a^3*d^3 + 3*a
^3*c*d^2 - 3*a^3*c^2*d))/(2*a^3*(c + d)^(1/2)*(c - d)^(7/2))))/(a^3*f*(c +
d)^(1/2)*(c - d)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec{\left(e+fx \right)}}{c\sec^{3}{\left(e+fx \right)}+3c\sec^{2}{\left(e+fx \right)}+3c\sec{\left(e+fx \right)}+c+d\sec^{4}{\left(e+fx \right)}+3d\sec^{3}{\left(e+fx \right)}+3d\sec^{2}{\left(e+fx \right)}+d\sec{\left(e+fx \right)}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**3 + 3*c*sec(e + f*x)**2 + 3*c*sec(e
+ f*x) + c + d*sec(e + f*x)**4 + 3*d*sec(e + f*x)**3 + 3*d*sec(e + f*x)**2
+ d*sec(e + f*x)), x)/a**3
```

3.232 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=288

$$\frac{(2c^2-12cd+45d^2)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)(c+d\sec(e+fx))}+\frac{d\left(2c^3-12c^2d+43cd^2+72d^3\right)\tan(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sec(e+fx))}-\frac{2d^3(4c+3d)\cdots}{a^3f(\cdots}$$

```
[Out] -2*d^3*(4*c+3*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/a^3/(c
-d)^(9/2)/(c+d)^(3/2)/f+1/15*d*(2*c^3-12*c^2*d+43*c*d^2+72*d^3)*tan(f*x+e)/
a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e)
)^3/(c+d*sec(f*x+e))+1/15*(2*c-9*d)*tan(f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))
^2/(c+d*sec(f*x+e))+1/15*(2*c^2-12*c*d+45*d^2)*tan(f*x+e)/(c-d)^3/f/(a^3+a^
3*sec(f*x+e))/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.49, antiderivative size = 325, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 103, 152, 12, 93, 205}

$$\frac{\left(-12c^2d+2c^3+43cd^2+72d^3\right)\tan(e+fx)}{15f(c-d)^4(c+d)\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3(4c+3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{9/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{\cdots}{f\left(c^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 6*d)*Tan[e + f*x])/(5*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^3) + (
(2*c^2 - 10*c*d - 27*d^2)*Tan[e + f*x])/(15*a*(c - d)^3*(c + d)*f*(a + a*Se
c[e + f*x])^2) + (2*d^3*(4*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(
9/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (
(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^3)*Tan[e + f*x])/(15*(c - d)^4*(c + d)*
f*(a^3 + a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e
+ f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{7/2}(c+dx)^2}\,dx, x, \sec(e+fx\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} - \frac{\tan(e+fx}{\left(c^2-\right.}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} - \frac{d\tan(}{\left(c^2-d^2\right)f(a+a\sec(e+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)}{15a(c-d)^3(c+d)f(a+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)}{15a(c-d)^3(c+d)f(a+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)}{15a(c-d)^3(c+d)f(a+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)}{15a(c-d)^3(c+d)f(a+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)}{15a(c-d)^3(c+d)f(a+}$$

Mathematica [C] time = 7.09, size = 1772, normalized size = 6.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((4*c + 3*d)*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^5*(((
16*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Si
n[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*
d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos
[2*e] - I*Sin[2*e]]) + (16*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]
```

```
*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[
c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d)*(a + a*Sec[
e + f*x])^3*(c + d*Sec[e + f*x])^2) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*
x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-55*c^5*Sin[(f*x)/2] + 135*c^4*d*Sin[(f
*x)/2] - 20*c^3*d^2*Sin[(f*x)/2] - 810*c^2*d^3*Sin[(f*x)/2] - 450*c*d^4*Sin
[(f*x)/2] + 150*d^5*Sin[(f*x)/2] + 47*c^5*Sin[(3*f*x)/2] - 137*c^4*d*Sin[(3
*f*x)/2] + 88*c^3*d^2*Sin[(3*f*x)/2] + 812*c^2*d^3*Sin[(3*f*x)/2] + 690*c*d
^4*Sin[(3*f*x)/2] + 75*d^5*Sin[(3*f*x)/2] - 50*c^5*Sin[e - (f*x)/2] + 130*c
^4*d*Sin[e - (f*x)/2] - 10*c^3*d^2*Sin[e - (f*x)/2] - 1030*c^2*d^3*Sin[e -
(f*x)/2] - 990*c*d^4*Sin[e - (f*x)/2] - 150*d^5*Sin[e - (f*x)/2] + 50*c^5*S
in[e + (f*x)/2] - 130*c^4*d*Sin[e + (f*x)/2] + 10*c^3*d^2*Sin[e + (f*x)/2]
+ 1030*c^2*d^3*Sin[e + (f*x)/2] + 765*c*d^4*Sin[e + (f*x)/2] - 150*d^5*Sin[
e + (f*x)/2] - 55*c^5*Sin[2*e + (f*x)/2] + 135*c^4*d*Sin[2*e + (f*x)/2] - 2
0*c^3*d^2*Sin[2*e + (f*x)/2] - 810*c^2*d^3*Sin[2*e + (f*x)/2] - 675*c*d^4*S
in[2*e + (f*x)/2] - 150*d^5*Sin[2*e + (f*x)/2] - 30*c^5*Sin[e + (3*f*x)/2]
+ 90*c^4*d*Sin[e + (3*f*x)/2] - 60*c^3*d^2*Sin[e + (3*f*x)/2] - 360*c^2*d^3
*Sin[e + (3*f*x)/2] - 30*c*d^4*Sin[e + (3*f*x)/2] + 75*d^5*Sin[e + (3*f*x)/
2] + 47*c^5*Sin[2*e + (3*f*x)/2] - 137*c^4*d*Sin[2*e + (3*f*x)/2] + 88*c^3*
d^2*Sin[2*e + (3*f*x)/2] + 812*c^2*d^3*Sin[2*e + (3*f*x)/2] + 525*c*d^4*Sin
[2*e + (3*f*x)/2] - 75*d^5*Sin[2*e + (3*f*x)/2] - 30*c^5*Sin[3*e + (3*f*x)/
2] + 90*c^4*d*Sin[3*e + (3*f*x)/2] - 60*c^3*d^2*Sin[3*e + (3*f*x)/2] - 360*
c^2*d^3*Sin[3*e + (3*f*x)/2] - 195*c*d^4*Sin[3*e + (3*f*x)/2] - 75*d^5*Sin[
3*e + (3*f*x)/2] + 20*c^5*Sin[e + (5*f*x)/2] - 76*c^4*d*Sin[e + (5*f*x)/2]
+ 106*c^3*d^2*Sin[e + (5*f*x)/2] + 346*c^2*d^3*Sin[e + (5*f*x)/2] + 219*c*d
^4*Sin[e + (5*f*x)/2] + 15*d^5*Sin[e + (5*f*x)/2] - 15*c^5*Sin[2*e + (5*f*x
)/2] + 45*c^4*d*Sin[2*e + (5*f*x)/2] - 30*c^3*d^2*Sin[2*e + (5*f*x)/2] - 90
*c^2*d^3*Sin[2*e + (5*f*x)/2] + 75*c*d^4*Sin[2*e + (5*f*x)/2] + 15*d^5*Sin[
2*e + (5*f*x)/2] + 20*c^5*Sin[3*e + (5*f*x)/2] - 76*c^4*d*Sin[3*e + (5*f*x)
/2] + 106*c^3*d^2*Sin[3*e + (5*f*x)/2] + 346*c^2*d^3*Sin[3*e + (5*f*x)/2] +
 144*c*d^4*Sin[3*e + (5*f*x)/2] - 15*d^5*Sin[3*e + (5*f*x)/2] - 15*c^5*Sin[
4*e + (5*f*x)/2] + 45*c^4*d*Sin[4*e + (5*f*x)/2] - 30*c^3*d^2*Sin[4*e + (5*
f*x)/2] - 90*c^2*d^3*Sin[4*e + (5*f*x)/2] - 15*d^5*Sin[4*e + (5*f*x)/2] + 7
*c^5*Sin[2*e + (7*f*x)/2] - 27*c^4*d*Sin[2*e + (7*f*x)/2] + 38*c^3*d^2*Sin[
2*e + (7*f*x)/2] + 72*c^2*d^3*Sin[2*e + (7*f*x)/2] + 15*c*d^4*Sin[2*e + (7*
f*x)/2] + 15*c*d^4*Sin[3*e + (7*f*x)/2] + 7*c^5*Sin[4*e + (7*f*x)/2] - 27*c
^4*d*Sin[4*e + (7*f*x)/2] + 38*c^3*d^2*Sin[4*e + (7*f*x)/2] + 72*c^2*d^3*Si
n[4*e + (7*f*x)/2]))/(120*c*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3*(c
+ d*Sec[e + f*x])^2)
```

fricas [B] time = 0.54, size = 1693, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/30*(15*(4*c*d^4 + 3*d^5 + (4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2
*d^3 + 13*c*d^4 + 3*d^5)*cos(f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*c
os(f*x + e)^2 + (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(c^2 - d^2
)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2
)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*
c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^
4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2 + 99*c^3*d^3 - 23*c^
2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 29*c^5*d + 51*c^4*d^2
+ 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(f*x + e)^2 + (2*c^6 -
 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3*c^6*d^2 + 5*a^3*c^5*
d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^3*c*d^7)*f*cos(f*x +
e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*a^3*c^4*d^4 - 8*a^3*c
^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3*(a^3*c^8 - 2*a^3*c^7
*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*
c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^
3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x +
e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d
^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f), -1/15*(15*(4*c*d^4 + 3*d^5 +
(4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2*d^3 + 13*c*d^4 + 3*d^5)*cos(
f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (4*c^2*d^3 +
15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(
d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^5*d - 12*c^4*d^2 + 4
1*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2
 + 99*c^3*d^3 - 23*c^2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 2
9*c^5*d + 51*c^4*d^2 + 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(
f*x + e)^2 + (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141
*c*d^5 - 171*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3
*c^6*d^2 + 5*a^3*c^5*d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^
3*c*d^7)*f*cos(f*x + e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*
a^3*c^4*d^4 - 8*a^3*c^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3
*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2
*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c
^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*
a^3*d^8)*f*cos(f*x + e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*
c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f)]
```

giac [B] time = 0.97, size = 951, normalized size = 3.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan
(1/2*f*x + 1/2*e)^2 - c - d)) + 120*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*
f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(-c^2 + d^2)) - (3*a^12*c^8*tan(
1/2*f*x + 1/2*e)^5 - 24*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 84*a^12*c^6*d^2
*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 + 210*a^1
2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^5
+ 84*a^12*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 24*a^12*c*d^7*tan(1/2*f*x + 1/2*
e)^5 + 3*a^12*d^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8*tan(1/2*f*x + 1/2*e)
^3 + 100*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 420*a^12*c^6*d^2*tan(1/2*f*x +
 1/2*e)^3 + 980*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 1400*a^12*c^4*d^4*tan
(1/2*f*x + 1/2*e)^3 + 1260*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 700*a^12*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 220*a^12*c*d^7*tan(1/2*f*x + 1/2*e)^3 - 30*
a^12*d^8*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^8*tan(1/2*f*x + 1/2*e) - 180*a^
12*c^7*d*tan(1/2*f*x + 1/2*e) + 1020*a^12*c^6*d^2*tan(1/2*f*x + 1/2*e) - 31
80*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e) + 5850*a^12*c^4*d^4*tan(1/2*f*x + 1/2*
e) - 6540*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e) + 4380*a^12*c^2*d^6*tan(1/2*f*x
 + 1/2*e) - 1620*a^12*c*d^7*tan(1/2*f*x + 1/2*e) + 255*a^12*d^8*tan(1/2*f*x
 + 1/2*e))/(a^15*c^10 - 10*a^15*c^9*d + 45*a^15*c^8*d^2 - 120*a^15*c^7*d^3
+ 210*a^15*c^6*d^4 - 252*a^15*c^5*d^5 + 210*a^15*c^4*d^6 - 120*a^15*c^3*d^7
 + 45*a^15*c^2*d^8 - 10*a^15*c*d^9 + a^15*d^10))/f
```

maple [A] time = 0.94, size = 284, normalized size = 0.99

$$\frac{\frac{\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2}{5}-\frac{2\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{5}+\frac{\left(\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^2}{5}-\frac{2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^2}{3}+\frac{8\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{3}-2\left(\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^2+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2-6cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+17\tan}{\left(c^2-2cd+d^2\right)(c-d)^2}}{4fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/4/f/a^3*(1/(c^2-2*c*d+d^2)/(c-d)^2*(1/5*tan(1/2*e+1/2*f*x)^5*c^2-2/5*tan(
1/2*e+1/2*f*x)^5*c*d+1/5*tan(1/2*e+1/2*f*x)^5*d^2-2/3*tan(1/2*e+1/2*f*x)^3*
c^2+8/3*tan(1/2*e+1/2*f*x)^3*c*d-2*tan(1/2*e+1/2*f*x)^3*d^2+tan(1/2*e+1/2*f
*x)*c^2-6*c*d*tan(1/2*e+1/2*f*x)+17*tan(1/2*e+1/2*f*x)*d^2)+16*d^3/(c-d)^4*
(-1/2*d/(c+d)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)
```

```
^2*d-c-d)-1/2*(4*c+3*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x
)*(c-d)/((c+d)*(c-d))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.12, size = 464, normalized size = 1.61

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20\,a^3\,f\,(c-d)^2}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{2\left(c^2-d^2\right)\left(\frac{1}{a^3\,(c-d)^2}-\frac{c^2-d^2}{2\,a^3\,(c-d)^4}\right)}{(c-d)^2}-\frac{3}{2\,a^3\,(c-d)^2}+\frac{(c+d)^2}{4\,a^3\,(c-d)^4}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{1}{3\,a^3\,(c-d)^2}-\frac{c^2-}{6\,a^3\,(}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^2) - (tan(e/2 + (f*x)/2)*((2*(c^2 -
d^2)*(1/(a^3*(c - d)^2) - (c^2 - d^2)/(2*a^3*(c - d)^4)))/(c - d)^2 - 3/(2*
a^3*(c - d)^2) + (c + d)^2/(4*a^3*(c - d)^4)))/f - (tan(e/2 + (f*x)/2)^3*(1
/(3*a^3*(c - d)^2) - (c^2 - d^2)/(6*a^3*(c - d)^4)))/f + (2*d^4*tan(e/2 + (
f*x)/2))/(f*(c + d)*(a^3*c^5 - tan(e/2 + (f*x)/2)^2*(a^3*c^5 - a^3*d^5 + 5*
a^3*c*d^4 - 5*a^3*c^4*d - 10*a^3*c^2*d^3 + 10*a^3*c^3*d^2) + a^3*d^5 - 3*a^
3*c*d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2)) + (d^3*atan((c^5*ta
n(e/2 + (f*x)/2)*1i - d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*
5i - c^4*d*tan(e/2 + (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2
*tan(e/2 + (f*x)/2)*10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(4*c + 3*d)*2i)/(a^
3*f*(c + d)^(3/2)*(c - d)^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^2\sec^3\left(e+fx\right)+3c^2\sec^2\left(e+fx\right)+3c^2\sec\left(e+fx\right)+c^2+2cd\sec^4\left(e+fx\right)+6cd\sec^3\left(e+fx\right)+6cd\sec^2\left(e+fx\right)+2cd\sec\left(e+fx\right)+d^2\sec^5\left(e+fx\right)+3d^2\sec^4\left(\right.}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**3 + 3*c**2*sec(e + f*x)**2 + 3*c*
*2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**4 + 6*c*d*sec(e + f*x)**3 + 6*
c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**5 + 3*d**2*se
c(e + f*x)**4 + 3*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**3
```

3.233 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=368

$$\frac{(2c^2-15cd+76d^2)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)(c+d\sec(e+fx))^2}-\frac{d^3(20c^2+30cd+13d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3f(c-d)^{11/2}(c+d)^{5/2}}+\frac{d(4c^3}{30a}$$

```
[Out] -d^3*(20*c^2+30*c*d+13*d^2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1
/2))/a^3/(c-d)^(11/2)/(c+d)^(5/2)/f+1/30*d*(4*c^3-30*c^2*d+146*c*d^2+195*d^
3)*tan(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))^2+1/5*tan(f*x+e)/(c-d)/f
/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2+1/15*(2*c-11*d)*tan(f*x+e)/a/(c-d)^2
/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2+1/15*(2*c^2-15*c*d+76*d^2)*tan(f*x
+e)/(c-d)^3/f/(a^3+a^3*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/30*d*(4*c^4-30*c^3*
d+142*c^2*d^2+525*c*d^3+304*d^4)*tan(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sec(
f*x+e))
```

Rubi [A] time = 0.71, antiderivative size = 414, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 103, 151, 152, 12, 93, 205}

$$\frac{d^3(20c^2+30cd+13d^2)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{11/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(142c^2d^2-30c^3d+4c^4+525cd^3+304d^4)\tan(}{30f(c-d)^5(c+d)^2\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 39*c*d + 22*d^2)*Tan[e + f*x])/(10*(c - d)^3*(c + d)^2*f*(a + a*S
ec[e + f*x])^3) + ((4*c^3 - 26*c^2*d - 184*c*d^2 - 109*d^3)*Tan[e + f*x])/(
30*a*(c - d)^4*(c + d)^2*f*(a + a*Sec[e + f*x])^2) + (d^3*(20*c^2 + 30*c*d
+ 13*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(11/2)*(c + d)^(5/2)*f*Sqrt
[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((4*c^4 - 30*c^3*d + 142*c
^2*d^2 + 525*c*d^3 + 304*d^4)*Tan[e + f*x])/(30*(c - d)^5*(c + d)^2*f*(a^3
+ a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x
])^3*(c + d*Sec[e + f*x])^2) - (3*d*(2*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^
2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{7/2}(c+dx)^3}\,dx, x, \sec(e+f\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{\tan(e}{2}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{}{2\left(c^2\right.}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} - \frac{d}{2\left(c^2-d^2\right)f(a+a\,\text{se}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184c\right.}{30a(c-d)^4(c+d)^2}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184c\right.}{30a(c-d)^4(c+d)^2}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184c\right.}{30a(c-d)^4(c+d)^2}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184c\right.}{30a(c-d)^4(c+d)^2}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184c\right.}{30a(c-d)^4(c+d)^2}$$

Mathematica [C] time = 7.83, size = 1096, normalized size = 2.98

$$\frac{4\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec\left(\frac{e}{2}\right)\left(23d\sin\left(\frac{e}{2}\right)-8c\sin\left(\frac{e}{2}\right)\right)\sec^6(e+fx)}{15(d-c)^4f(\sec(e+fx)a+a)^3(c+d\sec(e+fx))^3} + \frac{\left(20c^2+30dc+13d^2\right)\cos^6}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (4*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*
c*Sin[e/2] + 23*d*Sin[e/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(c + d
*Sec[e + f*x])^3) + ((20*c^2 + 30*c*d + 13*d^2)*Cos[e/2 + (f*x)/2]^6*(d + c
*Cos[e + f*x])^3*Sec[e + f*x]^6*(((-8*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(S
qrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*S
qrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*
Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (8*d^3*ArcTan[Sec
[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e]
)/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c
*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])
))/((-c + d)^5*(c + d)^2*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (
2*Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*Sin[(f*
x)/2])/(5*(-c + d)^3*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) + (4*
Cos[e/2 + (f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*c*S
in[(f*x)/2] + 23*d*Sin[(f*x)/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(
c + d*Sec[e + f*x])^3) - (8*Cos[e/2 + (f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec
[e/2]*Sec[e + f*x]^6*(7*c^2*Sin[(f*x)/2] - 44*c*d*Sin[(f*x)/2] + 127*d^2*Si
n[(f*x)/2]))/(15*(-c + d)^5*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3
) + (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])*Sec[e]*Sec[e + f*x]^6*(d^6
*Sin[e] - c*d^5*Sin[f*x]))/(c^2*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3
*(c + d*Sec[e + f*x])^3) - (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*S
ec[e]*Sec[e + f*x]^6*(-11*c^2*d^5*Sin[e] - 6*c*d^6*Sin[e] + 2*d^7*Sin[e] +
10*c^3*d^4*Sin[f*x] + 6*c^2*d^5*Sin[f*x] - c*d^6*Sin[f*x]))/(c^2*(-c + d)^5
*(c + d)^2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (2*Cos[e/2 +
(f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*Tan[e/2])/(5*(-c + d)^3*f*
(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3)
```

fricas [B] time = 0.62, size = 2677, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/60*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13*
c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^6
)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 13
*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
+ 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)*
cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
```

*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d
^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(4*c^6*d^2 - 30*c^
5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14
*c^8 - 60*c^7*d + 78*c^6*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 41
9*c^2*d^6 - 90*c*d^7 + 15*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^
6*d^2 + 1056*c^5*d^3 + 1626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7
 - 45*d^8)*cos(f*x + e)^3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2
400*c^4*d^4 + 1197*c^3*d^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x +
 e)^2 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 -
 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11
 - 3*a^3*c^10*d + 8*a^3*c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5
*d^6 - 3*a^3*c^3*d^8 + a^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*
c^10*d - 6*a^3*c^9*d^2 + 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 +
12*a^3*c^5*d^6 + 16*a^3*c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d
^10)*f*cos(f*x + e)^4 + (3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^
3*c^8*d^3 + 30*a^3*c^7*d^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d
^7 - a^3*c^3*d^8 - 15*a^3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)
^3 + (a^3*c^11 + 3*a^3*c^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d
^4 - 18*a^3*c^6*d^5 - 46*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17
*a^3*c^2*d^9 - 3*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d
 - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*
c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3
*a^3*d^11)*f*cos(f*x + e) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)
, -1/30*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13
*c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^
6)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 1
3*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
 + 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)
*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) +
c)/((c^2 - d^2)*sin(f*x + e))) - (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 55
5*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14*c^8 - 60*c^7*d + 78*c^6
*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 419*c^2*d^6 - 90*c*d^7 + 1
5*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^6*d^2 + 1056*c^5*d^3 + 1
626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7 - 45*d^8)*cos(f*x + e)^
3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2400*c^4*d^4 + 1197*c^3*d
^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x + e)^2 + (8*c^7*d - 48*c^
6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 - 459*c^2*d^6 - 1733*c*d^
7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11 - 3*a^3*c^10*d + 8*a^3*
c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5*d^6 - 3*a^3*c^3*d^8 + a
^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*c^10*d - 6*a^3*c^9*d^2 +
 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 + 12*a^3*c^5*d^6 + 16*a^3*
c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d^10)*f*cos(f*x + e)^4 +
(3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^3*c^8*d^3 + 30*a^3*c^7*d
^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d^7 - a^3*c^3*d^8 - 15*a^

```
3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 + (a^3*c^11 + 3*a^3*c
^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 18*a^3*c^6*d^5 - 46
*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 3*a^3*c*d
^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*
c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7
+ 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e
) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^
4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)]
```

giac [B] time = 0.62, size = 1419, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3
 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*sqrt(-c^2 + d^2)) -
 (3*a^12*c^12*tan(1/2*f*x + 1/2*e)^5 - 36*a^12*c^11*d*tan(1/2*f*x + 1/2*e)^
5 + 198*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^5 - 660*a^12*c^9*d^3*tan(1/2*f*x
 + 1/2*e)^5 + 1485*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^5 - 2376*a^12*c^7*d^5*
tan(1/2*f*x + 1/2*e)^5 + 2772*a^12*c^6*d^6*tan(1/2*f*x + 1/2*e)^5 - 2376*a^
12*c^5*d^7*tan(1/2*f*x + 1/2*e)^5 + 1485*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e)^
5 - 660*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^5 + 198*a^12*c^2*d^10*tan(1/2*f*x
 + 1/2*e)^5 - 36*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^12*tan(1/2*f
*x + 1/2*e)^5 - 10*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*c^11*d*tan(1
/2*f*x + 1/2*e)^3 - 990*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^3 + 3850*a^12*c^
9*d^3*tan(1/2*f*x + 1/2*e)^3 - 9900*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^3 + 1
7820*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e)^3 - 23100*a^12*c^6*d^6*tan(1/2*f*x +
 1/2*e)^3 + 21780*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e)^3 - 14850*a^12*c^4*d^8*
tan(1/2*f*x + 1/2*e)^3 + 7150*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^3 - 2310*a^
12*c^2*d^10*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^3
 - 40*a^12*d^12*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^12*tan(1/2*f*x + 1/2*e)
- 270*a^12*c^11*d*tan(1/2*f*x + 1/2*e) + 2340*a^12*c^10*d^2*tan(1/2*f*x + 1
/2*e) - 11850*a^12*c^9*d^3*tan(1/2*f*x + 1/2*e) + 38475*a^12*c^8*d^4*tan(1/
2*f*x + 1/2*e) - 84780*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e) + 131040*a^12*c^6*
d^6*tan(1/2*f*x + 1/2*e) - 144180*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e) + 11272
5*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e) - 61350*a^12*c^3*d^9*tan(1/2*f*x + 1/2*
e) + 22140*a^12*c^2*d^10*tan(1/2*f*x + 1/2*e) - 4770*a^12*c*d^11*tan(1/2*f*
x + 1/2*e) + 465*a^12*d^12*tan(1/2*f*x + 1/2*e))/(a^15*c^15 - 15*a^15*c^14*
d + 105*a^15*c^13*d^2 - 455*a^15*c^12*d^3 + 1365*a^15*c^11*d^4 - 3003*a^15*
c^10*d^5 + 5005*a^15*c^9*d^6 - 6435*a^15*c^8*d^7 + 6435*a^15*c^7*d^8 - 5005
```

```
*a^15*c^6*d^9 + 3003*a^15*c^5*d^10 - 1365*a^15*c^4*d^11 + 455*a^15*c^3*d^12
 - 105*a^15*c^2*d^13 + 15*a^15*c*d^14 - a^15*d^15) + 60*(10*c^2*d^4*tan(1/2
*f*x + 1/2*e)^3 - 3*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 7*d^6*tan(1/2*f*x + 1/2*
e)^3 - 10*c^2*d^4*tan(1/2*f*x + 1/2*e) - 15*c*d^5*tan(1/2*f*x + 1/2*e) - 5*
d^6*tan(1/2*f*x + 1/2*e))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4
*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*(c*tan(1/2*f*x
+ 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

maple [A] time = 0.97, size = 365, normalized size = 0.99

$$\frac{\frac{\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^{2}}{5}-\frac{2\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{5}+\frac{\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^{2}}{5}-\frac{2\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c^{2}}{3}+\frac{10\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)cd}{3}-\frac{8\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d^{2}}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^{2}-8cd\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+31\tan}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\left(c^{2}-2cd+d^{2}\right)}}{4f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/4/f/a^3*(1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c^2-2*c*d+d^2)*(1/5*tan(1/2*e+1/2*f
*x)^5*c^2-2/5*tan(1/2*e+1/2*f*x)^5*c*d+1/5*tan(1/2*e+1/2*f*x)^5*d^2-2/3*tan
(1/2*e+1/2*f*x)^3*c^2+10/3*tan(1/2*e+1/2*f*x)^3*c*d-8/3*tan(1/2*e+1/2*f*x)^
3*d^2+tan(1/2*e+1/2*f*x)*c^2-8*c*d*tan(1/2*e+1/2*f*x)+31*tan(1/2*e+1/2*f*x)
*d^2)+16*d^3/(c-d)^5*((-1/4*d*(10*c^2-3*c*d-7*d^2)/(c^2+2*c*d+d^2)*tan(1/2*
e+1/2*f*x)^3+5/4*d*(2*c+d)/(c+d)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*
c-tan(1/2*e+1/2*f*x)^2*d-c-d)^2-1/4*(20*c^2+30*c*d+13*d^2)/(c^2+2*c*d+d^2)/
((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.36, size = 655, normalized size = 1.78

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20\,a^3\,f\,(c-d)^3}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3\,(c+d)^2}{4\,a^3\,(c-d)^5}-\frac{5}{2\,a^3\,(c-d)^3}+\frac{3\,(c+d)\left(\frac{5}{4\,a^3\,(c-d)^3}-\frac{3\,(c+d)}{4\,a^3\,(c-d)^4}\right)}{c-d}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{5}{12\,a^3\,(c-d)^3}-\frac{c-}{4\,a^3\,(}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^3) - (tan(e/2 + (f*x)/2)*((3*(c + d)
^2)/(4*a^3*(c - d)^5) - 5/(2*a^3*(c - d)^3) + (3*(c + d)*(5/(4*a^3*(c - d)^
3) - (3*(c + d))/(4*a^3*(c - d)^4)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(5
/(12*a^3*(c - d)^3) - (c + d)/(4*a^3*(c - d)^4)))/f - ((tan(e/2 + (f*x)/2)^
3*(3*c*d^5 + 7*d^6 - 10*c^2*d^4))/(c + d)^2 + (5*tan(e/2 + (f*x)/2)*(2*c*d^
4 + d^5))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^3*c^7 + 2*a^3*d^7 - 10*a^3
*c*d^6 - 10*a^3*c^6*d + 18*a^3*c^2*d^5 - 10*a^3*c^3*d^4 - 10*a^3*c^4*d^3 +
18*a^3*c^5*d^2) - tan(e/2 + (f*x)/2)^4*(a^3*c^7 - a^3*d^7 + 7*a^3*c*d^6 - 7
*a^3*c^6*d - 21*a^3*c^2*d^5 + 35*a^3*c^3*d^4 - 35*a^3*c^4*d^3 + 21*a^3*c^5*
d^2) - a^3*c^7 + a^3*d^7 - 3*a^3*c*d^6 + 3*a^3*c^6*d + a^3*c^2*d^5 + 5*a^3*
c^3*d^4 - 5*a^3*c^4*d^3 - a^3*c^5*d^2)) + (d^3*atan((c^6*tan(e/2 + (f*x)/2)
*1i + d^6*tan(e/2 + (f*x)/2)*1i - c*d^5*tan(e/2 + (f*x)/2)*6i - c^5*d*tan(e
/2 + (f*x)/2)*6i + c^2*d^4*tan(e/2 + (f*x)/2)*15i - c^3*d^3*tan(e/2 + (f*x)
/2)*20i + c^4*d^2*tan(e/2 + (f*x)/2)*15i)/((c + d)^(1/2)*(c - d)^(11/2)))*(
30*c*d + 20*c^2 + 13*d^2)*1i)/(a^3*f*(c + d)^(5/2)*(c - d)^(11/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^3\sec^3(e+fx)+3c^3\sec^2(e+fx)+3c^3\sec(e+fx)+c^3+3c^2d\sec^4(e+fx)+9c^2d\sec^3(e+fx)+9c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^5(e+fx)+9cd}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x)**3 + 3*c**3*sec(e + f*x)**2 + 3*c*
*3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**4 + 9*c**2*d*sec(e + f*x)**
3 + 9*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x
)**5 + 9*c*d**2*sec(e + f*x)**4 + 9*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**6 + 3*d**3*sec(e + f*x)**5 + 3*d**3*sec(e +
 f*x)**4 + d**3*sec(e + f*x)**3), x)/a**3
```

3.234 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}\,f}$$

```
[Out] 2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*a^(1/2)/f/d^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3980, 206}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[d]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

Mathematica [A] time = 0.23, size = 102, normalized size = 1.67

$$\frac{\sqrt{2}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c\cos(e+fx)+d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)}{\sqrt{d}f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(S
qrt[d]*f*Sqrt[c + d*Sec[e + f*x]])
```

fricas [B] time = 0.67, size = 307, normalized size = 5.03

$$\left[\frac{\sqrt{\frac{a}{d}}\log\left(-\frac{8\,acd\cos\left(fx+e\right)+\left(ac^2-6\,acd+ad^2\right)\cos\left(fx+e\right)^3+4\left(2\,d^2\cos\left(fx+e\right)+\left(cd-d^2\right)\cos\left(fx+e\right)^2\right)\sqrt{\frac{a}{d}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2}\right.}{2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(a/d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f
*x + e)^3 + 4*(2*d^2*cos(f*x + e) + (c*d - d^2)*cos(f*x + e)^2)*sqrt(a/d)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
```

```
e))*sin(f*x + e) + 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(c
os(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-a/d)*arctan(-2*d*sqrt(-a/d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e)/((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)
*cos(f*x + e)))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{\sqrt{d \sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

maple [B] time = 2.19, size = 301, normalized size = 4.93

$$\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)\left(\ln\left(-\frac{2\left(\sqrt{2}\,\sqrt{-d}\,\sqrt{-\frac{2\left(d+c\cos\left(fx+e\right)\right)}{1+\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)-c\sin\left(fx+\cdots\right.\right.}{-1+\cos\left(fx+e\right)+\cdots}\right.\right.}{f\sin\left(fx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)
*cos(f*x+e)*(-1+cos(f*x+e))*(ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))
/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c*cos(f*x+e)+d*
cos(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e)))-ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d
+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c
*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e))))/sin(f*x+e)^2/(-2*
(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*2^(1/2)/(-d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{\sqrt{d \sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)),
x)
```

3.235 $\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/2)+2*arctanh(a^(1/2)*d^(1/
2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*d^(1/2)/f/a^(1
/2)
```

Rubi [A] time = 0.46, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3981, 3983, 203, 3980, 206}

$$\frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) + (2*Sqrt[d]*
ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d
*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3981

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> -Dist[(b*c - a*d)/d,
 Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x],
x] + Dist[b/d, Int[(Csc[e + f*x]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e
 + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - (-c+d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec\cdots}}$$

$$= -\frac{(2(c-d))\,\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{f} - \frac{(2d)\cdots}{\cdots}$$

$$= \frac{\sqrt{2}\,\sqrt{c-d}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f} + \frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{a+a\sec\cdots}}\right.}{\sqrt{a}\cdots}$$

Mathematica [A] time = 17.02, size = 187, normalized size = 1.34

$$\frac{\sqrt{c}\,\sin(e+fx)\sqrt{c+d\sec(e+fx)}\left(2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{c\cos(e+fx)+d}}{\sqrt{d}\,\sqrt{c-c\cos(e+fx)}}\right) - \sqrt{2}\,\sqrt{c-d}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{c\cos(e+fx)+d}}{\sqrt{c-d}\,\sqrt{c-c\cos(e+fx)}}\right)\right)}{f\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c-c\cos(e+fx)}\,\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[c]*(-(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + c*Cos[e +
f*x]])/(Sqrt[c - d]*Sqrt[c - c*Cos[e + f*x]])]) + 2*Sqrt[d]*ArcTanh[(Sqrt[c
]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c - c*Cos[e + f*x]])])*Sqrt[c + d
*Sec[e + f*x]]*Sin[e + f*x])/(f*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 0.77, size = 1048, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*cos(f*x + e) + c
 - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(d/a)*log(-((c^2 - 6*c
*d + d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sq
rt(d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/c
os(f*x + e))*sin(f*x + e) + 8*c*d*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(
f*x + e)^2 + 8*d^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/f, 1/2*(2*sqrt(2)*s
qrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*s
in(f*x + e))) + sqrt(d/a)*log(-((c^2 - 6*c*d + d^2)*cos(f*x + e)^3 + 4*((c
- d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(d/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*c*d
*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(f*x + e)^2 + 8*d^2)/(cos(f*x + e)
^3 + cos(f*x + e)^2)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqr
t(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2
*(c + d)*cos(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2
*sqrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*co
s(f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f, (sqrt(2)*sqrt((c - d)/a)*ar
ctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + s
qrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*cos(
f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\,\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

maple [B] time = 1.99, size = 503, normalized size = 3.59

$$\frac{\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)\left(d\ln\left(-\frac{2\left(\sqrt{2}\,\sqrt{-d}\,\sqrt{-\frac{2\left(d+c\cos\left(fx+e\right)\right)}{1+\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)-c\sin\left(fx+\right.\right.}{-1+\cos\left(fx+e\right)+}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)
*cos(f*x+e)*(-1+cos(f*x+e))*(d*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e
))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c*cos(f*x+e)+
d*cos(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e)))*(c-d)^(1/2)-d*ln(2*(2^(1/2)*(
-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e
)-d*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(c
-d)^(1/2)+ln(((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*
x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*2^(1/2)*(-d)^(1
/2)*c-ln(((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)
-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*2^(1/2)*(-d)^(1/2)*
d)/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/a/(c-d)^(1/2)*2^
(1/2)/(-d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\,\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)),
x)
```

3.236 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3983, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}\,f}$$

Mathematica [A] time = 0.22, size = 107, normalized size = 1.37

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c\cos(e+fx)+d}\,\tan^{-1}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)}{f\sqrt{c-d}\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]]*Cos[(e +
 f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(Sqrt[c - d]*f*Sqrt[a*(1 +
Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

fricas [A] time = 0.54, size = 246, normalized size = 3.15

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{ac-ad}}\log\left(-\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sqrt{-\frac{1}{ac-ad}}\cos(fx+e)\sin(fx+e)-(3c-d)\cos(fx+e)^2-2(c+d)\cos(fx+e)+c-3d}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*log(-(2*sqrt(2)*(c - d)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*c
 - a*d))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*c
```

```
os(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))/f, -sqrt(2)*a
rctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/(sqrt(a*c -
 a*d)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}\,\sqrt{d\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

maple [B] time = 1.95, size = 170, normalized size = 2.18

$$\frac{2\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\,\cos(fx+e)\left(-1+\cos(fx+e)\right)\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\,\ln\left(-\frac{\sqrt{c-d}\,\cos(fx+e)-\sin(fx+e)\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}-\sqrt{c-d}}{\sin(fx+e)}\right)}{f\sin(fx+e)^2\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}\,a\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*((d+c*co
s(f*x+e))/cos(f*x+e))^(1/2)*ln(-((c-d)^(1/2)*cos(f*x+e)-sin(f*x+e)*(-2*(d+c
*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-(c-d)^(1/2))/sin(f*x+e))/sin(f*x+e)^2/(-
2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/a/(c-d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}\,\sqrt{d\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)))
, x)
```

3.237 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] -arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(
c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)+2*arctanh(a^(1/2)*d^(1
/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/f/a^(1/2)/d^(
1/2)
```

Rubi [A] time = 0.53, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3985, 3983, 203, 3980, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]])/(Sqrt[a]*Sqrt[c - d]*f)) + (2*ArcTa
nh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[
e + f*x]])])/(Sqrt[a]*Sqrt[d]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3985

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> -Dist[a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/Sqrt[c + d*Csc[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(\cdots}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\,\mathrm{Subs}\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}\,f} + \frac{2\tanh^{-1}\left(\frac{\cdots}{\sqrt{a+a\sec\cdots}}\right.}{\sqrt{a}\cdots}$$

Mathematica [A] time = 0.28, size = 171, normalized size = 1.21

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c\cos(e+fx)+d}\left(\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)-\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(\cdots}{\sqrt{c\cos(e+f\cdots}}\right.\right.}{\sqrt{d}\,f\sqrt{c-d}\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]
```

```
[Out] (2*(-(Sqrt[d]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]) + Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d
 + c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x]
)/(Sqrt[c - d]*Sqrt[d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]
])
```

fricas [A] time = 0.81, size = 1100, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(a*
d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 +
4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) +
 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + co
s(f*x + e)^2)))/(a*d*f), 1/2*(2*sqrt(2)*a*d*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a*c - a*d) + sqrt(a*d)*log(-(8*a*c*d
*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f
*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*a*d^2 + (a*c^
2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/
(a*d*f), 1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
sqrt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 +
 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
 2*sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c - a
*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f), (sqrt(2)*
a*d*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a
*c - a*d) + sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/
((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)^2}{\sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

maple [B] time = 2.03, size = 403, normalized size = 2.86

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\,\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\,\cos(fx+e)\,(-1+\cos(fx+e))\left(-\ln\left(-\frac{\sqrt{c-d}\,\cos(fx+e)-\sin(fx+e)\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}-\cdots}{\sin(fx+e)}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)
*cos(f*x+e)*(-1+cos(f*x+e))*(-ln(-((c-d)^(1/2)*cos(f*x+e)-sin(f*x+e)*(-2*(d
+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-(c-d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^
(1/2)+ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos
(f*x+e)+sin(f*x+e)))*(c-d)^(1/2)-ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+
e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+e)
-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(c-d)^(1/2))/sin(f*x+e)^2/(-2
*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)^2}{\sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x
))), x)
```

3.238 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

```
[Out] 2*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(
1/2)/f/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {3967, 205}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Se
c[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\,\sqrt{c+d}\,f}$$

Mathematica [A] time = 0.23, size = 94, normalized size = 1.54

$$\frac{\sqrt{2}\,\sqrt{\cos(e+fx)}\,\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\,\sqrt{\cos(e+fx)}}\right)}{\sqrt{d}\,f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
+ f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(
Sqrt[d]*Sqrt[c + d]*f)
```

fricas [B] time = 0.67, size = 343, normalized size = 5.62

$$\left[\frac{\sqrt{-\frac{a}{cd+d^2}}\,\log\left(-\frac{\left(ac^2+8\,acd+8\,ad^2\right)\cos\left(fx+e\right)^3+ad^2+\left(ac^2+2\,acd\right)\cos\left(fx+e\right)^2-4\left(\left(c^2d+3\,cd^2+2\,d^3\right)\cos\left(fx+e\right)^2-\left(cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{a}{cd+d^2}}}{c^2\cos\left(fx+e\right)^3+\left(c^2+2\,cd\right)\cos\left(fx+e\right)^2+d^2+\left(2\,cd+d^2\right)\cos\left(fx+e\right)}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(-a/(c*d + d^2))*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3
+ a*d^2 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*((c^2*d + 3*c*d^2 + 2*d^3)*c
os(f*x + e)^2 - (c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x +
```

```
e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2
)*cos(f*x + e)))/f, sqrt(a/(c*d + d^2))*arctan(2*(c*d + d^2)*sqrt(a/(c*d +
d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a
*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)2*a*sqrt(-a)*sign(cos(f*x+exp(1)))*a
tan(1/2*(c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1)
)))^2-d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))
^2+a*c+3*a*d)/sqrt(2)/sqrt(-d^2-c*d)/a)/sqrt(-d^2-c*d)/a/f
```

maple [B] time = 1.50, size = 433, normalized size = 7.10

$$-\frac{\left(\ln\left(-\frac{2\left(\sqrt{\frac{d}{c-d}}\,\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{2}\,c\sin\left(fx+e\right)-\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,d\sin\left(fx+e\right)+\sqrt{\left(c+d\right)\left(c-d\right)}\,\cos\left(fx+e\right)-c\sin\left(fx+e\right)+d\sin\left(fx+e\right)-\sqrt{}\cdots\right.}{c\cos\left(fx+e\right)-d\cos\left(fx+e\right)-\sqrt{\left(c+d\right)\left(c-d\right)}\,\sin\left(fx+e\right)-c+d}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -1/2/f*(ln(-2*((d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2)
*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d
)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c
+d))-ln(2*(-(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*c*
sin(f*x+e)+2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*s
in(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(
c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)
))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)
*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{d \sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

3.239 $\int \frac{(g \sec(e+fx))^{3/2} \sqrt{a+a \sec(e+fx)}}{c+d \sec(e+fx)} \, dx$

Optimal. Leaf size=149

$$\frac{2\sqrt{a}\, g^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\, \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}\,\sqrt{g \sec(e+fx)}}\right)}{df} - \frac{2\sqrt{a}\,\sqrt{c}\, g^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\, \tan(e+fx)}{\sqrt{c+d}\,\sqrt{a \sec(e+fx)+a}\,\sqrt{g \sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

```
[Out] 2*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2))*a^(1/2)/d/f-2*g^(3/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*
x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)*c^(1/
2)/d/f/(c+d)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3970, 3802, 208, 3965}

$$\frac{2\sqrt{a}\, g^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\, \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}\,\sqrt{g \sec(e+fx)}}\right)}{df} - \frac{2\sqrt{a}\,\sqrt{c}\, g^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\, \tan(e+fx)}{\sqrt{c+d}\,\sqrt{a \sec(e+fx)+a}\,\sqrt{g \sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]])]])/(d*f) - (2*Sqrt[a]*Sqrt[c]*g^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])]])/(d*Sqrt[c + d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3802

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/f, Subst[Int[1/(b - d*x^2), x], x, (b*C
ot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[(a*d)/b, 0]
```

Rule 3965

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b*g
)/f, Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3970

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[g/d,
Int[Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(c*g)/d, I
nt[(Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{g\int\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx}{d} - \frac{(cg)\int\frac{\sqrt{g\sec(e+fx)}\sqrt{a+a}}{c+d\sec(e+f}}{d}$$

$$= -\frac{\left(2ag^2\right)\mathrm{Subst}\left(\int\frac{1}{a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df} + \frac{\left(2acg^2\right)}{}$$

$$= \frac{2\sqrt{a}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df} - \frac{2\sqrt{a}\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\right.}{d}$$

Mathematica [C] time = 1.37, size = 427, normalized size = 2.87

$$\frac{\left(\sqrt{2}-2i\right)g^2\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(i\left(2\sqrt{c+d}\log\left(2\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{2}\right)+2\sqrt{c}\log\left(\sqrt{2}\sqrt{c+}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I + Sqrt[2])*g^2*(2*Sqrt[c + d]*ArcTan[(Cos[(e + f*x)/4] - (-1 + Sqrt[
2])*Sin[(e + f*x)/4])/((1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]
```

```
+ 2*Sqrt[c + d]*ArcTan[(Cos[(e + f*x)/4] - (1 + Sqrt[2])*Sin[(e + f*x)/4])/
((-1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])] + I*(2*Sqrt[c + d]*Lo
g[Sqrt[2] + 2*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[2 - Sqrt[2]*Cos[(e + f*x)
/2] - Sqrt[2]*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[2 + Sqrt[2]*Cos[(e + f*x)
/2] - Sqrt[2]*Sin[(e + f*x)/2]] + 2*Sqrt[c]*Log[Sqrt[2]*Sqrt[c + d] - 2*Sqr
t[c]*Sin[(e + f*x)/2]] - 2*Sqrt[c]*Log[Sqrt[2]*Sqrt[c + d] + 2*Sqrt[c]*Sin[
(e + f*x)/2]]))*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(4*(I + Sqrt[2
])*d*Sqrt[c + d]*f*Sqrt[g*Sec[e + f*x]])
```

fricas [A] time = 7.93, size = 1126, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*g/(c + d))*g*log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*
d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)
*cos(f*x + e))*sqrt(a*c*g/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a
*c^2 + 8*a*c*d + a*d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + sqrt(a*g)*g*log((a*g*cos(f*x + e)
^3 - 7*a*g*cos(f*x + e)^2 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) +
8*a*g)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/(d*f), -1/2*(2*sqrt(-a*c*g/(c +
d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*
sin(f*x + e))) - sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2
 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 +
cos(f*x + e)^2)))/(d*f), 1/2*(2*sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) + sqrt(a*c*g/(c + d))*g*
log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*d)*g*cos(f*x + e)^2 + 4*((c^
2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x
 + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a*c^2 + 8*a*c*d + a*d^2)*g)/(
c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos
(f*x + e))))/(d*f), (sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*g*cos(
f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) - sqrt(-a*c*g/(c + d))*g*arctan(1/2
*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c + d))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*sin(f*x + e))))/
(d*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

maple [B] time = 2.17, size = 566, normalized size = 3.80

$$2\left(\frac{g}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\left(\cos^{2}\left(fx+e\right)\right)\left(-1+\cos\left(fx+e\right)\right)^{2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\sqrt{\frac{c}{c-d}}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\,\left(\cos\left(fx+e\right)+1+\sin\left(f\right.\right.}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(-1+cos(f*x+e))^2*(a*(1+cos(f*x+e))/c
os(f*x+e))^(1/2)*((c/(c-d))^(1/2)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos
(f*x+e)+1+sin(f*x+e)))*((c+d)*(c-d))^(1/2)+(c/(c-d))^(1/2)*arctanh(1/2*(1/(
1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e)))*((c+d)*(c-d))^(1/2)+c*ln(2
*(2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*c*sin(f*x+e)-2*(c/(c-d))^(1/2)
*(1/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+((c+d)*(c-
d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c
*cos(f*x+e)+d*cos(f*x+e)+c-d))-c*ln(-2*(2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e))
)^(1/2)*c*sin(f*x+e)-2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e
)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)+((c+d)*(c-d))^(1
/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)))/sin(f
*x+e)^4/(1/(1+cos(f*x+e)))^(3/2)*(c-d)/((c+d)*(c-d))^(1/2)/(c-d+((c+d)*(c-d
))^(1/2))/(-c+d+((c+d)*(c-d))^(1/2))/(c/(c-d))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
),x)
```

```
[Out] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(g*sec(e + f*x))**(3/2)/(c + d*sec(e +
f*x)), x)
```

3.240 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

```
[Out] arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)
/f/a^(1/2)-2*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))
^(1/2))*d^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3972, 3795, 203, 3967, 205}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]])
/(Sqrt[a]*(c - d)*f) - (2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sec[e + f*x]])]])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 3972

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[(Cs
c[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{ac+ad+\cdots}\right.}{(c\cdots}$$

$$= \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} - \frac{2\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{c+d}\,f}$$

Mathematica [C] time = 33.71, size = 229015, normalized size = 1877.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.78, size = 963, normalized size = 7.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(-d/(a*c + a*d))*log(
-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 - 4*(
(c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*
c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*(sqrt(2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(d/(a*c + a*d))*arctan(2*(c + d)*sqrt(d/(
a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)))/((c - d)*f), -1/
2*(sqrt(-d/(a*c + a*d))*log(-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 +
 2*c*d)*cos(f*x + e)^2 - 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d
^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3
+ (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqr
t(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f), -(sqrt(d/(a*c + a*d))*arctan(2*
(c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)) +
 sqrt(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
```

```
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assump
tions on a variableWarning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableUnable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
```

```
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
```

```
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at
zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration o
f abs or sign assumes constant sign by intervals (correct if the argument i
s real):Check [abs(t_nostep^2-1)]Evaluation time: 1.02index.cc index_m i_le
x_is_greater Error: Bad Argument Value
```

maple [B] time = 1.48, size = 520, normalized size = 4.26

$$\frac{\left(2\sqrt{(c+d)(c-d)}\ \ln\left(-\frac{-\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}\ \sin(fx+e)+\cos(fx+e)-1}{\sin(fx+e)}\right)\sqrt{\frac{d}{c-d}}+d\sqrt{2}\ \ln\left(-\frac{2\left(\sqrt{\frac{d}{c-d}}\ \sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}}\ \sqrt{2}\ c\sin(fx+e)-\sqrt{2}\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(2*((c+d)*(c-d))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(d/(c-d))^(1/2)+d*2^(1/2)*ln(-2*((d/(c-d)
)^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(
d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-
d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(
f*x+e)-d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))-d*2^(1/2)*ln(2*(-(
d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*c*sin(f*x+e)+2^
(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c
+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/
(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)))*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(d/(c-d))^(1
/2)/(c-d)/((c+d)*(c-d))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}\,(d\sec(fx+e)+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+fx)\,\sqrt{a+\frac{a}{\cos(e+fx)}}\,\left(c+\frac{d}{\cos(e+fx)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

3.241 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=124

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d
)/f/a^(1/2)+2*c*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+
e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3976, 3795, 203, 3967, 205}

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[a]*(c - d)*f)) + (2*c*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[
c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[d]*Sqrt[c + d]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
```

```
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 3976

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\right.}{(c-}$$

$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} + \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{d}\,\sqrt{c+d}\,f}$$

Mathematica [A] time = 0.41, size = 141, normalized size = 1.14

$$-\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{d}\,\sqrt{c+d}\,\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)-\sqrt{2}\,c\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\,\sqrt{\cos(e+fx)}}\right)\right)}{\sqrt{d}\,f(c-d)\sqrt{c+d}\,\sqrt{\cos(e+fx)}\,\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*(Sqrt[d]*Sqrt[c + d]*ArcTan[Sin[(e + f*x)/2]/Sqrt[Cos[e + f*x]]] - Sqrt
[2]*c*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f
*x]])])*Cos[(e + f*x)/2])/((c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[Cos[e + f*x]]
*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 0.87, size = 1041, normalized size = 8.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a*
c*d - a*d^2)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (
a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x
 + e)^2 - d*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
 e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)
*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f)
, -1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x +
e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
a*c*d + a*d^2)*c*arctan(2*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d +
(a*c + a*d)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f), 1/2*(sqrt(-a*c*d - a*d^2
)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (a*c^2 + 2*a
*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x + e)^2 - d
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*
c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x +
e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqrt(2)*(a*c*d + a*d^2)*arcta
n(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin
(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f), (sqrt(a*c*d + a*d^2)*c*arctan(2
*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))
) + sqrt(2)*(a*c*d + a*d^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm ="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste

```
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
```

to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t

```
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of ab
s or sign assumes constant sign by intervals (correct if the argument is re
al):Check [abs(t_nostep^2-1)]Evaluation time: 1.07index.cc index_m i_lex_is
_greater Error: Bad Argument Value
```

maple [B] time = 1.43, size = 520, normalized size = 4.19

$$-\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\ \sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\ \left(2\sqrt{\left(c+d\right)\left(c-d\right)}\ \ln\left(-\frac{-\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\ \sin\left(fx+e\right)+\cos\left(fx+e\right)-1}{\sin\left(fx+e\right)}\right)\sqrt{\frac{d}{c-d}}\ +c\sqrt{2}\ \ln\left(-\frac{2}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*(2*((c+d)*(c-d))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(d/(c-d))^(1/2)+c*2^(1/2)*ln(-2*((d/(c-d))
^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d
/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d
))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f
*x+e)-d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))-c*2^(1/2)*ln(2*(-(d
/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*c*sin(f*x+e)+2^(
1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+
d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(
((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)))/a/((c+d)*(c
-d))^(1/2)/(c-d)/(d/(c-d))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{2}}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))),
 x)
```

3.242 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=167

$$\frac{2\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)}$$

```
[Out] -g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)+2*g^(3/2)*arctanh(a^(1/2)
*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e
))^(1/2))*c^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3974, 3808, 208, 3965}

$$\frac{2\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] -((Sqrt[2]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*S
ec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[c]*
g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3965

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b*g
)/f, Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3974

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[(a*g
)/(b*c - a*d), Int[Sqrt[g*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] +
Dist[(c*g)/(b*c - a*d), Int[(Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])
/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = -\frac{g\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{(cg)\int \frac{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$

$$= \frac{\left(2g^2\right)\text{Subst}\left(\int \frac{1}{2a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{\left(2cg^2\right)\ldots}{\ldots}$$

$$= -\frac{\sqrt{2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} + \frac{2\sqrt{c}\,g^{3/2}\tanh^{-1}\ldots}{\ldots}$$

Mathematica [A] time = 0.39, size = 198, normalized size = 1.19

$$\frac{g\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{g\sec(e+fx)}\left(\sqrt{2}\sqrt{c}\left(\log\left(\sqrt{2}\sqrt{c+d}+2\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\ldots\right.\right.\right.}{f(c-d)\sqrt{c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (g*Cos[(e + f*x)/2]*(2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] - Sin[(e + f*x)/4]]
 - 2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] + Sin[(e + f*x)/4]] + Sqrt[2]*Sqrt[c]
*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c]*Sin[(e + f*x)/2]] + Log[Sqrt[2]*Sqrt
```

```
[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]]))*Sqrt[g*Sec[e + f*x]])/((c - d)*Sqrt
[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 1.26, size = 1103, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x
 + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f
*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x +
 e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/c
os(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 + 8*c*d +
 d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d
+ d^2)*cos(f*x + e))))/((c - d)*f), 1/2*(2*sqrt(2)*g*sqrt(-g/a)*arctan(sqrt
(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))
*cos(f*x + e)/(g*sin(f*x + e))) - sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*
x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2
 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)
*g*cos(f*x + e) + (8*c^2 + 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c
*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*
(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x + e)^
2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt
(-c*g/(a*c + a*d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))
*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(
f*x + e))/(c*g*sin(f*x + e))))/((c - d)*f), (sqrt(2)*g*sqrt(-g/a)*arctan(sq
rt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e
))*cos(f*x + e)/(g*sin(f*x + e))) + sqrt(-c*g/(a*c + a*d))*g*arctan(1/2*(c*
cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(c*g*sin(f*x + e))))/((c
- d)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
```

i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/

```
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
```

```
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
```

```
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
```

```
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
```

```
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
```

```
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(c
os(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Evaluation time: 1.07index.cc index_m i_lex_i
s_greater Error: Bad Argument Value
```

maple [B] time = 2.22, size = 472, normalized size = 2.83

$$\frac{\left(\frac{g}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}} \sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\, \left(\cos^2\left(fx+e\right)\right)\left(-1+\cos\left(fx+e\right)\right)^2 \left(\operatorname{arcsinh}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sqrt{2}\,\sqrt{\frac{c}{c-d}}\,\sqrt{(c+d)\,(c-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(g/cos(f*x+e))^(3/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(
-1+cos(f*x+e))^2*(arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2)*(c/(c-d))^(1/
2)*((c+d)*(c-d))^(1/2)-c*ln(2*(2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*c
*sin(f*x+e)-2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+c*sin(f
*x+e)-d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(((c
+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))+c*ln(-2*(2*(c/(
c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*c*sin(f*x+e)-2*(c/(c-d))^(1/2)*(1/(1+c
os(f*x+e)))^(1/2)*d*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2
)*cos(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x
+e)-d*cos(f*x+e)-c+d)))/sin(f*x+e)^4/(1/(1+cos(f*x+e)))^(3/2)/a/(c/(c-d))^(
1/2)/(c-d)/((c+d)*(c-d))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e +
 f*x))), x)
```

3.243 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=231

$$-\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,df(c-d)\sqrt{c+d}}+\frac{\sqrt{2}\,g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)}+\frac{2g^{5/2}\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{}$$

```
[Out] 2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2))/d/f/a^(1/2)+g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2
^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)
-2*c^(3/2)*g^(5/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(
g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/(c-d)/d/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3978, 3965, 208, 4023, 3808, 3802}

$$-\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,df(c-d)\sqrt{c+d}}+\frac{\sqrt{2}\,g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)}+\frac{2g^{5/2}\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]])])/(Sqrt[a]*d*f) + (Sqrt[2]*g^(5/2)*ArcTanh[(Sqrt[a]*
Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*f) - (2*c^(3/2)*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt
[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*d*Sqrt[c + d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3802

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/f, Subst[Int[1/(b - d*x^2), x], x, (b*C
ot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[
```

```
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3965

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b*g
)/f, Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3978

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[(c^2
*g^2)/(d*(b*c - a*d)), Int[(Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/
(c + d*Csc[e + f*x]), x], x] + Dist[g^2/(d*(b*c - a*d)), Int[(Sqrt[g*Csc[e
+ f*x]]*(a*c + (b*c - a*d)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\sec(e+fx)}\,(ac+(ac-ad)\sec(e+fx))}{\sqrt{a+a\sec(e+fx)}}\,dx}{a(c-d)d} - \frac{\left(c^2g^2\right)\int \frac{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\,\mathrm{se}}}{c+d\sec(e+fx)}}{a(c-d)d}$$

$$= \frac{g^2\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{g^2\int \sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}\,d}{ad}$$

$$= -\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)d\sqrt{c+d}\,f} - \frac{\left(2g^3\right)\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,df} + \frac{\sqrt{2}\,g^{5/2}\tanh^{-1}\left(\frac{}{\sqrt{2}\,\sqrt{g\,\mathrm{se}}}\right.}{\sqrt{a}\,(c-}$$

Mathematica [A] time = 0.38, size = 155, normalized size = 0.67

$$\frac{2g^2\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{g\sec(e+fx)}\left(\sqrt{2}\left((c-d)\sqrt{c+d}\,\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sin\left(\frac{1}{2}(e+\right.}{\sqrt{c+d}}\right.\right.\right.}{df(c-d)\sqrt{c+d}\,\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g^2*(d*Sqrt[c + d]*ArcTanh[Sin[(e + f*x)/2]] + Sqrt[2]*((c - d)*Sqrt[c +
 d]*ArcTanh[Sqrt[2]*Sin[(e + f*x)/2]] - c^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Si
n[(e + f*x)/2])/Sqrt[c + d]]))*Cos[(e + f*x)/2]*Sqrt[g*Sec[e + f*x]])/((c -
 d)*d*Sqrt[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 89.93, size = 1597, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*d*g^2*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*co
```

```
s(f*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)) + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d
)*g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*
cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2
+ 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2
+ (2*c*d + d^2)*cos(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(f*x + e)^
3 - 4*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g
)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(sqrt(2)*d*g^2*
sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f*x + e)^2 - 2*g*co
s(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*c*sqrt(-c*g/(a
*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(
-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
e))/(c*g*sin(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 - 4*(
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(
f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(2*sqrt(2)*d*g^2*sqrt(
-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - 2*(c - d)*g^2*sqrt(-g/a)
*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(f*x + e) - 2*g))
 + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*
g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*co
s(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 +
8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 +
(2*c*d + d^2)*cos(f*x + e))))/((c*d - d^2)*f), -(sqrt(2)*d*g^2*sqrt(-g/a)*a
rctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - (c - d)*g^2*sqrt(-g/a)*arctan(2
*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(f*x + e) - 2*g)) + c*sqrt
(-c*g/(a*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e
))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/co
s(f*x + e))/(c*g*sin(f*x + e))))/((c*d - d^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
```

```
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
```

```
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
```

```
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
```

```
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
```

```
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
```

```
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
```

/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Warning, integration of abs or sign assumes constant sign by i
ntervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)
))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos

```
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*p
i/t_nostep/2)Evaluation time: 1.57index.cc index_m i_lex_is_greater Error:
Bad Argument Value
```

maple [B] time = 2.20, size = 725, normalized size = 3.14

$$\frac{2\left(\frac{g}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\,\left(\cos^{3}\left(fx+e\right)\right)\left(-1+\cos\left(fx+e\right)\right)^{3}\left(\sqrt{2}\,\sqrt{\frac{c}{c-d}}\,\sqrt{\left(c+d\right)\left(c-d\right)}\,\operatorname{arcsinh}\left(\frac{-1+\cos(f}{\sin(fx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(g/cos(f*x+e))^(5/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^3*(
-1+cos(f*x+e))^3*(2^(1/2)*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*arcsinh((-1+c
```

```
os(f*x+e))/sin(f*x+e))*d-(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(1/2*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))*c+(c/(c-d))^(1/2)*((c+d)*
(c-d))^(1/2)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e))
)*d-(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2
)*(-cos(f*x+e)-1+sin(f*x+e)))*c+(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*arctanh
(1/2*(1/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e)))*d-c^2*ln(2*(2*(c/
(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*c*sin(f*x+e)-2*(c/(c-d))^(1/2)*(1/(1+
cos(f*x+e)))^(1/2)*d*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+((c+d)*(c-d))^(1/
2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*
x+e)+d*cos(f*x+e)+c-d))+c^2*ln(-2*(2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/
2)*c*sin(f*x+e)-2*(c/(c-d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+c*s
in(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)+((c+d)*(c-d))^(1/2))/
(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)))/sin(f*x+e)
^6/(1/(1+cos(f*x+e)))^(5/2)/a/(c/(c-d))^(1/2)/(-c+d+((c+d)*(c-d))^(1/2))/(c
-d+((c+d)*(c-d))^(1/2))/((c+d)*(c-d))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.244 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=250

$$\frac{\left(35acd+12bc^2+16bd^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{60f}+\frac{d\left(130ac^2d+45ad^3+24bc^3+116bcd^2\right)\tan(e+fx)\,\mathrm{s}}{120f}$$

```
[Out] 1/8*(8*a*c^4+24*a*c^2*d^2+3*a*d^4+16*b*c^3*d+12*b*c*d^3)*arctanh(sin(f*x+e)
)/f+1/30*(95*a*c^3*d+80*a*c*d^3+12*b*c^4+112*b*c^2*d^2+16*b*d^4)*tan(f*x+e)
/f+1/120*d*(130*a*c^2*d+45*a*d^3+24*b*c^3+116*b*c*d^2)*sec(f*x+e)*tan(f*x+e
)/f+1/60*(35*a*c*d+12*b*c^2+16*b*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*
(5*a*d+4*b*c)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f+1/5*b*(c+d*sec(f*x+e))^4*tan(
f*x+e)/f
```

Rubi [A] time = 0.50, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(95ac^3d+80acd^3+112bc^2d^2+12bc^4+16bd^4\right)\tan(e+fx)}{30f}+\frac{\left(24ac^2d^2+8ac^4+3ad^4+16bc^3d+12bcd^3\right)\tanh^{-1}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*ArcTanh[Sin[e
 + f*x]])/(8*f) + ((12*b*c^4 + 95*a*c^3*d + 112*b*c^2*d^2 + 80*a*c*d^3 + 16
*b*d^4)*Tan[e + f*x])/(30*f) + (d*(24*b*c^3 + 130*a*c^2*d + 116*b*c*d^2 + 4
5*a*d^3)*Sec[e + f*x]*Tan[e + f*x])/(120*f) + ((12*b*c^2 + 35*a*c*d + 16*b*
d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + ((4*b*c + 5*a*d)*(c + d*
Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + (b*(c + d*Sec[e + f*x])^4*Tan[e + f*
x])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} + \frac{1}{5}\int \sec(e+fx)($$
$$= \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+}{}$$
$$= \frac{\left(12bc^2+35acd+16bd^2\right)(c+d\sec(e+fx))^2\tan(e+}{60f}$$
$$= \frac{d\left(24bc^3+130ac^2d+116bcd^2+45ad^3\right)\sec(e+fx)\,t}{120f}$$
$$= \frac{d\left(24bc^3+130ac^2d+116bcd^2+45ad^3\right)\sec(e+fx)\,t}{120f}$$
$$= \frac{\left(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4\right)\tanh^{-1}(s}{8f}$$
$$= \frac{\left(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4\right)\tanh^{-1}(s}{8f}$$

Mathematica [A] time = 4.48, size = 201, normalized size = 0.80

$$\frac{15\left(a\left(8c^4+24c^2d^2+3d^4\right)+4bcd\left(4c^2+3d^2\right)\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(8\left(10d^2\left(2acd+b\left(3c^2+d^2\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (15*(4*b*c*d*(4*c^2 + 3*d^2) + a*(8*c^4 + 24*c^2*d^2 + 3*d^4))*ArcTanh[Sin[
e + f*x]] + Tan[e + f*x]*(15*d*(3*a*d*(8*c^2 + d^2) + 4*b*(4*c^3 + 3*c*d^2)
)*Sec[e + f*x] + 30*d^3*(4*b*c + a*d)*Sec[e + f*x]^3 + 8*(15*(4*a*c*d*(c^2
+ d^2) + b*(c^4 + 6*c^2*d^2 + d^4)) + 10*d^2*(2*a*c*d + b*(3*c^2 + d^2))*Ta
n[e + f*x]^2 + 3*b*d^4*Tan[e + f*x]^4)))/(120*f)
```

fricas [A] time = 0.47, size = 281, normalized size = 1.12

$$\frac{15\left(8\,ac^4+16\,bc^3d+24\,ac^2d^2+12\,bcd^3+3\,ad^4\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(8\,ac^4+16\,bc^3d+24\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2
+ 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*b*d^4
 + 8*(15*b*c^4 + 60*a*c^3*d + 60*b*c^2*d^2 + 40*a*c*d^3 + 8*b*d^4)*cos(f*x
+ e)^4 + 15*(16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*b*c^2*d^2 + 10*a*c*d^3 + 2*b*d^4)*cos(f*x + e)^2 + 30*(4*b*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(8*a*c^4+24*a*c^2*d^2+3*a*d^4+16*c^3*b*d+12*c*b*d^3)/16*ln(ab
s(tan((f*x+exp(1))/2)-1))+(8*a*c^4+24*a*c^2*d^2+3*a*d^4+16*c^3*b*d+12*c*b*d
^3)/16*ln(abs(tan((f*x+exp(1))/2)+1))-(480*tan((f*x+exp(1))/2)^9*a*c^3*d-36
0*tan((f*x+exp(1))/2)^9*a*c^2*d^2+480*tan((f*x+exp(1))/2)^9*a*c*d^3-75*tan(
(f*x+exp(1))/2)^9*a*d^4+120*tan((f*x+exp(1))/2)^9*c^4*b-240*tan((f*x+exp(1)
)/2)^9*c^3*b*d+720*tan((f*x+exp(1))/2)^9*c^2*b*d^2-300*tan((f*x+exp(1))/2)^
9*c*b*d^3+120*tan((f*x+exp(1))/2)^9*b*d^4-1920*tan((f*x+exp(1))/2)^7*a*c^3*
d+720*tan((f*x+exp(1))/2)^7*a*c^2*d^2-1280*tan((f*x+exp(1))/2)^7*a*c*d^3+30
*tan((f*x+exp(1))/2)^7*a*d^4-480*tan((f*x+exp(1))/2)^7*c^4*b+480*tan((f*x+e
xp(1))/2)^7*c^3*b*d-1920*tan((f*x+exp(1))/2)^7*c^2*b*d^2+120*tan((f*x+exp(1
))/2)^7*c*b*d^3-160*tan((f*x+exp(1))/2)^7*b*d^4+2880*tan((f*x+exp(1))/2)^5*
a*c^3*d+1600*tan((f*x+exp(1))/2)^5*a*c*d^3+720*tan((f*x+exp(1))/2)^5*c^4*b+
2400*tan((f*x+exp(1))/2)^5*c^2*b*d^2+464*tan((f*x+exp(1))/2)^5*b*d^4-1920*t
an((f*x+exp(1))/2)^3*a*c^3*d-720*tan((f*x+exp(1))/2)^3*a*c^2*d^2-1280*tan((
f*x+exp(1))/2)^3*a*c*d^3-30*tan((f*x+exp(1))/2)^3*a*d^4-480*tan((f*x+exp(1)
)/2)^3*c^4*b-480*tan((f*x+exp(1))/2)^3*c^3*b*d-1920*tan((f*x+exp(1))/2)^3*c
^2*b*d^2-120*tan((f*x+exp(1))/2)^3*c*b*d^3-160*tan((f*x+exp(1))/2)^3*b*d^4+
480*tan((f*x+exp(1))/2)*a*c^3*d+360*tan((f*x+exp(1))/2)*a*c^2*d^2+480*tan((
f*x+exp(1))/2)*a*c*d^3+75*tan((f*x+exp(1))/2)*a*d^4+120*tan((f*x+exp(1))/2)
*c^4*b+240*tan((f*x+exp(1))/2)*c^3*b*d+720*tan((f*x+exp(1))/2)*c^2*b*d^2+30
0*tan((f*x+exp(1))/2)*c*b*d^3+120*tan((f*x+exp(1))/2)*b*d^4)*1/120/(tan((f*
x+exp(1))/2)^2-1)^5)
```

maple [A] time = 1.69, size = 431, normalized size = 1.72

$$\frac{a\,c^{4}\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{4a\,c^{3}d\tan\left(fx+e\right)}{f}+\frac{3a\,c^{2}d^{2}\sec\left(fx+e\right)\tan\left(fx+e\right)}{f}+\frac{3a\,c^{2}d^{2}\ln\left(\sec\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x)
```

```
[Out] 1/f*a*c^4*ln(sec(f*x+e)+tan(f*x+e))+4/f*a*c^3*d*tan(f*x+e)+3/f*a*c^2*d^2*se
c(f*x+e)*tan(f*x+e)+3/f*a*c^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+8/3/f*a*c*d^3*t
an(f*x+e)+4/3/f*a*c*d^3*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a*d^4*tan(f*x+e)*sec(
f*x+e)^3+3/8/f*a*d^4*sec(f*x+e)*tan(f*x+e)+3/8/f*a*d^4*ln(sec(f*x+e)+tan(f*
x+e))+1/f*b*c^4*tan(f*x+e)+2/f*b*c^3*d*sec(f*x+e)*tan(f*x+e)+2/f*b*c^3*d*ln
(sec(f*x+e)+tan(f*x+e))+4/f*b*c^2*d^2*tan(f*x+e)+2/f*b*c^2*d^2*tan(f*x+e)*s
ec(f*x+e)^2+1/f*b*c*d^3*tan(f*x+e)*sec(f*x+e)^3+3/2/f*b*c*d^3*sec(f*x+e)*ta
n(f*x+e)+3/2/f*b*c*d^3*ln(sec(f*x+e)+tan(f*x+e))+8/15/f*b*d^4*tan(f*x+e)+1/
5/f*b*d^4*tan(f*x+e)*sec(f*x+e)^4+4/15/f*b*d^4*tan(f*x+e)*sec(f*x+e)^2
```

maxima [A] time = 0.92, size = 379, normalized size = 1.52

$$\frac{480\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)bc^{2}d^{2}+320\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)acd^{3}+16\left(3\,\tan\left(fx+e\right)^{5}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*b*d^4 - 60*b*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*b*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*b*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

mupad [B] time = 5.55, size = 555, normalized size = 2.22

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a\,c^4+2\,b\,c^3\,d+3\,a\,c^2\,d^2+\frac{3\,b\,c\,d^3}{2}+\frac{3\,a\,d^4}{8}\right)}{4\,a\,c^4+8\,b\,c^3\,d+12\,a\,c^2\,d^2+6\,b\,c\,d^3+\frac{3\,a\,d^4}{2}}\right)\left(2\,a\,c^4+4\,b\,c^3\,d+6\,a\,c^2\,d^2+3\,b\,c\,d^3+\frac{3\,a\,d^4}{4}\right)}{f}-\frac{\left(2\,b\,c^4-\frac{5\,a\,d^4}{4}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^4 + (3*a*d^4)/8 + 3*a*c^2*d^2 + (3*b*c*d^
3)/2 + 2*b*c^3*d))/(4*a*c^4 + (3*a*d^4)/2 + 12*a*c^2*d^2 + 6*b*c*d^3 + 8*b*
c^3*d))*(2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*b*c*d^3 + 4*b*c^3*d))/f -
(tan(e/2 + (f*x)/2)^5*(12*b*c^4 + (116*b*d^4)/15 + 40*b*c^2*d^2 + (80*a*c*d
^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*((5*a*d^4)/4 + 2*b*c^4 + 2*b*d^4 +
 6*a*c^2*d^2 + 12*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d + 5*b*c*d^3 + 4*b*c^3*d
) + tan(e/2 + (f*x)/2)^9*(2*b*c^4 - (5*a*d^4)/4 + 2*b*d^4 - 6*a*c^2*d^2 + 1
2*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d - 5*b*c*d^3 - 4*b*c^3*d) - tan(e/2 + (f
*x)/2)^3*((a*d^4)/2 + 8*b*c^4 + (8*b*d^4)/3 + 12*a*c^2*d^2 + 32*b*c^2*d^2 +
 (64*a*c*d^3)/3 + 32*a*c^3*d + 2*b*c*d^3 + 8*b*c^3*d) - tan(e/2 + (f*x)/2)^
7*(8*b*c^4 - (a*d^4)/2 + (8*b*d^4)/3 - 12*a*c^2*d^2 + 32*b*c^2*d^2 + (64*a*
c*d^3)/3 + 32*a*c^3*d - 2*b*c*d^3 - 8*b*c^3*d))/(f*(5*tan(e/2 + (f*x)/2)^2
- 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^4\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**4*sec(e + f*x), x)
```

3.245 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=180

$$\frac{d\left(20acd+6bc^2+9bd^2\right)\tan(e+fx)\sec(e+fx)}{24f}+\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\tan(e+fx)}{6f}+\frac{\left(8ac^3+12acd\right.}{}$$

```
[Out] 1/8*(8*a*c^3+12*a*c*d^2+12*b*c^2*d+3*b*d^3)*arctanh(sin(f*x+e))/f+1/6*(4*a*
d*(4*c^2+d^2)+3*b*(c^3+4*c*d^2))*tan(f*x+e)/f+1/24*d*(20*a*c*d+6*b*c^2+9*b*
d^2)*sec(f*x+e)*tan(f*x+e)/f+1/12*(4*a*d+3*b*c)*(c+d*sec(f*x+e))^2*tan(f*x+
e)/f+1/4*b*(c+d*sec(f*x+e))^3*tan(f*x+e)/f
```

Rubi [A] time = 0.36, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\tan(e+fx)}{6f}+\frac{\left(8ac^3+12acd^2+12bc^2d+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{d\left(20acd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*ArcTanh[Sin[e + f*x]])/(8*f)
 + ((4*a*d*(4*c^2 + d^2) + 3*b*(c^3 + 4*c*d^2))*Tan[e + f*x])/(6*f) + (d*(6
*b*c^2 + 20*a*c*d + 9*b*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + ((3*b*c +
4*a*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + (b*(c + d*Sec[e + f*x]
)^3*Tan[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\int \sec(e+fx)(c$$

$$= \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{b(c+d}{}$$

$$= \frac{d\left(6bc^2+20acd+9bd^2\right)\sec(e+fx)\tan(e+fx)}{24f}+\frac{(3b}{}$$

$$= \frac{d\left(6bc^2+20acd+9bd^2\right)\sec(e+fx)\tan(e+fx)}{24f}+\frac{(3b}{}$$

$$= \frac{\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f}$$

$$= \frac{\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f}$$

Mathematica [A] time = 1.11, size = 143, normalized size = 0.79

$$\frac{3\left(4a\left(2c^3+3cd^2\right)+3bd\left(4c^2+d^2\right)\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(9d\left(4acd+b\left(4c^2+d^2\right)\right)\sec(e+fx)+\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*(3*b*d*(4*c^2 + d^2) + 4*a*(2*c^3 + 3*c*d^2))*ArcTanh[Sin[e + f*x]] + Ta
n[e + f*x]*(9*d*(4*a*c*d + b*(4*c^2 + d^2))*Sec[e + f*x] + 6*b*d^3*Sec[e +
f*x]^3 + 8*(3*a*d*(3*c^2 + d^2) + 3*b*(c^3 + 3*c*d^2) + d^2*(3*b*c + a*d)*T
an[e + f*x]^2)))/(24*f)
```

fricas [A] time = 0.46, size = 211, normalized size = 1.17

$$3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*b*d^3 + 8*(3*b*c^3 + 9*a*c^2*d + 6*b*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*b*c^2*d + 4*a*c*d^2 + b*d^3)*cos(f*x +
 e)^2 + 8*(3*b*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-8*a*c^3-12*a*c*d^2-12*c^2*b*d-3*b*d^3)/16*ln(abs(tan((f*x+ex
p(1))/2)-1))-(-8*a*c^3-12*a*c*d^2-12*c^2*b*d-3*b*d^3)/16*ln(abs(tan((f*x+ex
p(1))/2)+1))+(-72*tan((f*x+exp(1))/2)^7*a*c^2*d+36*tan((f*x+exp(1))/2)^7*a*
```

```
c*d^2-24*tan((f*x+exp(1))/2)^7*a*d^3-24*tan((f*x+exp(1))/2)^7*c^3*b+36*tan(
(f*x+exp(1))/2)^7*c^2*b*d-72*tan((f*x+exp(1))/2)^7*c*b*d^2+15*tan((f*x+exp(
1))/2)^7*b*d^3+216*tan((f*x+exp(1))/2)^5*a*c^2*d-36*tan((f*x+exp(1))/2)^5*a
*c*d^2+40*tan((f*x+exp(1))/2)^5*a*d^3+72*tan((f*x+exp(1))/2)^5*c^3*b-36*tan
((f*x+exp(1))/2)^5*c^2*b*d+120*tan((f*x+exp(1))/2)^5*c*b*d^2+9*tan((f*x+exp
(1))/2)^5*b*d^3-216*tan((f*x+exp(1))/2)^3*a*c^2*d-36*tan((f*x+exp(1))/2)^3*
a*c*d^2-40*tan((f*x+exp(1))/2)^3*a*d^3-72*tan((f*x+exp(1))/2)^3*c^3*b-36*ta
n((f*x+exp(1))/2)^3*c^2*b*d-120*tan((f*x+exp(1))/2)^3*c*b*d^2+9*tan((f*x+ex
p(1))/2)^3*b*d^3+72*tan((f*x+exp(1))/2)*a*c^2*d+36*tan((f*x+exp(1))/2)*a*c*
d^2+24*tan((f*x+exp(1))/2)*a*d^3+24*tan((f*x+exp(1))/2)*c^3*b+36*tan((f*x+e
xp(1))/2)*c^2*b*d+72*tan((f*x+exp(1))/2)*c*b*d^2+15*tan((f*x+exp(1))/2)*b*d
^3)*1/24/(tan((f*x+exp(1))/2)^2-1)^4)
```

maple [A] time = 1.37, size = 290, normalized size = 1.61

$$\frac{a c^3 \ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{3a c^2 d \tan\left(fx+e\right)}{f}+\frac{3ac d^2 \sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{3ac d^2 \ln\left(\sec\left(fx+\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*a*c^3*ln(sec(f*x+e)+tan(f*x+e))+3/f*a*c^2*d*tan(f*x+e)+3/2/f*a*c*d^2*se
c(f*x+e)*tan(f*x+e)+3/2/f*a*c*d^2*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*a*d^3*tan
(f*x+e)+1/3/f*a*d^3*tan(f*x+e)*sec(f*x+e)^2+1/f*c^3*b*tan(f*x+e)+3/2/f*b*c^
2*d*sec(f*x+e)*tan(f*x+e)+3/2/f*b*c^2*d*ln(sec(f*x+e)+tan(f*x+e))+2/f*c*d^2
*b*tan(f*x+e)+1/f*c*d^2*b*tan(f*x+e)*sec(f*x+e)^2+1/4/f*b*d^3*tan(f*x+e)*se
c(f*x+e)^3+3/8/f*b*d^3*sec(f*x+e)*tan(f*x+e)+3/8/f*b*d^3*ln(sec(f*x+e)+tan(
f*x+e))
```

maxima [A] time = 0.34, size = 266, normalized size = 1.48

$$\frac{48\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)bcd^2+16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ad^3-3\,bd^3\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right.}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*b*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*b*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
```

```
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*b*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

mupad [B] time = 5.49, size = 395, normalized size = 2.19

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a\,c^3+\frac{3\,b\,c^2\,d}{2}+\frac{3\,a\,c\,d^2}{2}+\frac{3\,b\,d^3}{8}\right)}{4\,a\,c^3+6\,b\,c^2\,d+6\,a\,c\,d^2+\frac{3\,b\,d^3}{2}}\right)\left(2\,a\,c^3+3\,b\,c^2\,d+3\,a\,c\,d^2+\frac{3\,b\,d^3}{4}\right)}{f}-\frac{\left(2\,a\,d^3+2\,b\,c^3-\frac{5\,b\,d^3}{4}-3\,a\,c\,d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^3 + (3*b*d^3)/8 + (3*a*c*d^2)/2 + (3*b*c^
2*d)/2))/(4*a*c^3 + (3*b*d^3)/2 + 6*a*c*d^2 + 6*b*c^2*d))*(2*a*c^3 + (3*b*d
^3)/4 + 3*a*c*d^2 + 3*b*c^2*d))/f - (tan(e/2 + (f*x)/2)^7*(2*a*d^3 + 2*b*c^
3 - (5*b*d^3)/4 - 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*d^2 - 3*b*c^2*d) + tan(e/2
+ (f*x)/2)^3*((10*a*d^3)/3 + 6*b*c^3 - (3*b*d^3)/4 + 3*a*c*d^2 + 18*a*c^2*d
 + 10*b*c*d^2 + 3*b*c^2*d) - tan(e/2 + (f*x)/2)^5*((10*a*d^3)/3 + 6*b*c^3 +
 (3*b*d^3)/4 - 3*a*c*d^2 + 18*a*c^2*d + 10*b*c*d^2 - 3*b*c^2*d) - tan(e/2 +
 (f*x)/2)*(2*a*d^3 + 2*b*c^3 + (5*b*d^3)/4 + 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*
d^2 + 3*b*c^2*d))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*t
an(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^{3}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**3*sec(e + f*x), x)
```

3.246 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=115

$$\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\tan(e+fx)}{3f}+\frac{\left(a\left(2c^2+d^2\right)+2bcd\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(3ad+2bc)\tan(e+fx)\sec(e+$$

```
[Out] 1/2*(2*b*c*d+a*(2*c^2+d^2))*arctanh(sin(f*x+e))/f+2/3*(3*a*c*d+b*(c^2+d^2))
*tan(f*x+e)/f+1/6*d*(3*a*d+2*b*c)*sec(f*x+e)*tan(f*x+e)/f+1/3*b*(c+d*sec(f*
x+e))^2*tan(f*x+e)/f
```

Rubi [A] time = 0.19, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\tan(e+fx)}{3f}+\frac{\left(a\left(2c^2+d^2\right)+2bcd\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(3ad+2bc)\tan(e+fx)\sec(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]])/(2*f) + (2*(3*a*c*d + b
*(c^2 + d^2))*Tan[e + f*x])/(3*f) + (d*(2*b*c + 3*a*d)*Sec[e + f*x]*Tan[e +
 f*x])/(6*f) + (b*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f} + \frac{1}{3}\int \sec(e+fx)($$

$$= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f} + \frac{b(c+d\sec(e}{}$$

$$= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f} + \frac{b(c+d\sec(e}{}$$

$$= \frac{\left(2bcd+a\left(2c^2+d^2\right)\right)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{d(2bc+}{}$$

$$= \frac{\left(2bcd+a\left(2c^2+d^2\right)\right)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{2\left(3acd\right.}{}$$

Mathematica [A] time = 0.61, size = 88, normalized size = 0.77

$$\frac{3\left(a\left(2c^2+d^2\right)+2bcd\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(3d(ad+2bc)\sec(e+fx)+12acd+6b\left(c^2+d^2\right)+2\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (3*(2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(12*a*c
*d + 6*b*(c^2 + d^2) + 3*d*(2*b*c + a*d)*Sec[e + f*x] + 2*b*d^2*Tan[e + f*x
]^2))/(6*f)
```

fricas [A] time = 0.47, size = 150, normalized size = 1.30

$$\frac{3\left(2\,ac^2+2\,bcd+ad^2\right)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^2+2\,bcd+ad^2\right)\cos\left(fx+e\right)^3\log\left(-\sin\left(fx\right.\right.}{12\,f\cos\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
b*d^2 + 2*(3*b*c^2 + 6*a*c*d + 2*b*d^2)*cos(f*x + e)^2 + 3*(2*b*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-(2*a*c^2+a*d^2+2*c*b*d)/4*ln(abs(tan((f*x+exp(1))/2)-1))+(2*a
*c^2+a*d^2+2*c*b*d)/4*ln(abs(tan((f*x+exp(1))/2)+1))+(-12*tan((f*x+exp(1))/
2)^5*a*c*d+3*tan((f*x+exp(1))/2)^5*a*d^2-6*tan((f*x+exp(1))/2)^5*c^2*b+6*ta
n((f*x+exp(1))/2)^5*c*b*d-6*tan((f*x+exp(1))/2)^5*b*d^2+24*tan((f*x+exp(1))
/2)^3*a*c*d+12*tan((f*x+exp(1))/2)^3*c^2*b+4*tan((f*x+exp(1))/2)^3*b*d^2-12
*tan((f*x+exp(1))/2)*a*c*d-3*tan((f*x+exp(1))/2)*a*d^2-6*tan((f*x+exp(1))/2
)*c^2*b-6*tan((f*x+exp(1))/2)*c*b*d-6*tan((f*x+exp(1))/2)*b*d^2)*1/6/(tan((
f*x+exp(1))/2)^2-1)^3)
```

maple [A] time = 1.12, size = 174, normalized size = 1.51

$$\frac{a\,c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{2acd\tan\left(fx+e\right)}{f}+\frac{a\,d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{a\,d^2\ln\left(\sec\left(fx+e\right)+\mathrm{t}\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x)

[Out] 1/f*a*c^2*ln(sec(f*x+e)+tan(f*x+e))+2/f*a*c*d*tan(f*x+e)+1/2/f*a*d^2*sec(f*x+e)*tan(f*x+e)+1/2/f*a*d^2*ln(sec(f*x+e)+tan(f*x+e))+1/f*c^2*b*tan(f*x+e)+1/f*b*c*d*sec(f*x+e)*tan(f*x+e)+1/f*b*c*d*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*d^2*b*tan(f*x+e)+1/3/f*d^2*b*tan(f*x+e)*sec(f*x+e)^2

maxima [A] time = 0.33, size = 165, normalized size = 1.43

$$\frac{4\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)bd^{2}-6\,bcd\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*d^2 - 6*b*c*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*b*c^2*tan(f*x + e) + 24*a*c*d*tan(f*x + e))/f

mupad [B] time = 5.21, size = 227, normalized size = 1.97

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(ac^2+bcd+\frac{ad^2}{2}\right)}{4ac^2+4bcd+2ad^2}\right)\left(2ac^2+2bcd+ad^2\right)}{f}-\frac{\left(2bc^2-ad^2+2bd^2+4acd-2bcd\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)

[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^2 + (a*d^2)/2 + b*c*d))/(4*a*c^2 + 2*a*d^2 + 4*b*c*d))*(2*a*c^2 + a*d^2 + 2*b*c*d))/f - (tan(e/2 + (f*x)/2)*(a*d^2 + 2*b*c^2 + 2*b*d^2 + 4*a*c*d + 2*b*c*d) - tan(e/2 + (f*x)/2)^3*(4*b*c^2 + (4*b*d^2)/3 + 8*a*c*d) + tan(e/2 + (f*x)/2)^5*(2*b*c^2 - a*d^2 + 2*b*d^2 + 4*a*c*d - 2*b*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^{2}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2*sec(e + f*x), x)
```

3.247 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=61

$$\frac{(ad+bc)\tan(e+fx)}{f}+\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] 1/2*(2*a*c+b*d)*arctanh(sin(f*x+e))/f+(a*d+b*c)*tan(f*x+e)/f+1/2*b*d*sec(f*
x+e)*tan(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{(ad+bc)\tan(e+fx)}{f}+\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*ArcTanh[Sin[e + f*x]])/(2*f) + ((b*c + a*d)*Tan[e + f*x])/f
+ (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[n,
-1]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{bd\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\int \sec(e+fx)(2ac+bd$$

$$=\frac{bd\sec(e+fx)\tan(e+fx)}{2f}+(bc+ad)\int \sec^2(e+fx)\,d$$

$$=\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\sec(e+fx)\tan(e}{2f}$$

$$=\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{(bc+ad)\tan(e+fx)}{f}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 1.23

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{f}+\frac{ad\tan(e+fx)}{f}+\frac{bc\tan(e+fx)}{f}+\frac{bd\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (b*d*ArcTanh[Sin[e + f*x]])/(2*f) + (b*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

fricas [A] time = 0.46, size = 96, normalized size = 1.57

$$\frac{(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(bd+2\,(b}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + b*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + b*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(b*d + 2*(b*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-2*a*c-d*b)/4*ln(abs(tan((f*x+exp(1))/2)-1))-(-2*a*c-d*b)/4*l
n(abs(tan((f*x+exp(1))/2)+1))-(2*tan((f*x+exp(1))/2)^3*a*d+2*tan((f*x+exp(1
))/2)^3*c*b-tan((f*x+exp(1))/2)^3*d*b-2*tan((f*x+exp(1))/2)*a*d-2*tan((f*x+
exp(1))/2)*c*b-tan((f*x+exp(1))/2)*d*b)*1/2/(tan((f*x+exp(1))/2)^2-1)^2)
```

maple [A] time = 0.92, size = 86, normalized size = 1.41

$$\frac{ca\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{da\tan\left(fx+e\right)}{f}+\frac{cb\tan\left(fx+e\right)}{f}+\frac{bd\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{bd\ln\left(\sec\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*c*a*ln(sec(f*x+e)+tan(f*x+e))+1/f*d*a*tan(f*x+e)+1/f*c*b*tan(f*x+e)+1/2
*b*d*sec(f*x+e)*tan(f*x+e)/f+1/2/f*b*d*ln(sec(f*x+e)+tan(f*x+e))
```

maxima [A] time = 0.33, size = 88, normalized size = 1.44

$$-\frac{bd\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-4\,b}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(b*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*b*c*tan(f
*x + e) - 4*a*d*tan(f*x + e))/f
```

mupad [B] time = 2.79, size = 104, normalized size = 1.70

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(2\,a\,c+b\,d)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,a\,d+2\,b\,c+b\,d)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,a\,d+2\,b\,c-b\,d)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (atanh(tan(e/2 + (f*x)/2))*(2*a*c + b*d))/f + (tan(e/2 + (f*x)/2)*(2*a*d +
2*b*c + b*d) - tan(e/2 + (f*x)/2)^3*(2*a*d + 2*b*c - b*d))/(f*(tan(e/2 + (f
*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))*sec(e + f*x), x)
```

3.248 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{b\tanh^{-1}(\sin(e+fx))}{df}-\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\,\sqrt{c+d}}$$

```
[Out] b*arctanh(sin(f*x+e))/d/f-2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*
x)/(c+d)^(1/2))/d/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{b\tanh^{-1}(\sin(e+fx))}{df}-\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\,\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/(d*f) - (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(
e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*d*Sqrt[c + d]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{b\int \sec(e+fx)\,dx}{d} + \frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d}$$

$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{(bc-ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2}$$

$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e\right.\right.}{d^2f}$$

$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}$$

Mathematica [A] time = 0.21, size = 112, normalized size = 1.47

$$\frac{\frac{2(bc-ad)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right.\right.\right.}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*(b*c - a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[
c^2 - d^2] + b*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(d*f)
```

fricas [A] time = 1.08, size = 316, normalized size = 4.16

$$\left[-\frac{(bc-ad)\sqrt{c^2-d^2}\,\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}\,(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-\left(bc^2-bd^2\right)\log\left(\right.}{2\left(c^2d-d^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*
cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^
2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (b*c^2 - b*d^2)
*log(sin(f*x + e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d -
d^3)*f), -1/2*(2*(b*c - a*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*c
os(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (b*c^2 - b*d^2)*log(sin(f*x
+ e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d - d^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-b*1/2/d*ln(abs(tan((f*x+exp(1))/2)-1))+b*1/2/d*ln(abs(tan((f*
x+exp(1))/2)+1))+(-2*a*d+2*c*b)/d*1/2/sqrt(-c^2+d^2)*(atan((c*tan((f*x+exp(
1))/2)-d*tan((f*x+exp(1))/2))/sqrt(-c^2+d^2))+pi*sign(2*c-2*d)*floor((f*x+e
xp(1))/2/pi+1/2)))
```

maple [A] time = 0.59, size = 135, normalized size = 1.78

$$\frac{2a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{f\sqrt{(c+d)\,(c-d)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)cb}{fd\sqrt{(c+d)\,(c-d)}}-\frac{b\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fd}+\frac{b\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{fd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f*a/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1
/2))-2/f/d/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d
))^(1/2))*c*b-1/f/d*b*ln(tan(1/2*e+1/2*f*x)-1)+1/f/d*b*ln(tan(1/2*e+1/2*f*x
)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.73, size = 573, normalized size = 7.54

$$\frac{a\,c^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}}-\frac{a\,d^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(c^2-d^2\right)}^{3/2}}-\frac{2\,b\,d\,\mathrm{atanh}\left(\right.}{f\,\left(c^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (a*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (a*d^2*lo
g((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*b*d*atanh(sin(e
/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (a*log((c*cos(e/2 + (f
*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e
/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) + (b*c*d*log((c*sin
(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*b*c^2*atanh(sin(e/2 + (
f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c^2 - d^2)) - (b*c^3*log((c*sin(e/2 + (f
*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e
/2 + (f*x)/2)))/(d*f*(c^2 - d^2)^(3/2)) + (b*c*log((c*cos(e/2 + (f*x)/2) +
d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x
)/2))*((c + d)*(c - d))^(1/2))/(d*f*(c^2 - d^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

3.249 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=99

$$\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}+\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}}$$

```
[Out] 2*(a*c-b*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(c-d)^(3/2)
/(c+d)^(3/2)/f+(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}+\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d
)^(3/2)*(c + d)^(3/2)*f) + ((b*c - a*d)*Tan[e + f*x])/((c^2 - d^2)*f*(c + d
*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{\int \frac{(-ac+bd)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{(ac-bd)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c^2-d^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{(ac-bd)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))} + \frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x,\right.}{d\left(c^2-d^2\right)f}$$

$$= \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))}$$

Mathematica [A] time = 0.39, size = 97, normalized size = 0.98

$$\frac{\frac{(bc-ad)\sin(e+fx)}{(c-d)(c+d)(c\cos(e+fx)+d)} - \frac{2(ac-bd)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2
 - d^2)^(3/2) + ((b*c - a*d)*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e +
f*x])))/f
```

fricas [A] time = 0.49, size = 389, normalized size = 3.93

$$\left[\frac{\left(acd - bd^2 + \left(ac^2 - bcd\right)\cos\left(fx + e\right)\right)\sqrt{c^2 - d^2}\,\log\left(\frac{2\,cd\cos\left(fx+e\right) - \left(c^2 - 2\,d^2\right)\cos\left(fx+e\right)^2 + 2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+}{c^2\cos\left(fx+e\right)^2 + 2\,cd\cos\left(fx+e\right) + d^2}\right.}{2\left(\left(c^5 - 2\,c^3d^2 + cd^4\right)f\cos\left(fx + e\right) + \left(c^4d - 2\,c^2d^3 + d^5\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2
*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos
(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(
f*x + e) + d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^
5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x + e) + (c^4*d - 2*c^2*d^3 + d^5)*f), ((a*c
*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c
^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^3 - a*c^2
*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x +
e) + (c^4*d - 2*c^2*d^3 + d^5)*f)]
```

giac [A] time = 0.66, size = 179, normalized size = 1.81

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,c-2\,d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)\left(ac-bd\right)}{\left(c^2-d^2\right)\sqrt{-c^2+d^2}}+\frac{bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)\left(c^2-d^2\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - b*d)/((c^2 -
 d^2)*sqrt(-c^2 + d^2)) + (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
```

```
*e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c^2 -
d^2)))/f
```

maple [A] time = 0.66, size = 132, normalized size = 1.33

$$\frac{\frac{2(da-cb)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{(c^2-d^2)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}+\frac{2(ca-bd)\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(2*(a*d-b*c)/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1
/2*e+1/2*f*x)^2*d-c-d)+2*(a*c-b*d)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(
tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 2.12, size = 106, normalized size = 1.07

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,(a\,c-b\,d)}{f\,{(c+d)}^{3/2}\,{(c-d)}^{3/2}}-\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(a\,d-b\,c)}{f\,(c+d)\,(c-d)\,\left((d-c)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(a*c - b*d))/(f*
(c + d)^(3/2)*(c - d)^(3/2)) - (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(c + d
)*(c - d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{\left(c+d\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**2, x)
```

3.250 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3} dx$

Optimal. Leaf size=166

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{5/2}}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}+\frac{(bc-ad)\tan(e+fx}{2f\left(c^2-d^2\right)(c+d\sec(e+}$$

```
[Out] -(3*b*c*d-a*(2*c^2+d^2))*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2)
)/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(
f*x+e))^2-1/2*(3*a*c*d-b*(c^2+2*d^2))*tan(f*x+e)/(c^2-d^2)^2/f/(c+d*sec(f*x
+e))
```

Rubi [A] time = 0.30, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{5/2}}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}+\frac{(bc-ad)\tan(e+fx}{2f\left(c^2-d^2\right)(c+d\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/((c - d)^(5/2)*(c + d)^(5/2)*f)) + ((b*c - a*d)*Tan[e + f*x])/(2*(
c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) - ((3*a*c*d - b*(c^2 + 2*d^2))*Tan[e +
 f*x])/(2*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2(ac-bd)-(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} +$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} -$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} -$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} -$$

$$= \frac{\left(2ac^2-3bcd+ad^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)\tan(e+}{2\left(c^2-d^2\right)f(c+d\sec(}$$

Mathematica [A] time = 0.98, size = 172, normalized size = 1.04

$$\frac{\frac{\left(ad\left(d^2-4c^2\right)+bc\left(2c^2+d^2\right)\right)\sin(e+fx)}{c(c-d)^2(c+d)^2(c\cos(e+fx)+d)} - \frac{2\left(a\left(2c^2+d^2\right)-3bcd\right)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} + \frac{d(ad-bc)\sin(e+fx)}{c(c-d)(c+d)(c\cos(e+fx)+d)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2]])/Sqrt[
c^2 - d^2]])/(c^2 - d^2)^(5/2) + (d*(-(b*c) + a*d)*Sin[e + f*x])/(c*(c - d)
*(c + d)*(d + c*Cos[e + f*x])^2) + ((a*d*(-4*c^2 + d^2) + b*c*(2*c^2 + d^2)
)*Sin[e + f*x])/(c*(c - d)^2*(c + d)^2*(d + c*Cos[e + f*x])))/(2*f)
```

fricas [B] time = 0.54, size = 752, normalized size = 4.53

$$\left[\frac{\left(2\,ac^{2}d^{2}-3\,bcd^{3}+ad^{4}+\left(2\,ac^{4}-3\,bc^{3}d+ac^{2}d^{2}\right)\cos\left(fx+e\right)^{2}+2\left(2\,ac^{3}d-3\,bc^{2}d^{2}+acd^{3}\right)\cos\left(fx+e\right)\right)\sqrt{c^{2}}}{4\left(\left(c^{8}-3\,c^{6}d^{2}+3\,c^{4}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*
cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(c
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e
)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3
*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d
^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 - 3*c^6*d^2 + 3*c^4*d^4 - c^2
*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*cos(f*
x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f), 1/2*((2*a*c^2*d^2 - 3*
b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a
*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt
(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^4*d -
3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^
3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 -
3*c^6*d^2 + 3*c^4*d^4 - c^2*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 +
3*c^3*d^5 - c*d^7)*f*cos(f*x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f)]
```

giac [B] time = 1.95, size = 418, normalized size = 2.52

$$\frac{\frac{\left(2\,ac^{2}-3\,bcd+ad^{2}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\left(c^{4}-2\,c^{2}d^{2}+d^{4}\right)\sqrt{-c^{2}+d^{2}}}-\frac{2\,bc^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-4\,ac^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-bc^{2}d\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
```

```
 d^2)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(-c^2 + d^2)) - (2*b*c^3*tan(1/2*f*x +
 1/2*e)^3 - 4*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 3*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + b*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a*
d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*c^3*tan(1
/2*f*x + 1/2*e) + 4*a*c^2*d*tan(1/2*f*x + 1/2*e) - b*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*a*c*d^2*tan(1/2*f*x + 1/2*e) - b*c*d^2*tan(1/2*f*x + 1/2*e) - a*d^
3*tan(1/2*f*x + 1/2*e) - 2*b*d^3*tan(1/2*f*x + 1/2*e))/((c^4 - 2*c^2*d^2 +
d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

maple [A] time = 0.59, size = 236, normalized size = 1.42

$$\frac{-\frac{2\left(-\frac{\left(4acd+a\,d^{2}-2c^{2}b-bcd-2d^{2}b\right)\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{2(c-d)\left(c^{2}+2cd+d^{2}\right)}+\frac{\left(4acd-a\,d^{2}-2c^{2}b+bcd-2d^{2}b\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2(c+d)\left(c^{2}-2cd+d^{2}\right)}\right)}{\left(\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^{2}}+\frac{\left(2a\,c^{2}+a\,d^{2}-3bcd\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{\left(c^{4}-2c^{2}d^{2}+d^{4}\right)\sqrt{(c+d)(c-d)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*(-2*(-1/2*(4*a*c*d+a*d^2-2*b*c^2-b*c*d-2*b*d^2)/(c-d)/(c^2+2*c*d+d^2)*t
an(1/2*e+1/2*f*x)^3+1/2*(4*a*c*d-a*d^2-2*b*c^2+b*c*d-2*b*d^2)/(c+d)/(c^2-2*
c*d+d^2)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d
-c-d)^2+(2*a*c^2+a*d^2-3*b*c*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arc
tanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 4.99, size = 250, normalized size = 1.51

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(a\,d^2+2\,b\,c^2+2\,b\,d^2-4\,a\,c\,d-b\,c\,d\right)}{(c+d)\left(c^2-2\,c\,d+d^2\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\left(2\,b\,c^2-a\,d^2+2\,b\,d^2-4\,a\,c\,d+b\,c\,d\right)}{(c+d)^2\,(c-d)}}{f\left(2\,c\,d-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-2\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(c^2-2\,c\,d+d^2\right)+c^2+d^2\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(2\,c-2\,d)\left(c^2-\right.}{2\,\sqrt{c+d}\,(c-d)^{5/2}}\right.}{f\,(c+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)*(a*d^2 + 2*b*c^2 + 2*b*d^2 - 4*a*c*d - b*c*d))/((c + d
)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*b*c^2 - a*d^2 + 2*b*d^2 -
 4*a*c*d + b*c*d))/((c + d)^2*(c - d)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2
*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (a
tanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*
(c - d)^(5/2)))*(2*a*c^2 + a*d^2 - 3*b*c*d))/(f*(c + d)^(5/2)*(c - d)^(5/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sec \left(e + fx\right)\right) \sec \left(e + fx\right)}{\left(c + d \sec \left(e + fx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**3, x)
```

3.251 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=237

$$\frac{\left(-5acd+2bc^2+3bd^2\right)\tan(e+fx)}{6f\left(c^2-d^2\right)^2(c+d\sec(e+fx))^2}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}+\frac{\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\tanh^{-1}\left(\sqrt{\cdots}\right.}{f(c-d)^{7/2}(c+d)^{7/2}}$$

```
[Out] (2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/
(c+d)^(1/2))/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/
f/(c+d*sec(f*x+e))^3+1/6*(-5*a*c*d+2*b*c^2+3*b*d^2)*tan(f*x+e)/(c^2-d^2)^2/
f/(c+d*sec(f*x+e))^2+1/6*(-11*a*c^2*d-4*a*d^3+2*b*c^3+13*b*c*d^2)*tan(f*x+e
)/(c^2-d^2)^3/f/(c+d*sec(f*x+e))
```

Rubi [A] time = 0.51, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{7/2}(c+d)^{7/2}}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{6f\left(c^2-d^2\right)^3(c+d\sec(e+fx))}+\frac{(-5ac\cdots}{6f\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/((c - d)^(7/2)*(c + d)^(7/2)*f) + ((b*c - a*d)*Tan[e +
 f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((2*b*c^2 - 5*a*c*d + 3*b
*d^2)*Tan[e + f*x])/(6*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x])^2) + ((2*b*c^3
- 11*a*c^2*d + 13*b*c*d^2 - 4*a*d^3)*Tan[e + f*x])/(6*(c^2 - d^2)^3*f*(c +
d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3(ac-bd)-2(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,d}{3\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} +$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} +$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} +$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} +$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} +$$

$$= \frac{\left(2ac^3-4bc^2d+3acd^2-bd^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f} + \frac{(bc-ad}{3\left(c^2-d^2\right)f(}$$

Mathematica [A] time = 1.08, size = 405, normalized size = 1.71

$$\sec^3(e+fx)(a+b\sec(e+fx))(c\cos(e+fx)+d)\left(\frac{24\left(a\left(2c^3+3cd^2\right)-bd\left(4c^2+d^2\right)\right)(c\cos(e+fx)+d)^3\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^3*(a + b*Sec[e + f*x])*((24*(-(b*d*(4*c^
2 + d^2)) + a*(2*c^3 + 3*c*d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
```

```
^2 - d^2]]*(d + c*Cos[e + f*x])^3)/Sqrt[c^2 - d^2] - 6*b*c^5*Sin[e + f*x] +
 18*a*c^4*d*Sin[e + f*x] - 18*b*c^3*d^2*Sin[e + f*x] + 39*a*c^2*d^3*Sin[e +
 f*x] - 51*b*c*d^4*Sin[e + f*x] + 18*a*d^5*Sin[e + f*x] - 12*b*c^4*d*Sin[2*
(e + f*x)] + 54*a*c^3*d^2*Sin[2*(e + f*x)] - 54*b*c^2*d^3*Sin[2*(e + f*x)]
+ 6*a*c*d^4*Sin[2*(e + f*x)] + 6*b*d^5*Sin[2*(e + f*x)] - 6*b*c^5*Sin[3*(e
+ f*x)] + 18*a*c^4*d*Sin[3*(e + f*x)] - 10*b*c^3*d^2*Sin[3*(e + f*x)] - 5*a
*c^2*d^3*Sin[3*(e + f*x)] + b*c*d^4*Sin[3*(e + f*x)] + 2*a*d^5*Sin[3*(e + f
*x)]))/(24*(-c^2 + d^2)^3*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4)
```

fricas [B] time = 0.57, size = 1238, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e
) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*s
in(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2))
 + 2*(2*b*c^5*d^2 - 11*a*c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6
+ 4*a*d^7 + (6*b*c^7 - 18*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d
^4 - 7*a*c^2*d^5 + b*c*d^6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c
^5*d^2 + 7*b*c^4*d^3 + 8*a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*
x + e))*sin(f*x + e))/((c^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)
*f*cos(f*x + e)^3 + 3*(c^10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9
)*f*cos(f*x + e)^2 + 3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^1
0)*f*cos(f*x + e) + (c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)
, 1/6*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d
^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2*b*c^5*d^2 - 11*a*
c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6 + 4*a*d^7 + (6*b*c^7 - 18
*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d^4 - 7*a*c^2*d^5 + b*c*d^
6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c^5*d^2 + 7*b*c^4*d^3 + 8*
a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*x + e))*sin(f*x + e))/((c
^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)*f*cos(f*x + e)^3 + 3*(c^
10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9)*f*cos(f*x + e)^2 + 3*(c
^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e) + (c^8*
d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)]
```

giac [B] time = 0.40, size = 726, normalized size = 3.06

$$-\frac{\frac{3\left(2\,ac^{3}-4\,bc^{2}d+3\,acd^{2}-bd^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\left(c^{6}-3\,c^{4}d^{2}+3\,c^{2}d^{4}-d^{6}\right)\sqrt{-c^{2}+d^{2}}}+\frac{6\,bc^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-18\,ac^{4}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(-c^2 +
 d^2)) + (6*b*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^4*d*tan(1/2*f*x + 1/2*e)^
5 - 6*b*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e)^5
+ 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^5
- 27*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a*c*d^4*tan(1/2*f*x + 1/2*e)^5 +
12*b*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 6*a*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b*d^
5*tan(1/2*f*x + 1/2*e)^5 - 12*b*c^5*tan(1/2*f*x + 1/2*e)^3 + 36*a*c^4*d*tan
(1/2*f*x + 1/2*e)^3 - 16*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 32*a*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 28*b*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^5*tan(1/2*
f*x + 1/2*e)^3 + 6*b*c^5*tan(1/2*f*x + 1/2*e) - 18*a*c^4*d*tan(1/2*f*x + 1/
2*e) + 6*b*c^4*d*tan(1/2*f*x + 1/2*e) - 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e) +
 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + 27*
b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2*e) + 12*b*c*d^
4*tan(1/2*f*x + 1/2*e) - 6*a*d^5*tan(1/2*f*x + 1/2*e) - 3*b*d^5*tan(1/2*f*x
 + 1/2*e))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*(c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

maple [A] time = 0.76, size = 376, normalized size = 1.59

$$\frac{-\frac{2\left(-\frac{\left(6a\,c^{2}d+3ac\,d^{2}+2a\,d^{3}-2c^{3}b-2b\,c^{2}d-6bc\,d^{2}-b\,d^{3}\right)\left(\tan^{5}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{2(c-d)\left(c^{3}+3c^{2}d+3c\,d^{2}+d^{3}\right)}+\frac{2\left(9a\,c^{2}d+a\,d^{3}-3c^{3}b-7bc\,d^{2}\right)\left(\tan^{3}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3\left(c^{2}+2cd+d^{2}\right)\left(c^{2}-2cd+d^{2}\right)}-\frac{\left(6a\,c^{2}d-3ac\,d^{2}+2a\,d^{3}-2c^{3}b+2b\,c^{2}d-6bc\,d^{2}+b\,d^{3}\right)\tan\left(\cdots\right)}{2(c+d)\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)}\cdots\right)}{\left(\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)^{3}}\cdots}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 1/f*(-2*(-1/2*(6*a*c^2*d+3*a*c*d^2+2*a*d^3-2*b*c^3-2*b*c^2*d-6*b*c*d^2-b*d^
3)/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*e+1/2*f*x)^5+2/3*(9*a*c^2*d+a*d^
```

```
3-3*b*c^3-7*b*c*d^2)/(c^2+2*c*d+d^2)/(c^2-2*c*d+d^2)*tan(1/2*e+1/2*f*x)^3-1
/2*(6*a*c^2*d-3*a*c*d^2+2*a*d^3-2*b*c^3+2*b*c^2*d-6*b*c*d^2+b*d^3)/(c+d)/(c
^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*e+1/2*f*x))/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2
*e+1/2*f*x)^2*d-c-d)^3+(2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(
c-d))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 6.39, size = 439, normalized size = 1.85

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(2\,b\,c^3-2\,a\,d^3+b\,d^3-3\,a\,c\,d^2-6\,a\,c^2\,d+6\,b\,c\,d^2+2\,b\,c^2\,d\right)}{{\left(c+d\right)}^3\,\left(c-d\right)}+\frac{4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(-3\,b\,c^3+9\,a\,c^2\,d-7\,b\,c\,d^2+a\,d^3\right)}{3\,{\left(c+d\right)}^2\,\left(c^2-2\,c\,d+d^2\right)}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d^2+3\,c^2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(2*b*c^3 - 2*a*d^3 + b*d^3 - 3*a*c*d^2 - 6*a*c^2*d +
 6*b*c*d^2 + 2*b*c^2*d))/((c + d)^3*(c - d)) + (4*tan(e/2 + (f*x)/2)^3*(a*d
^3 - 3*b*c^3 + 9*a*c^2*d - 7*b*c*d^2))/(3*(c + d)^2*(c^2 - 2*c*d + d^2)) -
(tan(e/2 + (f*x)/2)*(2*a*d^3 - 2*b*c^3 + b*d^3 - 3*a*c*d^2 + 6*a*c^2*d - 6*
b*c*d^2 + 2*b*c^2*d))/((c + d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/
2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(
3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/
2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (atanh((tan(e/2 + (f*x)/
2)*(2*c - 2*d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3))/(2*(c + d)^(1/2)*(c - d)^(7
/2)))*(2*a*c^3 - b*d^3 + 3*a*c*d^2 - 4*b*c^2*d))/(f*(c + d)^(7/2)*(c - d)^(
7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{\left(c+d\sec\left(e+fx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**4, x)
```

3.252 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=247

$$\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}+\frac{2(bc-ad)^4\tan}{b^4f\sqrt{}}$$

```
[Out] 1/2*d^3*(-a*d+4*b*c)*arctanh(sin(f*x+e))/b^2/f+d*(-a*d+2*b*c)*(a^2*d^2-2*a*
b*c*d+2*b^2*c^2)*arctanh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/
2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/b^4/f/(a-b)^(1/2)/(a+b)^(1/2)+d^4*tan(f*
x+e)/b/f+d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*tan(f*x+e)/b^3/f+1/2*d^3*(-a*d+4
*b*c)*sec(f*x+e)*tan(f*x+e)/b^2/f+1/3*d^4*tan(f*x+e)^3/b/f
```

Rubi [A] time = 0.44, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3988, 2952, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}+\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{d^3(4bc-ad)\tan}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*(4*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d*(2*b*c - a*d)*(2*b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*
d)^4*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*
Sqrt[a + b]*f) + (d^4*Tan[e + f*x])/(b*f) + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a
^2*d^2)*Tan[e + f*x])/(b^3*f) + (d^3*(4*b*c - a*d)*Sec[e + f*x]*Tan[e + f*x
])/(2*b^2*f) + (d^4*Tan[e + f*x]^3)/(3*b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^4\sec^4(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^4}{b^4(b+a\cos(e+fx))} + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\sec(e+f}{b^4}\right.$$

$$= \frac{d^4\int\sec^4(e+fx)\,dx}{b} + \frac{(bc-ad)^4\int\frac{1}{b+a\cos(e+fx)}\,dx}{b^4} + \frac{\left(d^3(4bc-ad)\right)\int \mathrm{se}}{b^2}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} + \frac{d^3(4bc-ad)\,\mathrm{s}}{}$$

$$= \frac{d^3(4bc-ad)\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\mathrm{t}}{b^4f}$$

Mathematica [B] time = 4.69, size = 580, normalized size = 2.35

$$\frac{\cos^3(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^4\left(\frac{4bd^2\left(3a^2d^2-12abcd+2b^2\left(9c^2+d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{4bd^2\left(3a^2d^2-12abcd+2b^2(9\right.}{\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{c}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^3*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((-24*(b*c - a*
d)^4*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- 6*d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2
+ d^2))*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*d*(-8*a^2*b*c*d^2 + 2*
a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2 + d^2))*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + (b^2*d^3*(-3*a*d + b*(12*c + d)))/(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (2*b^3*d^4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 + (4*b*d^2*(-12*a*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d
^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (2*b^3*d^4*S
in[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (b^2*d^3*(-3*a*d
 + b*(12*c + d)))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*b*d^2*(-12*a
*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])))/(12*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*
x]))
```

fricas [B] time = 172.78, size = 1093, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*sqrt(a^2 - b^2)*cos(f*x + e)^3*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2
)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*
a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + 3*(8*(a^2*b^3
 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*
c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(sin(f*x + e) + 1)
 - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b -
 a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(-
sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*(18*(a^2*b^3 - b^5)*c^2*d^
2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b^3 - 2*b^5)*d^4)*cos(f*x +
 e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3), 1/12*(12*(b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(
f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*
(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x +
e)^3*log(sin(f*x + e) + 1) - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b
^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)
*d^4)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*
(18*(a^2*b^3 - b^5)*c^2*d^2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b
^3 - 2*b^5)*d^4)*cos(f*x + e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a
*b^4)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-2*c^4*b^4+8*c^3*a*b^3*d-12*c^2*a^2*b^2*d^2+8*c*a^3*b*d^3-2*a
^4*d^4)*1/2/b^4/sqrt(-a^2+b^2)*(atan((a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(
1))/2))/sqrt(-a^2+b^2))+pi*sign(2*a-2*b)*floor((f*x+exp(1))/2/pi+1/2))-(8*c
```

```
^3*b^3*d-12*c^2*a*b^2*d^2+8*c*a^2*b*d^3+4*c*b^3*d^3-2*a^3*d^4-a*b^2*d^4)*1/
4/b^4*ln(abs(tan((f*x+exp(1))/2)-1))+(8*c^3*b^3*d-12*c^2*a*b^2*d^2+8*c*a^2*
b*d^3+4*c*b^3*d^3-2*a^3*d^4-a*b^2*d^4)*1/4/b^4*ln(abs(tan((f*x+exp(1))/2)+1
))+(-36*tan((f*x+exp(1))/2)^5*c^2*b^2*d^2+24*tan((f*x+exp(1))/2)^5*c*a*b*d^
3+12*tan((f*x+exp(1))/2)^5*c*b^2*d^3-6*tan((f*x+exp(1))/2)^5*a^2*d^4-3*tan(
(f*x+exp(1))/2)^5*a*b*d^4-6*tan((f*x+exp(1))/2)^5*b^2*d^4+72*tan((f*x+exp(1
))/2)^3*c^2*b^2*d^2-48*tan((f*x+exp(1))/2)^3*c*a*b*d^3+12*tan((f*x+exp(1))/
2)^3*a^2*d^4+4*tan((f*x+exp(1))/2)^3*b^2*d^4-36*tan((f*x+exp(1))/2)*c^2*b^2
*d^2+24*tan((f*x+exp(1))/2)*c*a*b*d^3-12*tan((f*x+exp(1))/2)*c*b^2*d^3-6*ta
n((f*x+exp(1))/2)*a^2*d^4+3*tan((f*x+exp(1))/2)*a*b*d^4-6*tan((f*x+exp(1))/
2)*b^2*d^4)*1/6/b^3/(tan((f*x+exp(1))/2)^2-1)^3)
```

maple [B] time = 0.90, size = 1066, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x)
```

```
[Out] 2/f/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2
))*c^4-1/3/f*d^4/b/(tan(1/2*e+1/2*f*x)-1)^3-1/f*d^4/b/(tan(1/2*e+1/2*f*x)-1
)-1/2/f*d^4/b/(tan(1/2*e+1/2*f*x)-1)^2-1/3/f*d^4/b/(tan(1/2*e+1/2*f*x)+1)^3
-1/f*d^4/b/(tan(1/2*e+1/2*f*x)+1)+1/2/f*d^4/b/(tan(1/2*e+1/2*f*x)+1)^2-8/f/
b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))
*a*c^3*d-8/f/b^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b
)*(a+b))^(1/2))*a^3*c*d^3+12/f/b^2/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/
2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^2*c^2*d^2-2/f*d^3/b/(tan(1/2*e+1/2*f*x)
+1)^2*c+1/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)+1)^2*a-1/2/f*d^4/b^2*ln(tan(1/2*e
+1/2*f*x)+1)*a+4/f*d/b*ln(tan(1/2*e+1/2*f*x)+1)*c^3+2/f*d^3/b*ln(tan(1/2*e+
1/2*f*x)+1)*c-1/f*d^4/b^3/(tan(1/2*e+1/2*f*x)+1)*a^2-1/2/f*d^4/b^2/(tan(1/2
*e+1/2*f*x)+1)*a-6/f*d^2/b/(tan(1/2*e+1/2*f*x)+1)*c^2+2/f*d^3/b/(tan(1/2*e+
1/2*f*x)+1)*c-1/f*d^4/b^4*ln(tan(1/2*e+1/2*f*x)+1)*a^3-6/f*d^2/b/(tan(1/2*e
+1/2*f*x)-1)*c^2+4/f*d^3/b^2/(tan(1/2*e+1/2*f*x)-1)*a*c-4/f*d^3/b^3*ln(tan(
1/2*e+1/2*f*x)-1)*a^2*c+6/f*d^2/b^2*ln(tan(1/2*e+1/2*f*x)-1)*a*c^2+4/f*d^3/
b^3*ln(tan(1/2*e+1/2*f*x)+1)*a^2*c-6/f*d^2/b^2*ln(tan(1/2*e+1/2*f*x)+1)*a*c
^2+4/f*d^3/b^2/(tan(1/2*e+1/2*f*x)+1)*a*c+2/f/b^4/((a-b)*(a+b))^(1/2)*arcta
nh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^4*d^4+2/f*d^3/b/(tan(1/2
*e+1/2*f*x)-1)*c-1/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)-1)^2*a+2/f*d^3/b/(tan(1/
2*e+1/2*f*x)-1)^2*c+1/f*d^4/b^4*ln(tan(1/2*e+1/2*f*x)-1)*a^3+1/2/f*d^4/b^2*
ln(tan(1/2*e+1/2*f*x)-1)*a-4/f*d/b*ln(tan(1/2*e+1/2*f*x)-1)*c^3-2/f*d^3/b*l
n(tan(1/2*e+1/2*f*x)-1)*c-1/f*d^4/b^3/(tan(1/2*e+1/2*f*x)-1)*a^2-1/2/f*d^4/
b^2/(tan(1/2*e+1/2*f*x)-1)*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 11.32, size = 9987, normalized size = 40.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] (atan((((((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16
*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*
d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^
3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b
^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2
)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*
(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a
*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (8*ta
n(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*
b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 +
 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b
^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 19
2*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 -
104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d
^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*
a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c
^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 -
376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*
b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d
^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*
b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^
2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*1i)/b^4 - (((((
8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d
+ 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*
b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3
*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2
- 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x)/2)*(8*a*b^10
- 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4
```

*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3* a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2* d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6* d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5* c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2 *b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5* d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880 *a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5* c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6 *b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3 *(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*1i)/b^4)/((16*(4*a^11*d^12 - 6*a^10*b*d^12 + 16*b^11*c^11*d - a^6*b^5*d^12 + 2*a^7*b^4*d^12 - 5*a^8*b^ 3*d^12 + 6*a^9*b^2*d^12 - 16*b^11*c^6*d^6 - 64*b^11*c^8*d^4 + 8*b^11*c^9*d^ 3 - 64*b^11*c^10*d^2 + 72*a*b^10*c^5*d^7 + 32*a*b^10*c^6*d^6 + 368*a*b^10*c ^7*d^5 + 62*a*b^10*c^8*d^4 + 440*a*b^10*c^9*d^3 - 24*a*b^10*c^10*d^2 + 12*a ^5*b^6*c*d^11 - 24*a^6*b^5*c*d^11 + 60*a^7*b^4*c*d^11 - 72*a^8*b^3*c*d^11 + 72*a^9*b^2*c*d^11 - 129*a^2*b^9*c^4*d^8 - 144*a^2*b^9*c^5*d^7 - 936*a^2*b^ 9*c^6*d^6 - 496*a^2*b^9*c^7*d^5 - 1422*a^2*b^9*c^8*d^4 - 240*a^2*b^9*c^9*d^ 3 + 88*a^2*b^9*c^10*d^2 + 116*a^3*b^8*c^3*d^9 + 258*a^3*b^8*c^4*d^8 + 1384* a^3*b^8*c^5*d^7 + 1336*a^3*b^8*c^6*d^6 + 2848*a^3*b^8*c^7*d^5 + 1148*a^3*b^ 8*c^8*d^4 - 208*a^3*b^8*c^9*d^3 - 54*a^4*b^7*c^2*d^10 - 232*a^4*b^7*c^3*d^9 - 1301*a^4*b^7*c^4*d^8 - 1952*a^4*b^7*c^5*d^7 - 3888*a^4*b^7*c^6*d^6 - 249 6*a^4*b^7*c^7*d^5 + 276*a^4*b^7*c^8*d^4 + 108*a^5*b^6*c^2*d^10 + 788*a^5*b^ 6*c^3*d^9 + 1756*a^5*b^6*c^4*d^8 + 3744*a^5*b^6*c^5*d^7 + 3360*a^5*b^6*c^6* d^6 - 224*a^5*b^6*c^7*d^5 - 294*a^6*b^5*c^2*d^10 - 1008*a^6*b^5*c^3*d^9 - 2 556*a^6*b^5*c^4*d^8 - 3072*a^6*b^5*c^5*d^7 + 112*a^6*b^5*c^6*d^6 + 360*a^7* b^4*c^2*d^10 + 1216*a^7*b^4*c^3*d^9 + 1968*a^7*b^4*c^4*d^8 - 32*a^7*b^4*c^5 *d^7 - 384*a^8*b^3*c^2*d^10 - 880*a^8*b^3*c^3*d^9 + 4*a^8*b^3*c^4*d^8 + 264 *a^9*b^2*c^2*d^10 - 16*a*b^10*c^11*d - 48*a^10*b*c*d^11))/b^9 + (((((8*(4*b ^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^ 2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^ 4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10* c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a* b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2)*(8*a*b^10 - 16*a ^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2 *c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^ 6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a

```
*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 -
24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7
+ 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c
^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 -
304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b
^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^
6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*
a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c
^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b
^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*
d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (((((8*(4*b^13*c^4 - 8*a*b
^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*
a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c
^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*
b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32
*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x)/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*
b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4
*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*
d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b
^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b
^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6
- 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 +
192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^
7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c
*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^
2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6
*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 80
0*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5
*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6
 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^
6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a
^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d)
+ a^3*d^4 - 4*a^2*b*c*d^3))/b^4))*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c
*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*2i)/(b^4*f) - ((tan(e/2 + (f*x)/
2)*(2*a^2*d^4 + 2*b^2*d^4 + 4*b^2*c*d^3 + 12*b^2*c^2*d^2 - a*b*d^4 - 8*a*b*
c*d^3))/b^3 - (4*tan(e/2 + (f*x)/2)^3*(3*a^2*d^4 + b^2*d^4 + 18*b^2*c^2*d^2
 - 12*a*b*c*d^3))/(3*b^3) + (tan(e/2 + (f*x)/2)^5*(2*a^2*d^4 + 2*b^2*d^4 -
4*b^2*c*d^3 + 12*b^2*c^2*d^2 + a*b*d^4 - 8*a*b*c*d^3))/b^3)/(f*(3*tan(e/2 +
 (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + (atan((
(((a + b)*(a - b))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*
a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13
*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^
4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c
^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*
b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*
```

a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d
^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*
a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c
^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 -
1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*
b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d
^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7)
)/b^6 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c
^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b
^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^
2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c
*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^
12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^
4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b
^4))*(a*d - b*c)^4*1i)/(b^6 - a^2*b^4) + (((a + b)*(a - b))^(1/2)*((8*tan(e
/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7
*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16
*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*
c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a
*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104
*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7
- 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2
*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*
d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376
*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5
*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4
- 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8
*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6 - (((a + b)*(a - b))^(1/2)*(
a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 +
16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b
^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11
*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^
3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x
)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b
^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b^4))*(a*d - b*c)^4*1i)/(b^6 - a^2*
b^4))/((16*(4*a^11*d^12 - 6*a^10*b*d^12 + 16*b^11*c^11*d - a^6*b^5*d^12 + 2
*a^7*b^4*d^12 - 5*a^8*b^3*d^12 + 6*a^9*b^2*d^12 - 16*b^11*c^6*d^6 - 64*b^11
*c^8*d^4 + 8*b^11*c^9*d^3 - 64*b^11*c^10*d^2 + 72*a*b^10*c^5*d^7 + 32*a*b^1
0*c^6*d^6 + 368*a*b^10*c^7*d^5 + 62*a*b^10*c^8*d^4 + 440*a*b^10*c^9*d^3 - 2
4*a*b^10*c^10*d^2 + 12*a^5*b^6*c*d^11 - 24*a^6*b^5*c*d^11 + 60*a^7*b^4*c*d^
11 - 72*a^8*b^3*c*d^11 + 72*a^9*b^2*c*d^11 - 129*a^2*b^9*c^4*d^8 - 144*a^2*
b^9*c^5*d^7 - 936*a^2*b^9*c^6*d^6 - 496*a^2*b^9*c^7*d^5 - 1422*a^2*b^9*c^8*
d^4 - 240*a^2*b^9*c^9*d^3 + 88*a^2*b^9*c^10*d^2 + 116*a^3*b^8*c^3*d^9 + 258
*a^3*b^8*c^4*d^8 + 1384*a^3*b^8*c^5*d^7 + 1336*a^3*b^8*c^6*d^6 + 2848*a^3*b
^8*c^7*d^5 + 1148*a^3*b^8*c^8*d^4 - 208*a^3*b^8*c^9*d^3 - 54*a^4*b^7*c^2*d^

10 - 232*a^4*b^7*c^3*d^9 - 1301*a^4*b^7*c^4*d^8 - 1952*a^4*b^7*c^5*d^7 - 38
88*a^4*b^7*c^6*d^6 - 2496*a^4*b^7*c^7*d^5 + 276*a^4*b^7*c^8*d^4 + 108*a^5*b
^6*c^2*d^10 + 788*a^5*b^6*c^3*d^9 + 1756*a^5*b^6*c^4*d^8 + 3744*a^5*b^6*c^5
*d^7 + 3360*a^5*b^6*c^6*d^6 - 224*a^5*b^6*c^7*d^5 - 294*a^6*b^5*c^2*d^10 -
1008*a^6*b^5*c^3*d^9 - 2556*a^6*b^5*c^4*d^8 - 3072*a^6*b^5*c^5*d^7 + 112*a^
6*b^5*c^6*d^6 + 360*a^7*b^4*c^2*d^10 + 1216*a^7*b^4*c^3*d^9 + 1968*a^7*b^4*
c^4*d^8 - 32*a^7*b^4*c^5*d^7 - 384*a^8*b^3*c^2*d^10 - 880*a^8*b^3*c^3*d^9 +
4*a^8*b^3*c^4*d^8 + 264*a^9*b^2*c^2*d^10 - 16*a*b^10*c^11*d - 48*a^10*b*c*
d^11))/b^9 + (((a + b)*(a - b))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4
*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4
*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^
6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5
+ 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*
d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4
*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*
a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c
^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 +
800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b
^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d
^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*
a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64
*a^8*b*c*d^7))/b^6 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*b^13*c^4
- 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*
c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*
a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 +
16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*
d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)
*(a*d - b*c)^4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4))))
/(b^6 - a^2*b^4))*(a*d - b*c)^4)/(b^6 - a^2*b^4) - (((a + b)*(a - b))^(1/2)
*((8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8
- a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^
3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 +
48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d
^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c
*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*
b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4
- 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^
3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6
*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 4
16*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^
4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6
+ 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6 - (((a + b)*(a - b
))^(1/2)*(a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^
13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4
+ 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 1

```
6*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d
^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(
e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(8*a*b^10 - 16*a^2*b^9
 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b^4))*(a*d - b*c)^4)/(b^6
 - a^2*b^4)))*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*2i)/(f*(b^6 - a^2*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d \sec \left(e + f x\right)\right)^{4} \sec \left(e + f x\right)}{a + b \sec \left(e + f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

3.253 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\,\sqrt{a+b}}+\frac{d^2(3bc-ad)\tan(e+fx)}{b^2f}$$

```
[Out] 1/2*d^3*arctanh(sin(f*x+e))/b/f+d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*arctanh(sin
(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^
(1/2))/b^3/f/(a-b)^(1/2)/(a+b)^(1/2)+d^2*(-a*d+3*b*c)*tan(f*x+e)/b^2/f+1/2*
d^3*sec(f*x+e)*tan(f*x+e)/b/f
```

Rubi [A] time = 0.35, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3988, 2952, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{d^2(3bc-ad)\tan(e+fx)}{b^2f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*ArcTanh[Sin[e + f*x]])/(2*b*f) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcTanh[(Sqrt[a - b]*Tan[
(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*f) + (d^2*(3*b*c -
 a*d)*Tan[e + f*x])/(b^2*f) + (d^3*Sec[e + f*x]*Tan[e + f*x])/(2*b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^3\sec^3(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^3}{b^3(b+a\cos(e+fx))} + \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\sec(e+fx)}{b^3} + \frac{d^2(\cdots}{\cdots}\right.$$

$$= \frac{d^3\int \sec^3(e+fx)\,dx}{b} + \frac{(bc-ad)^3\int \frac{1}{b+a\cos(e+fx)}\,dx}{b^3} + \frac{\left(d^2(3bc-ad)\right)\int \cdots}{b^2}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{d^3\sec(e+fx)\tan(e+\cdots}{2bf}$$

$$= \frac{d^3\tanh^{-1}(\sin(e+fx))}{2bf} + \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx)\cdots}{b^3f}$$

Mathematica [B] time = 1.47, size = 389, normalized size = 2.29

$$\frac{\cos^2(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^3\left(-2d\left(2a^2d^2-6abcd+b^2\left(6c^2+d^2\right)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((8*(-(b*c) + a
*d)^3*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 - 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^3)/(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^2 + (4*b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - (b^2*d^3)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*
b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
))/(4*b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x]))
```

fricas [B] time = 37.60, size = 779, normalized size = 4.58

$$\left[-\frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{a^2-b^2}\,\cos\left(fx+e\right)^2\log\left(\frac{2\,ab\cos\left(fx+e\right)-\left(a^2-2\,b^2\right)\cos\left(fx+e\right)^2-2\,\sqrt{a^2-b^2}\left(b\cos\left(fx\right.\right.}{a^2\cos\left(fx+e\right)^2+2\,ab\cos\left(fx+e\right)+b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/4*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a^2 - b^2
)*cos(f*x + e)^2*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(
f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3
*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(sin(f*x
 + e) + 1) + (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 -
a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*((a^2*b^2 - b
^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2), 1/4*(4*(b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*
(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e)^2 + (6*(a^2*b
^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*co
s(f*x + e)^2*log(sin(f*x + e) + 1) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b -
a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e
) + 1) + 2*((a^2*b^2 - b^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b
^3)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-2*c^3*b^3+6*c^2*a*b^2*d-6*c*a^2*b*d^2+2*a^3*d^3)*1/2/b^3/sqr
t(-a^2+b^2)*(atan((a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(1))/2))/sqrt(-a^2+b
^2))+pi*sign(2*a-2*b)*floor((f*x+exp(1))/2/pi+1/2))+(-6*c^2*b^2*d+6*c*a*b*d
^2-2*a^2*d^3-b^2*d^3)*1/4/b^3*ln(abs(tan((f*x+exp(1))/2)-1))-(-6*c^2*b^2*d+
6*c*a*b*d^2-2*a^2*d^3-b^2*d^3)*1/4/b^3*ln(abs(tan((f*x+exp(1))/2)+1))-(6*ta
```

```
n((f*x+exp(1))/2)^3*c*b*d^2-2*tan((f*x+exp(1))/2)^3*a*d^3-tan((f*x+exp(1))/
2)^3*b*d^3-6*tan((f*x+exp(1))/2)*c*b*d^2+2*tan((f*x+exp(1))/2)*a*d^3-tan((f
*x+exp(1))/2)*b*d^3)*1/2/b^2/(tan((f*x+exp(1))/2)^2-1)^2)
```

maple [B] time = 0.67, size = 593, normalized size = 3.49

$$-\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)a^{3}d^{3}}{f\,b^{3}\sqrt{(a-b)\,(a+b)}}+\frac{6\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)a^{2}c\,d^{2}}{f\,b^{2}\sqrt{(a-b)\,(a+b)}}-\frac{6\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)a\,c^{2}d}{f\,b\sqrt{(a-b)\,(a+b)}}+\frac{2\,\mathrm{arctan}}{f\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x)
```

```
[Out] -2/f/b^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))
^(1/2))*a^3*d^3+6/f/b^2/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x
)/((a-b)*(a+b))^(1/2))*a^2*c*d^2-6/f/b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*ta
n(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a*c^2*d+2/f/((a-b)*(a+b))^(1/2)*arcta
nh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*c^3+1/2/f*d^3/b/(tan(1/2*e
+1/2*f*x)-1)^2-1/f*d^3/b^3*ln(tan(1/2*e+1/2*f*x)-1)*a^2+3/f*d^2/b^2*ln(tan(
1/2*e+1/2*f*x)-1)*a*c-3/f*d/b*ln(tan(1/2*e+1/2*f*x)-1)*c^2-1/2/f*d^3/b*ln(t
an(1/2*e+1/2*f*x)-1)+1/f*d^3/b^2/(tan(1/2*e+1/2*f*x)-1)*a-3/f*d^2/b/(tan(1/
2*e+1/2*f*x)-1)*c+1/2/f*d^3/b/(tan(1/2*e+1/2*f*x)-1)-1/2/f*d^3/b/(tan(1/2*e
+1/2*f*x)+1)^2+1/f*d^3/b^3*ln(tan(1/2*e+1/2*f*x)+1)*a^2-3/f*d^2/b^2*ln(tan(
1/2*e+1/2*f*x)+1)*a*c+3/f*d/b*ln(tan(1/2*e+1/2*f*x)+1)*c^2+1/2/f*d^3/b*ln(t
an(1/2*e+1/2*f*x)+1)+1/f*d^3/b^2/(tan(1/2*e+1/2*f*x)+1)*a-3/f*d^2/b/(tan(1/
2*e+1/2*f*x)+1)*c+1/2/f*d^3/b/(tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.66, size = 6730, normalized size = 39.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)*(b*d^3 - 2*a*d^3 + 6*b*c*d^2))/b^2 + (tan(e/2 + (f*x)/
2)^3*(2*a*d^3 + b*d^3 - 6*b*c*d^2))/b^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e
/2 + (f*x)/2)^2 + 1)) - (atan(((((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c
^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 1
3*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c
^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b
^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*
b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2
+ 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*
b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 +
24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a
*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*
a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*
b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*(
8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*
c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*
c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3 + (((8*tan(e/2 + (f*x)/2)*(
8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6
- 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7
*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6
*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*
b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 1
68*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4
*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4
+ 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((8*(4*b^10*c^3
 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 +
 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*
b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*t
an(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2)
 + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*
d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3)/((16*(4
*a^8*d^9 - 6*a^7*b*d^9 - 12*b^8*c^8*d - a^3*b^5*d^9 + 2*a^4*b^4*d^9 - 5*a^5
*b^3*d^9 + 6*a^6*b^2*d^9 + b^8*c^3*d^6 + 12*b^8*c^5*d^4 - 2*b^8*c^6*d^3 + 3
6*b^8*c^7*d^2 - 3*a*b^7*c^2*d^7 - 2*a*b^7*c^3*d^6 - 48*a*b^7*c^4*d^5 - 12*a
*b^7*c^5*d^4 - 178*a*b^7*c^6*d^3 + 12*a*b^7*c^7*d^2 + 3*a^2*b^6*c*d^8 - 6*a
^3*b^5*c*d^8 + 27*a^4*b^4*c*d^8 - 36*a^5*b^3*c*d^8 + 48*a^6*b^2*c*d^8 + 6*a
^2*b^6*c^2*d^7 + 77*a^2*b^6*c^3*d^6 + 66*a^2*b^6*c^4*d^5 + 384*a^2*b^6*c^5*
d^4 + 104*a^2*b^6*c^6*d^3 - 48*a^2*b^6*c^7*d^2 - 63*a^3*b^5*c^2*d^7 - 112*a
^3*b^5*c^3*d^6 - 474*a^3*b^5*c^4*d^5 - 324*a^3*b^5*c^5*d^4 + 76*a^3*b^5*c^6
*d^3 + 90*a^4*b^4*c^2*d^7 + 364*a^4*b^4*c^3*d^6 + 432*a^4*b^4*c^4*d^5 - 60*
a^4*b^4*c^5*d^4 - 174*a^5*b^3*c^2*d^7 - 324*a^5*b^3*c^3*d^6 + 24*a^5*b^3*c^
```

4*d^5 + 144*a^6*b^2*c^2*d^7 - 4*a^6*b^2*c^3*d^6 + 12*a*b^7*c^8*d - 36*a^7*b*c*d^8))/b^6 - (((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3 + (((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*tan(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3))*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*2i)/(b^3*f) - (atan(((((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3) + (((

a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3))/((16*(4*a^8*d^9 - 6*a^7*b*d^9 - 12*b^8*c^8*d - a^3*b^5*d^9 + 2*a^4*b^4*d^9 - 5*a^5*b^3*d^9 + 6*a^6*b^2*d^9 + b^8*c^3*d^6 + 12*b^8*c^5*d^4 - 2*b^8*c^6*d^3 + 36*b^8*c^7*d^2 - 3*a*b^7*c^2*d^7 - 2*a*b^7*c^3*d^6 - 48*a*b^7*c^4*d^5 - 12*a*b^7*c^5*d^4 - 178*a*b^7*c^6*d^3 + 12*a*b^7*c^7*d^2 + 3*a^2*b^6*c*d^8 - 6*a^3*b^5*c*d^8 + 27*a^4*b^4*c*d^8 - 36*a^5*b^3*c*d^8 + 48*a^6*b^2*c*d^8 + 6*a^2*b^6*c^2*d^7 + 77*a^2*b^6*c^3*d^6 + 66*a^2*b^6*c^4*d^5 + 384*a^2*b^6*c^5*d^4 + 104*a^2*b^6*c^6*d^3 - 48*a^2*b^6*c^7*d^2 - 63*a^3*b^5*c^2*d^7 - 112*a^3*b^5*c^3*d^6 - 474*a^3*b^5*c^4*d^5 - 324*a^3*b^5*c^5*d^4 + 76*a^3*b^5*c^6*d^3 + 90*a^4*b^4*c^2*d^7 + 364*a^4*b^4*c^3*d^6 + 432*a^4*b^4*c^4*d^5 - 60*a^4*b^4*c^5*d^4 - 174*a^5*b^3*c^2*d^7 - 324*a^5*b^3*c^3*d^6 + 24*a^5*b^3*c^4*d^5 + 144*a^6*b^2*c^2*d^7 - 4*a^6*b^2*c^3*d^6 + 12*a*b^7*c^8*d - 36*a^7*b*c*d^8))/b^6 - (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3) + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5

```
+ 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5
*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 -
 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6
*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((a + b)*(a - b))^(1/2)*(
a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*
b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3
+ 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 -
 24*a*b^9*c^2*d))/b^6 + (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d
- b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 -
 a^2*b^3)))/(b^5 - a^2*b^3)))*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*2i)/(f*
(b^5 - a^2*b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^3 \sec\left(e + fx\right)}{a + b\sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

3.254 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=103

$$\frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\,\sqrt{a+b}}+\frac{d^2\tan(e+fx)}{bf}$$

```
[Out] d*(-a*d+2*b*c)*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)
*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/b^2/f/(a-b)^(1/2)/(a+b)^(1/2)+d^2*tan(f*x+
e)/b/f
```

Rubi [A] time = 0.30, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3988, 2952, 2659, 208, 3770, 3767, 8}

$$\frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\,\sqrt{a+b}}+\frac{d^2\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[
(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*f
) + (d^2*Tan[e + f*x])/(b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^2\sec^2(e+fx)}{b+a\cos(e+fx)}\,dx$$
$$= \int \left(\frac{(bc-ad)^2}{b^2(b+a\cos(e+fx))} + \frac{d(2bc-ad)\sec(e+fx)}{b^2} + \frac{d^2\sec^2(e+fx)}{b}\right)$$
$$= \frac{d^2\int \sec^2(e+fx)\,dx}{b} + \frac{(bc-ad)^2\int \frac{1}{b+a\cos(e+fx)}\,dx}{b^2} + \frac{(d(2bc-ad))\int \sec}{b^2}$$
$$= \frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f} - \frac{d^2\,\mathrm{Subst}(\int 1\,dx, x, -\tan(e+fx))}{bf} + $$
$$= \frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,f}$$

Mathematica [A] time = 0.83, size = 135, normalized size = 1.31

$$\frac{d\left(bd\tan(e+fx) - (2bc-ad)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((-2*(b*c - a*d)^2*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-((2*b*c - a*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + b*d*Tan[e + f*x]))/(b^2*f)
```

fricas [B] time = 5.65, size = 518, normalized size = 5.03

$$\left[\frac{2\left(a^2b-b^3\right)d^2\sin\left(fx+e\right) + \left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{a^2-b^2}\,\cos\left(fx+e\right)\log\left(\frac{2\,ab\cos\left(fx+e\right) - \left(a^2-2\,b^2\right)\cos\left(fx+e\right)^2 + 2\,\sqrt{}}{a^2\cos\left(fx+e\right)^2 + 2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sq
rt(a^2 - b^2)*cos(f*x + e)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x
+ e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)
/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b - b^3)*c*d -
(a^3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*
d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4
)*f*cos(f*x + e)), 1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e) + (2*(a^2*b - b^3)*c*d - (a^
3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*d -
 (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4)*f
*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-2*c^2*b^2+4*c*a*b*d-2*a^2*d^2)*1/2/b^2/sqrt(-a^2+b^2)*(atan(
(a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(1))/2))/sqrt(-a^2+b^2))+pi*sign(2*a-2
*b)*floor((f*x+exp(1))/2/pi+1/2))-(2*c*b*d-a*d^2)*1/2/b^2*ln(abs(tan((f*x+e
xp(1))/2)-1))+(2*c*b*d-a*d^2)*1/2/b^2*ln(abs(tan((f*x+exp(1))/2)+1))-tan((f
*x+exp(1))/2)*d^2/b/(tan((f*x+exp(1))/2)^2-1))
```

maple [B] time = 0.64, size = 288, normalized size = 2.80

$$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)a^{2}d^{2}}{f\,b^{2}\sqrt{(a-b)\,(a+b)}}-\frac{4\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)acd}{f\,b\sqrt{(a-b)\,(a+b)}}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)c^{2}}{f\sqrt{(a-b)\,(a+b)}}-\frac{d^{2}}{f\,b\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x)
```

```
[Out] 2/f/b^2/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^
(1/2))*a^2*d^2-4/f/b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/(
(a-b)*(a+b))^(1/2))*a*c*d+2/f/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1
/2*f*x)/((a-b)*(a+b))^(1/2))*c^2-1/f*d^2/b/(tan(1/2*e+1/2*f*x)-1)+1/f*d^2/b
^2*ln(tan(1/2*e+1/2*f*x)-1)*a-2/f*d/b*ln(tan(1/2*e+1/2*f*x)-1)*c-1/f*d^2/b/
```

```
(tan(1/2*e+1/2*f*x)+1)-1/f*d^2/b^2*ln(tan(1/2*e+1/2*f*x)+1)*a+2/f*d/b*ln(ta
n(1/2*e+1/2*f*x)+1)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 7.32, size = 3559, normalized size = 34.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] - (2*d^2*tan(e/2 + (f*x)/2))/(b*f*(tan(e/2 + (f*x)/2)^2 - 1)) - (atan(((((a
 + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c
^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c
^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3
*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d
^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2
 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*
a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b
*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)
^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2) + (((a + b)*(a - b))
^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d
^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^
2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*
a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 - (((
a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2
 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/
b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(2*a*b^6
 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)^2)/(b^4 - a^2
*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2))/((64*(a^4*b*d^6 - a^5*d^6 - 2*b^5
*c^5*d + 4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^4*d^2 - 6*a^3*b^2*c*d^5
 - a^4*b*c^2*d^4 + 13*a^2*b^3*c^2*d^4 + 8*a^2*b^3*c^3*d^3 - 5*a^2*b^3*c^4*d
^2 - 12*a^3*b^2*c^2*d^4 + 4*a^3*b^2*c^3*d^3 + 2*a*b^4*c^5*d + 6*a^4*b*c*d^5
```

))/b^3 - (((a + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*
c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2
 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3
 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d
- 8*a^4*b*c*d^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^
2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a
*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(
1/2)*(a*d - b*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2))
)*(a*d - b*c)^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2)/(b^4 - a^2*b^2) + (((a + b
)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 -
 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d
^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2
*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))
/b^2 - (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a
^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*
b^5*c*d))/b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^
2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)^2)/
(b^4 - a^2*b^2))*(a*d - b*c)^2)/(b^4 - a^2*b^2)))*((a + b)*(a - b))^(1/2)*(
a*d - b*c)^2*2i)/(f*(b^4 - a^2*b^2)) - (d*atan(((d*(a*d - 2*b*c)*((32*tan(e
/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4
+ 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a
^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 +
 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 + (d*(a*d - 2*b*c)*((3
2*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^
4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*d*tan(e/2 + (f*
x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2)*1i)/b^2 +
 (d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4
- 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*
d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^
2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3)
)/b^2 - (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*
c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d
))/b^3 + (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^
3*b^4))/b^4))/b^2)*1i)/b^2)/((64*(a^4*b*d^6 - a^5*d^6 - 2*b^5*c^5*d + 4*b^5
*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^4*d^2 - 6*a^3*b^2*c*d^5 - a^4*b*c^2*d
^4 + 13*a^2*b^3*c^2*d^4 + 8*a^2*b^3*c^3*d^3 - 5*a^2*b^3*c^4*d^2 - 12*a^3*b^
2*c^2*d^4 + 4*a^3*b^2*c^3*d^3 + 2*a*b^4*c^5*d + 6*a^4*b*c*d^5))/b^3 - (d*(a
*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^
4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 -
12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2
+ 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2
+ (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 +
2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3
 - (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4)
)/b^4))/b^2))/b^2 + (d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b

```
^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*
d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*
d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3
*d - 8*a^4*b*c*d^3))/b^2 - (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a
*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*
c*d + 2*a^2*b^5*c*d))/b^3 + (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6
 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2))/b^2))*(a*d - 2*b*c)*2i)/(b^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^{2}\sec\left(e + fx\right)}{a + b\sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

3.255 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\,\sqrt{a+b}}+\frac{d\tanh^{-1}(\sin(e+fx))}{bf}$$

```
[Out] d*arctanh(sin(f*x+e))/b/f+2*(-a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*
x)/(a+b)^(1/2))/b/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\,\sqrt{a+b}}+\frac{d\tanh^{-1}(\sin(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(b*f) + (2*(b*c - a*d)*ArcTanh[(Sqrt[a - b]*Tan[(
e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx &= \frac{d\int \sec(e+fx)\,dx}{b}+\frac{(bc-ad)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{b}\\
&= \frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{(bc-ad)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b^2}\\
&= \frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{(2(bc-ad))\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(e\right.\right.}{b^2 f}\\
&= \frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,f}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 112, normalized size = 1.47

$$
\frac{\frac{2(ad-bc)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+d\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right.\right.\right.}{bf}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((2*(-(b*c) + a*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]))/(b*f)
```

fricas [A] time = 0.97, size = 309, normalized size = 4.07

$$\left[\frac{\left(a^2-b^2\right)d\log\left(\sin\left(fx+e\right)+1\right)-\left(a^2-b^2\right)d\log\left(-\sin\left(fx+e\right)+1\right)-\sqrt{a^2-b^2}\,(bc-ad)\log\left(\frac{2\,ab\cos\left(fx+e\right)-\left(a^2\right.}{}\right.}{2\left(a^2b-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x + e)
 + 1) - sqrt(a^2 - b^2)*(b*c - a*d)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)
*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a
^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/((a^2*b - b^3)*
f), 1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x +
 e) + 1) + 2*sqrt(-a^2 + b^2)*(b*c - a*d)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f
*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/((a^2*b - b^3)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-d*1/2/b*ln(abs(tan((f*x+exp(1))/2)-1))+d*1/2/b*ln(abs(tan((f*
x+exp(1))/2)+1))+(-2*c*b+2*a*d)/b*1/2/sqrt(-a^2+b^2)*(atan((a*tan((f*x+exp(
1))/2)-b*tan((f*x+exp(1))/2))/sqrt(-a^2+b^2))+pi*sign(2*a-2*b)*floor((f*x+e
xp(1))/2/pi+1/2)))
```

maple [A] time = 0.66, size = 135, normalized size = 1.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)da}{fb\sqrt{(a-b)\,(a+b)}}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)c}{f\sqrt{(a-b)\,(a+b)}}-\frac{d\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-1\right)}{fb}+\frac{d\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{fb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

```
[Out] -2/f/b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(
1/2))*d*a+2/f/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(
a+b))^(1/2))*c-1/f*d/b*ln(tan(1/2*e+1/2*f*x)-1)+1/f*d/b*ln(tan(1/2*e+1/2*f*
x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.81, size = 571, normalized size = 7.51

$$\frac{b^2\,c\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}-\frac{a^2\,c\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}-\frac{2\,b\,d\,\mathrm{atanh}\left(\dots\right.}{f\,\left(a^2-\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] (b^2*c*log((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (a^2*c*lo
g((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 -
b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (2*b*d*atanh(sin(e
/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)) + (c*log((a*cos(e/2 + (f
*x)/2) + b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e
/2 + (f*x)/2))*((a + b)*(a - b))^(1/2))/(f*(a^2 - b^2)) - (a*b*d*log((b*sin
(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) + (2*a^2*d*atanh(sin(e/2 + (
f*x)/2)/cos(e/2 + (f*x)/2)))/(b*f*(a^2 - b^2)) + (a^3*d*log((b*sin(e/2 + (f
*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e
/2 + (f*x)/2)))/(b*f*(a^2 - b^2)^(3/2)) - (a*d*log((a*cos(e/2 + (f*x)/2) +
b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x
)/2))*((a + b)*(a - b))^(1/2))/(b*f*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

3.256 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=121

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\,\sqrt{a+b}\,(bc-ad)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*b*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/(-a*d+b*c)/f/(a-b)^
(1/2)/(a+b)^(1/2)-2*d*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/(
-a*d+b*c)/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3988, 3001, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\,\sqrt{a+b}\,(bc-ad)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*(b*c - a*d)*f) - (2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c
 + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
```

```
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx &= \int \frac{\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}\,dx\\ &= \frac{b\int \frac{1}{b+a\cos(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{d+c\cos(e+fx)}\,dx}{bc-ad}\\ &= \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} - \frac{(2d)\,\mathrm{Subst}\left(\int\right.}{}\\ &= \frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(bc-ad)f} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}\,(bc-ad)f}\end{aligned}$$

Mathematica [A] time = 0.25, size = 119, normalized size = 0.98

$$-\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}\,(bc-ad)} - \frac{2d\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*b*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2
]*(b*c - a*d)*f) - (2*d*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/((-(b*c) + a*d)*Sqrt[c^2 - d^2]*f)
```

fricas [A] time = 2.95, size = 1040, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2 - b^2)*sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2
)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (b*c^2 - b*d^
2)*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*co
s(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b
^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -1/2*((a^2 - b^2)*
sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 +
2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos
(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))
))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 -
 a*b^2)*d^3)*f), -1/2*(2*(a^2 - b^2)*sqrt(-c^2 + d^2)*d*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^2 - b*d^2)*sq
rt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sq
rt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x
 + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c
^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -((a^2 - b^2)*sqrt(-c^2
 + d^2)*d*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*
x + e))) - (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*
b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

giac [B] time = 1.45, size = 526, normalized size = 4.35

$$\frac{\frac{\left(\sqrt{-c^2+d^2}\,b(c-2\,d)|c-d|+\sqrt{-c^2+d^2}\,ad|c-d|+\sqrt{-c^2+d^2}\,|-bc+ad||c-d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-\frac{2\,ac-2\,bd+\sqrt{-4\,(ac+bc+ad+bd)(ac-bc-ad+bd)+4\,(ac-bd)^2}}{ac-bc-ad+bd}}}\right)\right)}{(bc-ad)^2\left(c^2-2\,cd+d^2\right)+\left(c^3-2\,c^2d+cd^2\right)a|-bc+ad|-\left(c^2d-2\,cd^2+d^3\right)b|-bc+ad|}+}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((sqrt(-c^2 + d^2)*b*(c - 2*d)*abs(c - d) + sqrt(-c^2 + d^2)*a*d*abs(c - d)
 + sqrt(-c^2 + d^2)*abs(-b*c + a*d)*abs(c - d))*(pi*floor(1/2*(f*x + e)/pi
```

```
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d + sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((b*c - a*d)^2*(c^2 - 2*c*d + d^2) + (c^3 - 2*c^2*
d + c*d^2)*a*abs(-b*c + a*d) - (c^2*d - 2*c*d^2 + d^3)*b*abs(-b*c + a*d)) +
 (sqrt(-a^2 + b^2)*b*c*abs(a - b) + sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(a - b)
 - sqrt(-a^2 + b^2)*abs(-b*c + a*d)*abs(a - b))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d - sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((a^2 - 2*a*b + b^2)*(b*c - a*d)^2 - (a^3 - 2*a^2*
b + a*b^2)*c*abs(-b*c + a*d) + (a^2*b - 2*a*b^2 + b^3)*d*abs(-b*c + a*d)))/
f
```

maple [A] time = 0.63, size = 108, normalized size = 0.89

$$\frac{\frac{2d\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(da-cb)\sqrt{(c+d)(c-d)}}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(da-cb)\sqrt{(a-b)(a+b)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*(2*d/(a*d-b*c)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c
+d)*(c-d))^(1/2))-2*b/(a*d-b*c)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e
+1/2*f*x)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 4.36, size = 2665, normalized size = 22.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))),x)

[Out] (b*c^2*atan((b^5*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(
e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2
)^(3/2)*2i + b^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*
tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a
^2 - b^2)^(1/2)*1i - a^2*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i +
a^3*b^2*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2 + (f*
x)/2)*(a^2 - b^2)^(3/2)*2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2
i + a*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*tan(e/2 +
 (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(
1/2)*1i + a^2*b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a^3*b^2*c*d
*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (f*x)/2)*(a^
2 - b^2)^(3/2)*2i - a*b^4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i)/(a^6
*d^2 - b^6*c^2 + 2*a^2*b^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2*a^4*b^2*d^2)
)*(a^2 - b^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3
*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b*d^2*atan((b^5*c^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^
2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + b^5*d^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*tan(e/2 + (f*x)/2)*(a^2
- b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^2
*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*d^2*tan(e/2 + (f
*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*
2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a*b^2*c^2*tan(e/2 +
(f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1
/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^2*b^3*c*d*ta
n(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a^3*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2
 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i - a*b^
4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i)/(a^6*d^2 - b^6*c^2 + 2*a^2*b
^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2*a^4*b^2*d^2))*(a^2 - b^2)^(1/2)*2i)/
(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b
^2*c^2*d - a^2*b*c*d^2)) + (a^2*d*atan((a^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2)*1i - b^2*c^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*d^3*ta
n(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + b^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2)*2i - a^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a^2*c^
3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^2*d^3*tan(e/2 + (f*x)
/2)*(c^2 - d^2)^(1/2)*3i + b^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)
*1i - a*b*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*d^5*tan(e/2 + (
f*x)/2)*(c^2 - d^2)^(1/2)*2i + a^2*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/
2)*2i + a^2*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*c^4*d*tan(e
/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + a*b*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2)*2i + a*b*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a*b*c
*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*c*d^4*tan(e/2 + (f*x)/2)
*(c^2 - d^2)^(1/2)*2i)/(a^2*d^6 - b^2*c^6 - 2*a^2*c^2*d^4 + a^2*c^4*d^2 - b
^2*c^2*d^4 + 2*b^2*c^4*d^2))*(c^2 - d^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 +
```

```
a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2))
 - (b^2*d*atan((a^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^5*t
an(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*d^3*tan(e/2 + (f*x)/2)*(c^2 -
d^2)^(3/2)*2i + b^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a^2*c^2*d
^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a^2*c^3*d^2*tan(e/2 + (f*x)/2)
*(c^2 - d^2)^(1/2)*1i - b^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*3i
 + b^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a*b*d^3*tan(e/2 +
(f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2
)*2i + a^2*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + a^2*c*d^4*tan(e/
2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*c^4*d*tan(e/2 + (f*x)/2)*(c^2 - d^2
)^(1/2)*1i + a*b*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i + a*b*c^3*
d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a*b*c*d^2*tan(e/2 + (f*x)/2)*
(c^2 - d^2)^(3/2)*2i - a*b*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i)/(
a^2*d^6 - b^2*c^6 - 2*a^2*c^2*d^4 + a^2*c^4*d^2 - b^2*c^2*d^4 + 2*b^2*c^4*d
^2))*(c^2 - d^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 -
a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)\left(c + d\sec\left(e + fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

3.257 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}\,(bc-ad)^2}+\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{2d\left(-acd+2bc^2-bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan(}{\sqrt{c-}}\right.}{f(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2}$$

```
[Out] -2*d*(-a*c*d+2*b*c^2-b*d^2)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1
/2))/(c-d)^(3/2)/(c+d)^(3/2)/(-a*d+b*c)^2/f+d^2*sin(f*x+e)/(-a*d+b*c)/(c^2-
d^2)/f/(d+c*cos(f*x+e))+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^
(1/2))/(-a*d+b*c)^2/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3988, 3056, 3001, 2659, 208}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}\,(bc-ad)^2}+\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{2d\left(-acd+2bc^2-bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan(}{\sqrt{c-}}\right.}{f(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(b*c - a*d)^2*f) - (2*d*(2*b*c^2 - a*c*d - b*d^2)*ArcTanh[(Sqrt[c
 - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^(3/2)*(c + d)^(3/2)*(b*c - a
*d)^2*f) + (d^2*Sin[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*(d + c*Cos[e + f*x
]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))^2}\,dx$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\int \frac{-bcd-\left(acd-b\left(c^2-d^2\right)\right)\cos(e}{(b+a\cos(e+fx))(d+c\cos(e+}}{(bc-ad)\left(c^2-d^2\right.}$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{b^2\int \frac{1}{b+a\cos(e+fx)}\,dx}{(bc-ad)^2} + \cdots$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+b+(-a-}\right.}{(bc}$$

$$= \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)^2 f} - \frac{2d\left(2bc^2-acd-bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{(c-d)^{3/2}(c+d)^{3/2}(bc-}$$

Mathematica [A] time = 0.72, size = 229, normalized size = 1.22

$$\frac{-2b^2\left(c^2-d^2\right)^{3/2}(c\cos(e+fx)+d)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right) - d\sqrt{a^2-b^2}\left(d\sqrt{c^2-d^2}(ad-bc)\sin(e+fx)-2\left(\cdots\right.\right.}{f\sqrt{a^2-b^2}(c-d)(c+d)\sqrt{c^2-d^2}(bc-ad)^2(c\cos(e+f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (-2*b^2*(c^2 - d^2)^(3/2)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]]*(d + c*Cos[e + f*x]) - Sqrt[a^2 - b^2]*d*(-2*(2*b*c^2 - a*c*d - b*d^2)*
ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x]) +
 d*(-(b*c) + a*d)*Sqrt[c^2 - d^2]*Sin[e + f*x]))/(Sqrt[a^2 - b^2]*(c - d)*(
c + d)*(b*c - a*d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x]))
```

fricas [B] time = 167.79, size = 2863, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2
*c*d^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^
2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2
*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b -
b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b^3)*c
^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqrt(c^2
- d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2
- d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^
2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/(((a^2*b^2 -
 b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4
*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a
*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*b^2 - b^4)*c^
6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^
3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3
)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), 1/2*(2*(b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*
d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x + e))*sqrt(-a^2 + b^2)*
arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) +
 (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a
^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x +
e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2
- 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*c
os(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^
3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))
/(((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4
)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 -
 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*
b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^
4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(
a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), -1/2*(2*(2*(a^2*b - b^3)*c^
2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d -
(a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)
*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e)))
- (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d
^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*c
os(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2
 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - 2*((a^2*b - b^3)
*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)
*sin(f*x + e))/(((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a
^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 +
b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x
 + e) + ((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b
^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)
*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), ((b^2*c^4*d -
 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x +
```

```
e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 -
b^2)*sin(f*x + e))) - (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2
*b - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b
^3)*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f
*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + ((a^2*b - b^3)*c^3*d^2 - (a^3 -
a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/(((
a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^
5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(
a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*b^2
- b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^
3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*
b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f)]
```

giac [B] time = 2.50, size = 340, normalized size = 1.82

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-a^2+b^2}}+\frac{d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(bc^3-ac^2d-bcd^2+ad^3\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}-\cdots\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*b^2/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)) + d^2*tan(1/2*f*x + 1/2*e)/((b*c^3 - a*
c^2*d - b*c*d^2 + a*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)
^2 - c - d)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 +
 2*a*b*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)))/f
```

maple [A] time = 0.71, size = 208, normalized size = 1.11

$$\frac{-\frac{2d\left(-\frac{d(da-cb)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\left(c^2-d^2\right)\left(\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)c-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)d-c-d\right)}-\frac{\left(acd-2c^2b+d^2b\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}\right)}{(da-cb)^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(da-cb)^2\sqrt{(a-b)(a+b)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(-2*d/(a*d-b*c)^2*(-d*(a*d-b*c)/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e
+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)-(a*c*d-2*b*c^2+b*d^2)/(c+d)/(c-d)
/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2)))
+2*b^2/(a*d-b*c)^2/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a
-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 15.42, size = 20827, normalized size = 111.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)
)*(a*d^2 + b*c^2 - a*c*d - b*c*d)) - (d*atan(((d*((32*tan(e/2 + (f*x)/2)*(b
^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d +
3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 +
 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*
a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^
2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 -
 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2
*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a
^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*
c^2*d^3 + 2*a*b*c^3*d^2) + (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7
*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*
d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2
 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 1
1*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a
^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*
b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^
```

2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2
 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^
4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*
a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^
3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^
2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*
c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*
d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b
^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*
c^5*d - 3*a^2*b*c*d^5) + (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/
2)*(b*d^2 - 2*b*c^2 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2
*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d
^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7
*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 +
2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*
b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3
*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*
b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*
c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*
a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d
^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b
^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 -
 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c
^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a
^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^
3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^
5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c
^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^
8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3
*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 -
6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^
2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6
 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^
5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*1
i)/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c
^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*
c^3*d^5 - 6*a*b*c^5*d^3) + (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6
 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3
*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13
*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a
^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*
a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3
 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2
*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^
2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3

d^2) - (d((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*
d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 +
a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6
- 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8
*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^
5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b
*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*
a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d
^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*
b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4
*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*
d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b
^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 +
b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 -
b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*
b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d
^5) - (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2
+ a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*
b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8
+ 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^
4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a
*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*
b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4
*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b
*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^
5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 2
0*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2
*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3
*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6
- 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3
*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14
*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*
d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2
*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^
4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^8 - b^2*c^8 - 3*a^2
*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^
2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((
c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3
*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 +
3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))
((c + d)^3(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*1i)/(a^2*d^8 - b^2*c
^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*
d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5
d^3))/((64(b^5*d^5 - a*b^4*d^5 - b^5*c*d^4 + 2*b^5*c^4*d - 3*b^5*c^2*d^3
+ 2*b^5*c^3*d^2 + 2*a*b^4*c^2*d^3 - 5*a*b^4*c^3*d^2 - 2*a^2*b^3*c*d^4 + 2*a

^2*b^3*c^2*d^3 + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4 - 2*a*
b^4*c^4*d))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*
c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3
*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*
b^2*c^5*d - 3*a^2*b*c*d^5) + (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^
6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a
^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 +
13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6
*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 1
2*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d
^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a
^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*
c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c
^3*d^2) + (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^
8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6
 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^
6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 -
 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*
a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6
*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 1
6*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2
*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^
3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 +
 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^
7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5
*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6
+ b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3
 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^
2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c
*d^5) + (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c
^2 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^
2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d
^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*
c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8
*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*
a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a
^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6
*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*
b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 +
 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c
^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a
^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d
^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b
^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 -
14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^

8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3) - (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) - (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) - (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^

5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 +
16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^
3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^
3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5
 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2
*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 3
0*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d
 + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3
 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*
a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4
*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*
d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^
(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2
*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*
b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^
3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*
a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2
*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((c + d)^3*(c -
 d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*2i)/(f*(a^2*d^8 - b^2*c^8 - 3*a^2*c^
2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c
^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)) - (b^2
atan(((b^2(a^2 - b^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6
 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3
*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13
*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a
^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*
a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3
 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2
*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^
2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3
d^2) + (b^2(a^2 - b^2)^(1/2)*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^
7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2
*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^
2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 -
11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*
a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6
*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a
^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^
2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b
^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31
*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d
^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b
^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b
*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3
*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*

b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2
*c^5*d - 3*a^2*b*c*d^5) + (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a
*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a
^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d
^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*
c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 +
4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 -
6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8
*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*
a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*
a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*
d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*
b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 -
24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c
^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a
^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d
^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*
c^9*d + 2*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2
*a*b^3*c*d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*
c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4
*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a
^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a^2*b^2*c
^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) + (b^2*(a^2 - b^2)^(1/2)*((32
*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*
c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2
*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 -
11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4
*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d
^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d
^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c
^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4
+ 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) - (b^2*(a^2 - b^2)^(1/2)*((3
2*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^
9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 +
b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*
d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d
- 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 -
5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a
^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^
4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^
4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 +
11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d
^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b
^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 1
0*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*

c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) - (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))/((64*(b^5*d^5 - a*b^4*d^5 - b^5*c*d^4 + 2*b^5*c^4*d - 3*b^5*c^2*d^3 + 2*b^5*c^3*d^2 + 2*a*b^4*c^2*d^3 - 5*a*b^4*c^3*d^2 - 2*a^2*b^3*c*d^4 + 2*a^2*b^3*c^2*d^3 + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4 - 2*a*b^4*c^4*d))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) + (b^2*(a^2 - b^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) + (b^2*(a^2 - b^2)^(1/2)*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d

^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) + (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) - (b^2*(a^2 - b^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) - (b^2*(a^2 - b^2)^(1/2)*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*

```
c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c
*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^
5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^
4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^
3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^
4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*
c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a
^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^
5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/
(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^
3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d
^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d -
3*a^2*b*c*d^5) - (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a*b^6*c^10
 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^
10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^
7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 -
 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*
c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5
*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*
c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5
*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c
^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*
a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d
^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b
^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 +
36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^
2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a
^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2
*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*
d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 -
 a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*
b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^
2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2
*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))*(a^2 - b^2)^(1/2)*2i)/(f*(a^4*d^2 - b^4
*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2), x)
```

3.258 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=379

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\tan(e+fx)}{b^4f}-\frac{(bc-ad)^5\sin(e+fx)}{b^4f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d\right.}{b^5f}$$

```
[Out] 1/2*d^4*(-2*a*d+5*b*c)*arctanh(sin(f*x+e))/b^3/f+d^2*(-4*a^3*d^3+15*a^2*b*c
*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*arctanh(sin(f*x+e))/b^5/f+2*(-a*d+b*c)^5*ar
ctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/a/(a-b)^(3/2)/b^3/(a+b)^(
3/2)/f-(-a*d+b*c)^5*sin(f*x+e)/b^4/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c
)^4*(4*a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/a/b^5/f
/(a-b)^(1/2)/(a+b)^(1/2)+d^5*tan(f*x+e)/b^2/f+d^3*(3*a^2*d^2-10*a*b*c*d+10*
b^2*c^2)*tan(f*x+e)/b^4/f+1/2*d^4*(-2*a*d+5*b*c)*sec(f*x+e)*tan(f*x+e)/b^3/
f+1/3*d^5*tan(f*x+e)^3/b^2/f
```

Rubi [A] time = 0.67, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\tan(e+fx)}{b^4f}+\frac{d^2\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)\tanh^{-1}(\sin(e+fx))}{b^5f}-\frac{}{b^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*(5*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(2*b^3*f) + (d^2*(10*b^3*c^3 -
20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*ArcTanh[Sin[e + f*x]])/(b^5*f)
 + (2*(b*c - a*d)^5*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a
*(a - b)^(3/2)*b^3*(a + b)^(3/2)*f) + (2*(b*c - a*d)^4*(b*c + 4*a*d)*ArcTan
h[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^5*Sqrt[a +
b]*f) - ((b*c - a*d)^5*Sin[e + f*x])/(b^4*(a^2 - b^2)*f*(b + a*Cos[e + f*x]
)) + (d^5*Tan[e + f*x])/(b^2*f) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2
)*Tan[e + f*x])/(b^4*f) + (d^4*(5*b*c - 2*a*d)*Sec[e + f*x]*Tan[e + f*x])/(
2*b^3*f) + (d^5*Tan[e + f*x]^3)/(3*b^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^5\sec^4(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(\frac{(-bc+ad)^5}{ab^4(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^4(bc+4ad)}{ab^5(b+a\cos(e+fx))} + \frac{d^2\left(10b^3c^3-20a\right.}{}\right.$$

$$= \frac{d^5\int\sec^4(e+fx)\,dx}{b^2} + \frac{\left(d^4(5bc-2ad)\right)\int\sec^3(e+fx)\,dx}{b^3} - \frac{(bc-ad)^5\int}{}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)\tanh^{-1}(\sin(e+fx))}{b^5f} - \frac{}{b^4\left(a^2\right.}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2\right.}{b^5f}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2\right.}{b^5f}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2\right.}{b^5f}$$

Mathematica [B] time = 6.55, size = 1137, normalized size = 3.00

$$\frac{(b + a\cos(e + fx))\left(12d^5\sin(e + fx)b^5 + 60c^2d^3\sin(e + fx)b^5 + 6c^5\sin(2(e + fx))b^5 + 30cd^4\sin(2(e + fx))b^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*(b*c - a*d)^4*(-(a*b*c) - 4*a^2*d + 5*b^2*d)*ArcTanh[((-a + b)*Tan[(e +
 f*x)/2])/Sqrt[a^2 - b^2]]*Cos[e + f*x]^3*(b + a*Cos[e + f*x])^2*(c + d*Sec
[e + f*x])^5)/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)*f*(d + c*Cos[e + f*x])^5*(a
 + b*Sec[e + f*x])^2) + ((-20*b^3*c^3*d^2 + 40*a*b^2*c^2*d^3 - 30*a^2*b*c*d
^4 - 5*b^3*c*d^4 + 8*a^3*d^5 + 2*a*b^2*d^5)*Cos[e + f*x]^3*(b + a*Cos[e + f
*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(c + d*Sec[e + f*x])^5)/(2*
b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2) + ((20*b^3*c^3*d^2 - 4
0*a*b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*C
os[e + f*x]^3*(b + a*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]]*(c + d*Sec[e + f*x])^5)/(2*b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f
*x])^2) + ((b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^5*(-60*a^2*b^3*c^2*d^3
*Sin[e + f*x] + 60*b^5*c^2*d^3*Sin[e + f*x] + 45*a^3*b^2*c*d^4*Sin[e + f*x]
 - 45*a*b^4*c*d^4*Sin[e + f*x] - 12*a^4*b*d^5*Sin[e + f*x] + 12*b^5*d^5*Sin
[e + f*x] + 6*b^5*c^5*Sin[2*(e + f*x)] - 30*a*b^4*c^4*d*Sin[2*(e + f*x)] +
60*a^2*b^3*c^3*d^2*Sin[2*(e + f*x)] - 120*a^3*b^2*c^2*d^3*Sin[2*(e + f*x)]
+ 60*a*b^4*c^2*d^3*Sin[2*(e + f*x)] + 90*a^4*b*c*d^4*Sin[2*(e + f*x)] - 90*
a^2*b^3*c*d^4*Sin[2*(e + f*x)] + 30*b^5*c*d^4*Sin[2*(e + f*x)] - 24*a^5*d^5
*Sin[2*(e + f*x)] + 22*a^3*b^2*d^5*Sin[2*(e + f*x)] - 4*a*b^4*d^5*Sin[2*(e
+ f*x)] - 60*a^2*b^3*c^2*d^3*Sin[3*(e + f*x)] + 60*b^5*c^2*d^3*Sin[3*(e + f
*x)] + 45*a^3*b^2*c*d^4*Sin[3*(e + f*x)] - 45*a*b^4*c*d^4*Sin[3*(e + f*x)]
- 12*a^4*b*d^5*Sin[3*(e + f*x)] + 8*a^2*b^3*d^5*Sin[3*(e + f*x)] + 4*b^5*d^
5*Sin[3*(e + f*x)] + 3*b^5*c^5*Sin[4*(e + f*x)] - 15*a*b^4*c^4*d*Sin[4*(e +
 f*x)] + 30*a^2*b^3*c^3*d^2*Sin[4*(e + f*x)] - 60*a^3*b^2*c^2*d^3*Sin[4*(e
+ f*x)] + 30*a*b^4*c^2*d^3*Sin[4*(e + f*x)] + 45*a^4*b*c*d^4*Sin[4*(e + f*x
)] - 30*a^2*b^3*c*d^4*Sin[4*(e + f*x)] - 12*a^5*d^5*Sin[4*(e + f*x)] + 7*a^
3*b^2*d^5*Sin[4*(e + f*x)] + 2*a*b^4*d^5*Sin[4*(e + f*x)]))/(24*b^4*(-a^2 +
 b^2)*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-tan((f*x+exp(1))/2)*c^5*b^5+5*tan((f*x+exp(1))/2)*c^4*a*b^4*
d-10*tan((f*x+exp(1))/2)*c^3*a^2*b^3*d^2+10*tan((f*x+exp(1))/2)*c^2*a^3*b^2
*d^3-5*tan((f*x+exp(1))/2)*c*a^4*b*d^4+tan((f*x+exp(1))/2)*a^5*d^5)/(-a^2*b
^4+b^6)/(tan((f*x+exp(1))/2)^2*a-tan((f*x+exp(1))/2)^2*b-a-b)+(-60*tan((f*x
+exp(1))/2)^5*c^2*b^2*d^3+60*tan((f*x+exp(1))/2)^5*c*a*b*d^4+15*tan((f*x+ex
p(1))/2)^5*c*b^2*d^4-18*tan((f*x+exp(1))/2)^5*a^2*d^5-6*tan((f*x+exp(1))/2)
^5*a*b*d^5-6*tan((f*x+exp(1))/2)^5*b^2*d^5+120*tan((f*x+exp(1))/2)^3*c^2*b^
2*d^3-120*tan((f*x+exp(1))/2)^3*c*a*b*d^4+36*tan((f*x+exp(1))/2)^3*a^2*d^5+
4*tan((f*x+exp(1))/2)^3*b^2*d^5-60*tan((f*x+exp(1))/2)*c^2*b^2*d^3+60*tan((
f*x+exp(1))/2)*c*a*b*d^4-15*tan((f*x+exp(1))/2)*c*b^2*d^4-18*tan((f*x+exp(1
))/2)*a^2*d^5+6*tan((f*x+exp(1))/2)*a*b*d^5-6*tan((f*x+exp(1))/2)*b^2*d^5)*
1/6/b^4/(tan((f*x+exp(1))/2)^2-1)^3+(2*c^5*a*b^5-10*c^4*b^6*d-20*c^3*a^3*b^
3*d^2+40*c^3*a*b^5*d^2+40*c^2*a^4*b^2*d^3-60*c^2*a^2*b^4*d^3-30*c*a^5*b*d^4
+40*c*a^3*b^3*d^4+8*a^6*d^5-10*a^4*b^2*d^5)*1/2/(-a^2*b^5+b^7)/sqrt(-a^2+b^
2)*(atan((a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(1))/2))/sqrt(-a^2+b^2))+pi*s
ign(2*a-2*b)*floor((f*x+exp(1))/2/pi+1/2))-(20*c^3*b^3*d^2-40*c^2*a*b^2*d^3
+30*c*a^2*b*d^4+5*c*b^3*d^4-8*a^3*d^5-2*a*b^2*d^5)*1/4/b^5*ln(abs(tan((f*x+
exp(1))/2)-1))+(20*c^3*b^3*d^2-40*c^2*a*b^2*d^3+30*c*a^2*b*d^4+5*c*b^3*d^4-
8*a^3*d^5-2*a*b^2*d^5)*1/4/b^5*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 0.76, size = 1870, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x)
```

```
[Out] -1/f*d^5/b^2/(tan(1/2*e+1/2*f*x)-1)-1/2/f*d^5/b^2/(tan(1/2*e+1/2*f*x)-1)^2-
1/3/f*d^5/b^2/(tan(1/2*e+1/2*f*x)+1)^3-1/f*d^5/b^2/(tan(1/2*e+1/2*f*x)+1)+1
/2/f*d^5/b^2/(tan(1/2*e+1/2*f*x)+1)^2-1/3/f*d^5/b^2/(tan(1/2*e+1/2*f*x)-1)^
3-20/f/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)
/((a-b)*(a+b))^(1/2))*a^3*c^3*d^2-10/f/b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*
```

```
arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^4*d^5+40/f/(a-b)/(a
+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2
))*a*c^3*d^2+8/f/b^5/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*
e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^6*d^5-10/f*b/(a-b)/(a+b)/((a-b)*(a+b))^(1
/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*c^4*d-20/f/b^2/(a
^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a
-b)*a^3*c^2*d^3+20/f/b/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2
-tan(1/2*e+1/2*f*x)^2*b-a-b)*a^2*c^3*d^2+10/f/b^3/(a^2-b^2)*tan(1/2*e+1/2*f
*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a^4*c*d^4-60/f/b/(a
-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b)
)^(1/2))*a^2*c^2*d^3-30/f/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)
*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^5*c*d^4+40/f/b^3/(a-b)/(a+b)/((a
-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^4*
c^2*d^3+40/f/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/
2*f*x)/((a-b)*(a+b))^(1/2))*a^3*c*d^4-10/f/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*
tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a*c^4*d+2/f/(a-b)/(a+b)/((
a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*c^5
*a-2/f/b^4/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1
/2*f*x)^2*b-a-b)*a^5*d^5-15/f*d^4/b^4*ln(tan(1/2*e+1/2*f*x)-1)*a^2*c+20/f*d
^3/b^3*ln(tan(1/2*e+1/2*f*x)-1)*a*c^2+10/f*d^4/b^3/(tan(1/2*e+1/2*f*x)-1)*a
*c+15/f*d^4/b^4*ln(tan(1/2*e+1/2*f*x)+1)*a^2*c-20/f*d^3/b^3*ln(tan(1/2*e+1/
2*f*x)+1)*a*c^2+10/f*d^4/b^3/(tan(1/2*e+1/2*f*x)+1)*a*c+2/f*b/(a^2-b^2)*tan
(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*c^5-1/f
*d^5/b^3/(tan(1/2*e+1/2*f*x)-1)^2*a+5/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)-1)^2*
c+1/f*d^5/b^3/(tan(1/2*e+1/2*f*x)+1)^2*a-5/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)+
1)^2*c-3/f*d^5/b^4/(tan(1/2*e+1/2*f*x)+1)*a^2-1/f*d^5/b^3/(tan(1/2*e+1/2*f*
x)+1)*a-10/f*d^3/b^2/(tan(1/2*e+1/2*f*x)+1)*c^2+5/2/f*d^4/b^2/(tan(1/2*e+1/
2*f*x)+1)*c-1/f*d^5/b^3*ln(tan(1/2*e+1/2*f*x)+1)*a+4/f*d^5/b^5*ln(tan(1/2*e
+1/2*f*x)-1)*a^3+1/f*d^5/b^3*ln(tan(1/2*e+1/2*f*x)-1)*a-10/f*d^2/b^2*ln(tan
(1/2*e+1/2*f*x)-1)*c^3-5/2/f*d^4/b^2*ln(tan(1/2*e+1/2*f*x)-1)*c-3/f*d^5/b^4
/(tan(1/2*e+1/2*f*x)-1)*a^2-1/f*d^5/b^3/(tan(1/2*e+1/2*f*x)-1)*a-10/f*d^3/b
^2/(tan(1/2*e+1/2*f*x)-1)*c^2+5/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)-1)*c+10/f*d
^2/b^2*ln(tan(1/2*e+1/2*f*x)+1)*c^3+5/2/f*d^4/b^2*ln(tan(1/2*e+1/2*f*x)+1)*
c-4/f*d^5/b^5*ln(tan(1/2*e+1/2*f*x)+1)*a^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 16.95, size = 17256, normalized size = 45.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (atan(((((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^1
0*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^
10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 -
 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4
+ 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^
4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*
a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9
+ 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3
*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 +
 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 16
0*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*
b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7
*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 -
 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^
5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7
*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8
- 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680
*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*
b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^
3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7
+ 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*
d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d
^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^
4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^
12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2
*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b
^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30
*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^
3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3
+ 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^
17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(b
^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15
*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^
11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*(b^2*(a*d^5 +
20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4
```

```
))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3
*d^2) - 15*a^2*b*c*d^4)*1i)/b^5 + (((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 -
128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a
^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^
4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*
c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b
^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4
 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8
*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a
^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d
^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 +
 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*
a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^
9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d
^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 8
0*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5
*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7
*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 +
 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*
a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^
4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*
d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a
*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8
*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*
c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 -
16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b
^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 2
0*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^
4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^
16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*
d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80
*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12)
 - (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*
c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^
13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9
- a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 1
0*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5
- b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*1i)/b^5)/((16*(256*a^14*
d^15 - 128*a^13*b*d^15 + 20*a^6*b^8*d^15 - 20*a^7*b^7*d^15 + 124*a^8*b^6*d^
15 - 24*a^9*b^5*d^15 + 48*a^10*b^4*d^15 + 192*a^11*b^3*d^15 - 448*a^12*b^2*
d^15 + 125*b^14*c^6*d^9 + 1000*b^14*c^8*d^7 - 250*b^14*c^9*d^6 + 2000*b^14*
c^10*d^5 - 1000*b^14*c^11*d^4 - 600*a*b^13*c^5*d^10 - 125*a*b^13*c^6*d^9 -
6425*a*b^13*c^7*d^8 + 1100*a*b^13*c^8*d^7 - 16200*a*b^13*c^9*d^6 + 8100*a*b
^13*c^10*d^5 - 400*a*b^13*c^11*d^4 + 400*a*b^13*c^12*d^3 - 180*a^5*b^9*c*d^
14 + 180*a^6*b^8*c*d^14 - 1320*a^7*b^7*c*d^14 + 270*a^8*b^6*c*d^14 - 900*a^
```

9*b^5*c*d^14 - 2160*a^10*b^4*c*d^14 + 5280*a^11*b^3*c*d^14 + 1440*a^12*b^2*c*d^14 + 1170*a^2*b^12*c^4*d^11 + 600*a^2*b^12*c^5*d^10 + 17795*a^2*b^12*c^6*d^9 - 1375*a^2*b^12*c^7*d^8 + 57480*a^2*b^12*c^8*d^7 - 29740*a^2*b^12*c^9*d^6 - 400*a^2*b^12*c^10*d^5 - 2010*a^2*b^12*c^11*d^4 - 40*a^2*b^12*c^13*d^2 - 1180*a^3*b^11*c^3*d^12 - 1170*a^3*b^11*c^4*d^11 - 27754*a^3*b^11*c^5*d^10 - 995*a^3*b^11*c^6*d^9 - 117635*a^3*b^11*c^7*d^8 + 66680*a^3*b^11*c^8*d^7 + 17400*a^3*b^11*c^9*d^6 + 2604*a^3*b^11*c^10*d^5 + 400*a^3*b^11*c^11*d^4 + 80*a^3*b^11*c^12*d^3 + 645*a^4*b^10*c^2*d^13 + 1180*a^4*b^10*c^3*d^12 + 26690*a^4*b^10*c^4*d^11 + 4654*a^4*b^10*c^5*d^10 + 153580*a^4*b^10*c^6*d^9 - 103805*a^4*b^10*c^7*d^8 - 79760*a^4*b^10*c^8*d^7 + 5840*a^4*b^10*c^9*d^6 - 1600*a^4*b^10*c^10*d^5 + 340*a^4*b^10*c^11*d^4 - 645*a^5*b^9*c^2*d^13 - 16245*a^5*b^9*c^3*d^12 - 5690*a^5*b^9*c^4*d^11 - 133278*a^5*b^9*c^5*d^10 + 119980*a^5*b^9*c^6*d^9 + 188520*a^5*b^9*c^7*d^8 - 28880*a^5*b^9*c^8*d^7 - 1200*a^5*b^9*c^9*d^6 - 1584*a^5*b^9*c^10*d^5 + 6135*a^6*b^8*c^2*d^13 + 3645*a^6*b^8*c^3*d^12 + 77460*a^6*b^8*c^4*d^11 - 105562*a^6*b^8*c^5*d^10 - 279820*a^6*b^8*c^6*d^9 + 57980*a^6*b^8*c^7*d^8 + 21280*a^6*b^8*c^8*d^7 + 2800*a^6*b^8*c^9*d^6 - 1335*a^7*b^7*c^2*d^13 - 29515*a^7*b^7*c^3*d^12 + 69980*a^7*b^7*c^4*d^11 + 279768*a^7*b^7*c^5*d^10 - 74940*a^7*b^7*c^6*d^9 - 64460*a^7*b^7*c^7*d^8 - 2720*a^7*b^7*c^8*d^7 + 6960*a^8*b^6*c^2*d^13 - 33645*a^8*b^6*c^3*d^12 - 192920*a^8*b^6*c^4*d^11 + 69104*a^8*b^6*c^5*d^10 + 108320*a^8*b^6*c^6*d^9 + 1540*a^8*b^6*c^7*d^8 + 10980*a^9*b^5*c^2*d^13 + 91160*a^9*b^5*c^3*d^12 - 46520*a^9*b^5*c^4*d^11 - 118136*a^9*b^5*c^5*d^10 - 480*a^9*b^5*c^6*d^9 - 28380*a^10*b^4*c^2*d^13 + 22430*a^10*b^4*c^3*d^12 + 87600*a^10*b^4*c^4*d^11 + 64*a^10*b^4*c^5*d^10 - 7320*a^11*b^3*c^2*d^13 - 44220*a^11*b^3*c^3*d^12 + 14640*a^12*b^2*c^2*d^13 - 2880*a^13*b*c*d^14))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^

9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5 - (((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^

3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*2i)/(b^5*f) - ((tan(e/2 + (f*x)/2)^5*(18*b^5*c^5 - 72*a^5*d^5 + 2*b^5*d^5 + 16*a*b^4*d^5 + 12*a^4*b*d^5 - 15*b^5*c*d^4 - 14*a^2*b^3*d^5 + 38*a^3*b^2*d^5 - 60*b^5*c^2*d^3 + 180*a*b^4*c^2*d^3 - 165*a^2*b^3*c*d^4 - 45*a^3*b^2*c*d^4 + 60*a^2*b^3*c^2*d^3 + 180*a^2*b^3*c^3*d^2 - 360*a^3*b^2*c^2*d^3 + 45*a*b^4*c*d^4 - 90*a*b^4*c^4*d + 270*a^4*b*c*d^4))/(3*(a*b^4 - b^5)*(a + b)) - (tan(e/2 + (f*x)/2)^7*(2*b^5*c^5 - 8*a^5*d^5 - 2*b^5*d^5 + 4*a^4*b*d^5 + 5*b^5*c*d^4 - 2*a^2*b^3*d^5 + 6*a^3*b^2*d^5 - 20*b^5*c^2*d^3 + 20*a*b^4*c^2*d^3 - 25*a^2*b^3*c*d^4 - 15*a^3*b^2*c*d^4 + 20*a^2*b^3*c^2*d^3 + 20*a^2*b^3*c^3*d^2 - 40*a^3*b^2*c^2*d^3 + 15*a*b^4*c*d^4 - 10*a*b^4*c^4*d + 30*a^4*b*c*d^4))/((a*b^4 - b^5)*(a + b)) + (tan(e/2 + (f*x)/2)*(2*b^5*c^5 - 8*a^5*d^5 + 2*b^5*d^5 - 4*a^4*b*d^5 + 5*b^5*c*d^4 + 2*a^2*b^3*d^5 + 6*a^3*b^2*d^5 + 20*b^5*c^2*d^3 + 20*a*b^4*c^2*d^3 - 25*a^2*b^3*c*d^4 + 15*a^3*b^2*c*d^4 - 20*a^2*b^3*c^2*d^3 + 20*a^2*b^3*c^3*d^2 - 40*a^3*b^2*c^2*d^3 - 15*a*b^4*c*d^4 - 10*a*b^4*c^4*d + 30*a^4*b*c*d^4))/((a*b^4 - b^5)*(a + b)) + (tan(e/2 + (f*x)/2)^3*(72*a^5*d^5 - 18*b^5*c^5 + 2*b^5*d^5 - 16*a*b^4*d^5 + 12*a^4*b*d^5 + 15*b^5*c*d^4 - 14*a^2*b^3*d^5 - 38*a^3*b^2*d^5 - 60*b^5*c^2*d^3 - 180*a*b^4*c^2*d^3 + 165*a^2*b^3*c*d^4 - 45*a^3*b^2*c*d^4 + 60*a^2*b^3*c^2*d^3 - 180*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 45*a*b^4*c*d^4 + 90*a*b^4*c^4*d - 270*a^4*b*c*d^4))/(3*b^4*(a + b)*(a - b)))/(f*(a + b + tan(e/2 + (f*x)/2)^8*(a - b) - tan(e/2 + (f*x)/2)^2*(4*a + 2*b) - tan(e/2 + (f*x)/2)^6*(4*a - 2*b) + 6*a*tan(e/2 + (f*x)/2)^4)) + (atan((((a*d - b*c)^4*((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 -

800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*
b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^
6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6
 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 560
0*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10
*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^
10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^1
8*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5
 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*
a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a
*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 -
 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c
*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120
*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^
13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/
(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^4
((a + b)^3(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*(8*a*b^15 - 8*a^2*
b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/((a*b^10 + b^1
1 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(a*d - b*
c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2
*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d
 + a*b*c)*1i)/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5) + ((a*d - b*c)^4*((8
*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*
a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6
*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b
^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*
c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 160
0*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*
d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b
^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 96
0*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b
^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*
c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6
 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055
*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b
^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d
^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 -
800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*
b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^
6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6
 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 560
0*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10
*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^
10 + b^11 - a^2*b^9 - a^3*b^8) - (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^1
8*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5

+ 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*
a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a
*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 -
20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c
*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120
*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^
13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/
(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^4
((a + b)^3(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*(8*a*b^15 - 8*a^2*
b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/((a*b^10 + b^1
1 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(a*d - b*
c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2
*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d
+ a*b*c)*1i)/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))/((16*(256*a^14*d^15
- 128*a^13*b*d^15 + 20*a^6*b^8*d^15 - 20*a^7*b^7*d^15 + 124*a^8*b^6*d^15 -
24*a^9*b^5*d^15 + 48*a^10*b^4*d^15 + 192*a^11*b^3*d^15 - 448*a^12*b^2*d^15
+ 125*b^14*c^6*d^9 + 1000*b^14*c^8*d^7 - 250*b^14*c^9*d^6 + 2000*b^14*c^10
*d^5 - 1000*b^14*c^11*d^4 - 600*a*b^13*c^5*d^10 - 125*a*b^13*c^6*d^9 - 6425
*a*b^13*c^7*d^8 + 1100*a*b^13*c^8*d^7 - 16200*a*b^13*c^9*d^6 + 8100*a*b^13*
c^10*d^5 - 400*a*b^13*c^11*d^4 + 400*a*b^13*c^12*d^3 - 180*a^5*b^9*c*d^14 +
180*a^6*b^8*c*d^14 - 1320*a^7*b^7*c*d^14 + 270*a^8*b^6*c*d^14 - 900*a^9*b^
5*c*d^14 - 2160*a^10*b^4*c*d^14 + 5280*a^11*b^3*c*d^14 + 1440*a^12*b^2*c*d^
14 + 1170*a^2*b^12*c^4*d^11 + 600*a^2*b^12*c^5*d^10 + 17795*a^2*b^12*c^6*d^
9 - 1375*a^2*b^12*c^7*d^8 + 57480*a^2*b^12*c^8*d^7 - 29740*a^2*b^12*c^9*d^6
- 400*a^2*b^12*c^10*d^5 - 2010*a^2*b^12*c^11*d^4 - 40*a^2*b^12*c^13*d^2 -
1180*a^3*b^11*c^3*d^12 - 1170*a^3*b^11*c^4*d^11 - 27754*a^3*b^11*c^5*d^10 -
995*a^3*b^11*c^6*d^9 - 117635*a^3*b^11*c^7*d^8 + 66680*a^3*b^11*c^8*d^7 +
17400*a^3*b^11*c^9*d^6 + 2604*a^3*b^11*c^10*d^5 + 400*a^3*b^11*c^11*d^4 + 8
0*a^3*b^11*c^12*d^3 + 645*a^4*b^10*c^2*d^13 + 1180*a^4*b^10*c^3*d^12 + 2669
0*a^4*b^10*c^4*d^11 + 4654*a^4*b^10*c^5*d^10 + 153580*a^4*b^10*c^6*d^9 - 10
3805*a^4*b^10*c^7*d^8 - 79760*a^4*b^10*c^8*d^7 + 5840*a^4*b^10*c^9*d^6 - 16
00*a^4*b^10*c^10*d^5 + 340*a^4*b^10*c^11*d^4 - 645*a^5*b^9*c^2*d^13 - 16245
*a^5*b^9*c^3*d^12 - 5690*a^5*b^9*c^4*d^11 - 133278*a^5*b^9*c^5*d^10 + 11998
0*a^5*b^9*c^6*d^9 + 188520*a^5*b^9*c^7*d^8 - 28880*a^5*b^9*c^8*d^7 - 1200*a
^5*b^9*c^9*d^6 - 1584*a^5*b^9*c^10*d^5 + 6135*a^6*b^8*c^2*d^13 + 3645*a^6*b
^8*c^3*d^12 + 77460*a^6*b^8*c^4*d^11 - 105562*a^6*b^8*c^5*d^10 - 279820*a^6
*b^8*c^6*d^9 + 57980*a^6*b^8*c^7*d^8 + 21280*a^6*b^8*c^8*d^7 + 2800*a^6*b^8
*c^9*d^6 - 1335*a^7*b^7*c^2*d^13 - 29515*a^7*b^7*c^3*d^12 + 69980*a^7*b^7*c
^4*d^11 + 279768*a^7*b^7*c^5*d^10 - 74940*a^7*b^7*c^6*d^9 - 64460*a^7*b^7*c
^7*d^8 - 2720*a^7*b^7*c^8*d^7 + 6960*a^8*b^6*c^2*d^13 - 33645*a^8*b^6*c^3*d
^12 - 192920*a^8*b^6*c^4*d^11 + 69104*a^8*b^6*c^5*d^10 + 108320*a^8*b^6*c^6
*d^9 + 1540*a^8*b^6*c^7*d^8 + 10980*a^9*b^5*c^2*d^13 + 91160*a^9*b^5*c^3*d^
12 - 46520*a^9*b^5*c^4*d^11 - 118136*a^9*b^5*c^5*d^10 - 480*a^9*b^5*c^6*d^9
- 28380*a^10*b^4*c^2*d^13 + 22430*a^10*b^4*c^3*d^12 + 87600*a^10*b^4*c^4*d
^11 + 64*a^10*b^4*c^5*d^10 - 7320*a^11*b^3*c^2*d^13 - 44220*a^11*b^3*c^3*d^

12 + 14640*a^12*b^2*c^2*d^13 - 2880*a^13*b*c*d^14))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + ((a*d - b*c)^4*((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/((a*b^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5) - ((a*d - b*c)^4*((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9

```
 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4
*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 9
60*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*
a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^
9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*
d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4
000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*
b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c
^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 +
 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*
a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^
5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*
d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 -
5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*
c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8*(4*a*
b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 -
4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5
*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^
3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*
b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 14
0*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2
*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 +
200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b
^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (8*
tan(e/2 + (f*x)/2)*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b
^2*d + a*b*c)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^
11 - 8*a^6*b^10))/((a*b^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 +
3*a^4*b^7 - a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d -
 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a
- b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 -
a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a
*b*c)*2i)/(f*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^{5}\sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**5*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

3.259 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=297

$$\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}-\frac{(bc-ad)^4\sin(e+fx)}{b^3f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^3(3ad+bc)\tanh^{-1}}{ab^4f\sqrt{a-b}\sqrt{}}$$

```
[Out] 1/2*d^4*arctanh(sin(f*x+e))/b^2/f+d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*arcta
nh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/
(a+b)^(1/2))/a/(a-b)^(3/2)/b^2/(a+b)^(3/2)/f-(-a*d+b*c)^4*sin(f*x+e)/b^3/(a
^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c)^3*(3*a*d+b*c)*arctanh((a-b)^(1/2)*t
an(1/2*e+1/2*f*x)/(a+b)^(1/2))/a/b^4/f/(a-b)^(1/2)/(a+b)^(1/2)+2*d^3*(-a*d+
2*b*c)*tan(f*x+e)/b^3/f+1/2*d^4*sec(f*x+e)*tan(f*x+e)/b^2/f
```

Rubi [A] time = 0.53, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}-\frac{(bc-ad)^4\sin(e+fx)}{b^3f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2
*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*d)^4*ArcTanh[(Sqrt[a - b
]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b^2*(a + b)^(3/2)*f) + (
2*(b*c - a*d)^3*(b*c + 3*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*b^4*Sqrt[a + b]*f) - ((b*c - a*d)^4*Sin[e + f*x])/(b
^3*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (2*d^3*(2*b*c - a*d)*Tan[e + f*x])
/(b^3*f) + (d^4*Sec[e + f*x]*Tan[e + f*x])/(2*b^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^4\sec^3(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(-\frac{(-bc+ad)^4}{ab^3(b+a\cos(e+fx))^2} - \frac{(-bc+ad)^3(bc+3ad)}{ab^4(b+a\cos(e+fx))} + \frac{d^2\left(6b^2c^2-8a\right.}{}\right.$$

$$= \frac{d^4\int \sec^3(e+fx)\,dx}{b^2} - \frac{(bc-ad)^4\int \frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab^3} + \frac{\left(2d^3(2bc-ad)\right)}{}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} - \frac{(bc-ad)^4\sin(}{b^3\left(a^2-b^2\right)f(b+a}$$

$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+f}{b^4f}$$

$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+f}{b^4f}$$

$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+f}{b^4f}$$

Mathematica [A] time = 4.12, size = 511, normalized size = 1.72

$$\cos^2(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^4\left(-2d^2\left(6a^2d^2-16abcd+b^2\left(12c^2+d^2\right)\right)(a\cos(e+fx)+b)\,\mathrm{l}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((8*(-(b*c) + a
*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) - 2*d^2*(-16*a*b*c*d +
6*a^2*d^2 + b^2*(12*c^2 + d^2))*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(
b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^4*(b
+ a*Cos[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (8*b*d^3*(2*b*
c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - (b^2*d^4*(b + a*Cos[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^2 + (8*b*d^3*(2*b*c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]) + (4*b*(b*c - a*d)^4*Sin[e + f*x])/((-a
 + b)*(a + b))))/(4*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)2/f*((-tan((f*x+exp(1))/2)*c^4*b^4+4*tan((f*x+exp(1))/2)*c^3*a*b^3*
d-6*tan((f*x+exp(1))/2)*c^2*a^2*b^2*d^2+4*tan((f*x+exp(1))/2)*c*a^3*b*d^3-t
an((f*x+exp(1))/2)*a^4*d^4)/(-a^2*b^3+b^5)/(tan((f*x+exp(1))/2)^2*a-tan((f*
x+exp(1))/2)^2*b-a-b)-(8*tan((f*x+exp(1))/2)^3*c*b*d^3-4*tan((f*x+exp(1))/2
)^3*a*d^4-tan((f*x+exp(1))/2)^3*b*d^4-8*tan((f*x+exp(1))/2)*c*b*d^3+4*tan((
f*x+exp(1))/2)*a*d^4-tan((f*x+exp(1))/2)*b*d^4)*1/2/b^3/(tan((f*x+exp(1))/2
)^2-1)^2+(2*c^4*a*b^4-8*c^3*b^5*d-12*c^2*a^3*b^2*d^2+24*c^2*a*b^4*d^2+16*c*
a^4*b*d^3-24*c*a^2*b^3*d^3-6*a^5*d^4+8*a^3*b^2*d^4)*1/2/(-a^2*b^4+b^6)/sqrt
(-a^2+b^2)*(atan((a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(1))/2))/sqrt(-a^2+b^
2))+pi*sign(2*a-2*b)*floor((f*x+exp(1))/2/pi+1/2))+(-12*c^2*b^2*d^2+16*c*a*
b*d^3-6*a^2*d^4-b^2*d^4)*1/4/b^4*ln(abs(tan((f*x+exp(1))/2)-1))-(-12*c^2*b^
2*d^2+16*c*a*b*d^3-6*a^2*d^4-b^2*d^4)*1/4/b^4*ln(abs(tan((f*x+exp(1))/2)+1)
))
```

maple [B] time = 0.68, size = 1249, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x)
```

```
[Out] -8/f/b^2/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2
*f*x)^2*b-a-b)*a^3*c*d^3+1/2/f*d^4/b^2*ln(tan(1/2*e+1/2*f*x)+1)+1/2/f*d^4/b
^2/(tan(1/2*e+1/2*f*x)+1)+1/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)-1)^2-1/2/f*d^4/
b^2*ln(tan(1/2*e+1/2*f*x)-1)+1/2/f*d^4/b^2/(tan(1/2*e+1/2*f*x)-1)-1/2/f*d^4
/b^2/(tan(1/2*e+1/2*f*x)+1)^2+24/f/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(
(a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a*c^2*d^2+12/f/b/(a^2-b^2)*ta
n(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a^2*c^
2*d^2-6/f/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f
*x)/((a-b)*(a+b))^(1/2))*a^5*d^4+8/f/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*ar
ctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^3*d^4-8/f*b/(a-b)/(a+
b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2)
)*c^3*d-8/f/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+
1/2*f*x)^2*b-a-b)*a*c^3*d+2/f/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)
*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*c^4*a+2/f/b^3/(a^2-b^2)*tan(1/2*e+
1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a^4*d^4+2/f*d^
4/b^3/(tan(1/2*e+1/2*f*x)+1)*a-4/f*d^3/b^2/(tan(1/2*e+1/2*f*x)+1)*c-8/f*d^3
/b^3*ln(tan(1/2*e+1/2*f*x)+1)*a*c+2/f*b/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan
(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*c^4+8/f*d^3/b^3*ln(tan(1/2*e+
1/2*f*x)-1)*a*c-3/f*d^4/b^4*ln(tan(1/2*e+1/2*f*x)-1)*a^2-6/f*d^2/b^2*ln(tan
(1/2*e+1/2*f*x)-1)*c^2+2/f*d^4/b^3/(tan(1/2*e+1/2*f*x)-1)*a-4/f*d^3/b^2/(ta
n(1/2*e+1/2*f*x)-1)*c+3/f*d^4/b^4*ln(tan(1/2*e+1/2*f*x)+1)*a^2+6/f*d^2/b^2*
ln(tan(1/2*e+1/2*f*x)+1)*c^2-24/f/b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh
((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^2*c*d^3+16/f/b^3/(a-b)/(a+
b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2)
```

```
)*a^4*c*d^3-12/f/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*
e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^3*c^2*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 14.37, size = 12483, normalized size = 42.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (atan(((((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 +
 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^
4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2
*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b
^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*
a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2
*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*
b^9) - (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c
*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^
6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c^2*d^2) +
 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*
d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*
b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^
8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2
- 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*
d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^
5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a
^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6
*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96
*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6
*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5
 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^
```

6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3
*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*
d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d
^4 - 8*a*b*c*d^3)*1i)/b^4 - (((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3
*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16
*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*
b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*
b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16
*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*
d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b
^12 - a^2*b^10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2)
+ 3*a^2*d^4 - 8*a*b*c*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^
10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*
(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)
*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*
a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4
*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d
^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4
*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b
^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*
a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4
*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576
*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*
c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 +
 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6
*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*
d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9
*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 +
 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3)*1i)/b^4)/((16*(108*a^11*d^12 - 54*a^
10*b*d^12 + 4*a^3*b^8*d^12 - 4*a^4*b^7*d^12 + 41*a^5*b^6*d^12 - 9*a^6*b^5*d
^12 + 63*a^7*b^4*d^12 + 81*a^8*b^3*d^12 - 216*a^9*b^2*d^12 - 4*b^11*c^3*d^9
 - 96*b^11*c^5*d^7 + 32*b^11*c^6*d^6 - 576*b^11*c^7*d^5 + 384*b^11*c^8*d^4
+ 12*a*b^10*c^2*d^10 + 4*a*b^10*c^3*d^9 + 417*a*b^10*c^4*d^8 - 96*a*b^10*c^
5*d^7 + 3288*a*b^10*c^6*d^6 - 2256*a*b^10*c^7*d^5 + 144*a*b^10*c^8*d^4 - 19
2*a*b^10*c^9*d^3 - 12*a^2*b^9*c*d^11 + 12*a^3*b^8*c*d^11 - 252*a^4*b^7*c*d^
11 + 60*a^5*b^6*c*d^11 - 744*a^6*b^5*c*d^11 - 648*a^7*b^4*c*d^11 + 1872*a^8
*b^3*c*d^11 + 432*a^9*b^2*c*d^11 - 12*a^2*b^9*c^2*d^10 - 716*a^2*b^9*c^3*d^
9 + 63*a^2*b^9*c^4*d^8 - 7872*a^2*b^9*c^5*d^7 + 5784*a^2*b^9*c^6*d^6 + 192*
a^2*b^9*c^7*d^5 + 690*a^2*b^9*c^8*d^4 + 24*a^2*b^9*c^10*d^2 + 606*a^3*b^8*c
^2*d^10 + 76*a^3*b^8*c^3*d^9 + 10203*a^3*b^8*c^4*d^8 - 8592*a^3*b^8*c^5*d^7
 - 3752*a^3*b^8*c^6*d^6 - 480*a^3*b^8*c^7*d^5 - 144*a^3*b^8*c^8*d^4 - 32*a^
3*b^8*c^9*d^3 - 126*a^4*b^7*c^2*d^10 - 7680*a^4*b^7*c^3*d^9 + 8277*a^4*b^7*
c^4*d^8 + 11232*a^4*b^7*c^5*d^7 - 1552*a^4*b^7*c^6*d^6 + 384*a^4*b^7*c^7*d^
5 - 132*a^4*b^7*c^8*d^4 + 3318*a^5*b^6*c^2*d^10 - 5424*a^5*b^6*c^3*d^9 - 16
488*a^5*b^6*c^4*d^8 + 4128*a^5*b^6*c^5*d^7 + 464*a^5*b^6*c^6*d^6 + 384*a^5*

b^6*c^7*d^5 + 2394*a^6*b^5*c^2*d^10 + 13904*a^6*b^5*c^3*d^9 - 4860*a^6*b^5*
c^4*d^8 - 3264*a^6*b^5*c^5*d^7 - 400*a^6*b^5*c^6*d^6 - 6888*a^7*b^4*c^2*d^1
0 + 3472*a^7*b^4*c^3*d^9 + 5868*a^7*b^4*c^4*d^8 + 192*a^7*b^4*c^5*d^7 - 158
4*a^8*b^3*c^2*d^10 - 5504*a^8*b^3*c^3*d^9 - 36*a^8*b^3*c^4*d^8 + 2952*a^9*b
^2*c^2*d^10 - 864*a^10*b*c*d^11))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + ((
(((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^
12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^
4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48
*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3
 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c
*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32
*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8
*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3)*(8*
a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(
b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^
4 - 8*a*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a
*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 +
 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a
^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^
9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*
a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 -
 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^
2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 70
4*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*
c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4
- 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^
5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3
*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 80
0*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*
b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*
b*c*d^3))/b^4 + (((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b
^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^
4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24
*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d +
 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*
d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5
*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^
10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4
- 8*a*b*c*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b
^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c
^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)*(72*a^10*d^
8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8
- 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a
^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10
*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a

*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 1
60*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^
7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^
2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4
*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 94
4*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^
5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6
 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^
7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384
*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2)
 + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*
a*b*c*d^3)*2i)/(b^4*f) - ((tan(e/2 + (f*x)/2)^5*(6*a^4*d^4 + 2*b^4*c^4 + b^
4*d^4 + 3*a*b^3*d^4 - 3*a^3*b*d^4 - 8*b^4*c*d^3 - 5*a^2*b^2*d^4 + 8*a^2*b^2
*c*d^3 + 12*a^2*b^2*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 16*a^3*b*c*d^
3))/((a*b^3 - b^4)*(a + b)) + (tan(e/2 + (f*x)/2)*(6*a^4*d^4 + 2*b^4*c^4 +
b^4*d^4 - 3*a*b^3*d^4 + 3*a^3*b*d^4 + 8*b^4*c*d^3 - 5*a^2*b^2*d^4 - 8*a^2*b
^2*c*d^3 + 12*a^2*b^2*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 16*a^3*b*c*
d^3))/(b^3*(a + b)*(a - b)) - (2*tan(e/2 + (f*x)/2)^3*(6*a^4*d^4 + 2*b^4*c^
4 - b^4*d^4 - 3*a^2*b^2*d^4 + 12*a^2*b^2*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*
c^3*d - 16*a^3*b*c*d^3))/(b*(a*b^2 - b^3)*(a + b)))/(f*(a + b - tan(e/2 + (
f*x)/2)^2*(3*a + b) - tan(e/2 + (f*x)/2)^6*(a - b) + tan(e/2 + (f*x)/2)^4*(
3*a - b))) - (atan((((a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2
 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8
*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 +
17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*
b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288
*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 +
 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*
d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a
^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3
*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 153
6*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5
*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3
 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^
7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7
 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((
(8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12
*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4
 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a
*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 +
 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d
^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a
*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*t
an(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^
2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9

- 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^
2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d
+ a*b*c)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((a*d - b*c)^3*((a
+ b)^3*(a - b)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*
a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8
+ 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*
a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b
^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64
*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7
- 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c
^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 7
04*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7
*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4
- 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a
^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^
3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 8
00*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a
*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15
*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4
- 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*
a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*
a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3
- 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*
c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11
+ b^12 - a^2*b^10 - a^3*b^9) - (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b
)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 1
6*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7
- a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b
)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*
b^6 - a^6*b^4))/((16*(108*a^11*d^12 - 54*a^10*b*d^12 + 4*a^3*b^8*d^12 - 4*a
^4*b^7*d^12 + 41*a^5*b^6*d^12 - 9*a^6*b^5*d^12 + 63*a^7*b^4*d^12 + 81*a^8*b
^3*d^12 - 216*a^9*b^2*d^12 - 4*b^11*c^3*d^9 - 96*b^11*c^5*d^7 + 32*b^11*c^6
*d^6 - 576*b^11*c^7*d^5 + 384*b^11*c^8*d^4 + 12*a*b^10*c^2*d^10 + 4*a*b^10*
c^3*d^9 + 417*a*b^10*c^4*d^8 - 96*a*b^10*c^5*d^7 + 3288*a*b^10*c^6*d^6 - 22
56*a*b^10*c^7*d^5 + 144*a*b^10*c^8*d^4 - 192*a*b^10*c^9*d^3 - 12*a^2*b^9*c*
d^11 + 12*a^3*b^8*c*d^11 - 252*a^4*b^7*c*d^11 + 60*a^5*b^6*c*d^11 - 744*a^6
*b^5*c*d^11 - 648*a^7*b^4*c*d^11 + 1872*a^8*b^3*c*d^11 + 432*a^9*b^2*c*d^11
- 12*a^2*b^9*c^2*d^10 - 716*a^2*b^9*c^3*d^9 + 63*a^2*b^9*c^4*d^8 - 7872*a^
2*b^9*c^5*d^7 + 5784*a^2*b^9*c^6*d^6 + 192*a^2*b^9*c^7*d^5 + 690*a^2*b^9*c^
8*d^4 + 24*a^2*b^9*c^10*d^2 + 606*a^3*b^8*c^2*d^10 + 76*a^3*b^8*c^3*d^9 + 1
0203*a^3*b^8*c^4*d^8 - 8592*a^3*b^8*c^5*d^7 - 3752*a^3*b^8*c^6*d^6 - 480*a^
3*b^8*c^7*d^5 - 144*a^3*b^8*c^8*d^4 - 32*a^3*b^8*c^9*d^3 - 126*a^4*b^7*c^2*
d^10 - 7680*a^4*b^7*c^3*d^9 + 8277*a^4*b^7*c^4*d^8 + 11232*a^4*b^7*c^5*d^7

- 1552*a^4*b^7*c^6*d^6 + 384*a^4*b^7*c^7*d^5 - 132*a^4*b^7*c^8*d^4 + 3318*a^5*b^6*c^2*d^10 - 5424*a^5*b^6*c^3*d^9 - 16488*a^5*b^6*c^4*d^8 + 4128*a^5*b^6*c^5*d^7 + 464*a^5*b^6*c^6*d^6 + 384*a^5*b^6*c^7*d^5 + 2394*a^6*b^5*c^2*d^10 + 13904*a^6*b^5*c^3*d^9 - 4860*a^6*b^5*c^4*d^8 - 3264*a^6*b^5*c^5*d^7 - 400*a^6*b^5*c^6*d^6 - 6888*a^7*b^4*c^2*d^10 + 3472*a^7*b^4*c^3*d^9 + 5868*a^7*b^4*c^4*d^8 + 192*a^7*b^4*c^5*d^7 - 1584*a^8*b^3*c^2*d^10 - 5504*a^8*b^3*c^3*d^9 - 36*a^8*b^3*c^4*d^8 + 2952*a^9*b^2*c^2*d^10 - 864*a^10*b*c*d^11))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + ((a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) - ((a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 9

```
44*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b
^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^
6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a
^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 38
4*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*b^15*d^4 - 4*a*
b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4
 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6
*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^
2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d -
80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*
d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^
14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*tan(e/2 + (f*x)/2)*(a*
d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*(8*a*b^1
3 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^
8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*
d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 -
 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a
^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(
3*a^2*d - 4*b^2*d + a*b*c)*2i)/(f*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d \sec\left(e + fx\right)\right)^4 \sec\left(e + fx\right)}{\left(a + b \sec\left(e + fx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

3.260 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=228

$$-\frac{(bc-ad)^3\sin(e+fx)}{b^2f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(}{\sqrt{a+}}\right.}{ab^3f\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*arctanh(sin(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(
1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/a/(a-b)^(3/2)/b/(a+b)^(3/2)/f-(-a*d+b*
c)^3*sin(f*x+e)/b^2/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c)^2*(2*a*d+b*c)
*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/a/b^3/f/(a-b)^(1/2)/(a
+b)^(1/2)+d^3*tan(f*x+e)/b^2/f
```

Rubi [A] time = 0.47, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770, 3767, 8}

$$-\frac{(bc-ad)^3\sin(e+fx)}{b^2f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(}{\sqrt{a+}}\right.}{ab^3f\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcT
anh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b*(a + b)
^(3/2)*f) + (2*(b*c - a*d)^2*(b*c + 2*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*
x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^3*Sqrt[a + b]*f) - ((b*c - a*d)^3*Sin
[e + f*x])/(b^2*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (d^3*Tan[e + f*x])/(b
^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^3\sec^2(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(\frac{(-bc+ad)^3}{ab^2(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^2(bc+2ad)}{ab^3(b+a\cos(e+fx))} + \frac{d^2(3bc-2ad)\,\mathrm{s}}{b^3} \ldots\right.$$

$$= \frac{d^3\int \sec^2(e+fx)\,dx}{b^2} + \frac{\left(d^2(3bc-2ad)\right)\int \sec(e+fx)\,dx}{b^3} - \frac{(bc-ad)^3\int \ldots}{\ldots}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3 f} - \frac{(bc-ad)^3\sin(e+fx)}{b^2\left(a^2-b^2\right)f(b+a\cos(e+fx))} \ldots$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3 f} + \frac{2(bc-ad)^2(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a-\ldots}}{\ldots}\right.}{a\sqrt{a-b}\,b^3\sqrt{a+b}\,f \ldots}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3 f} + \frac{2(bc-ad)^2(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a-\ldots}}{\ldots}\right.}{a\sqrt{a-b}\,b^3\sqrt{a+b}\,f \ldots}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3 f} + \frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+f\ldots\right.}{\sqrt{a+b}}\right.}{a(a-b)^{3/2}b(a+b)^{3/2}f}$$

Mathematica [A] time = 1.80, size = 362, normalized size = 1.59

$$\frac{\cos(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^3\left(-\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)(a\cos(e+fx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + d^2 \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((-2*(b*c - a*d)^
2*(a*b*c + 2*a^2*d - 3*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + d^2*(-3*b*c + 2*a*d)*(b +
```

```
a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + d^2*(3*b*c - 2*
a*d)*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*d^3
*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]) + (b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]) + (b*(b*c - a*d)^3*Sin[e + f*x])/((-a + b)*(a + b))))/(b^3*f*
(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^2)
```

fricas [B] time = 96.70, size = 1326, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5
- 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 -
2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(
b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b
*cos(f*x + e) + b^2)) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*
a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d
^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1)
 - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^
3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a^4*b^2 - 2*a
^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a
^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 + (a^4*b^4 -
2*a^2*b^6 + b^8)*f*cos(f*x + e)), 1/2*(2*((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*
(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5 - 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^
4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d
^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e))) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 -
 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^
4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f
*x + e) + 1) - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
 a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a
^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a
^4*b^2 - 2*a^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*
c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*co
s(f*x + e))*sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 +
 (a^4*b^4 - 2*a^2*b^6 + b^8)*f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*((-3*c*b*d^2+2*a*d^3)*1/2/b^3*ln(abs(tan((f*x+exp(1))/2)-1))+(3
*c*b*d^2-2*a*d^3)*1/2/b^3*ln(abs(tan((f*x+exp(1))/2)+1))+(-2*c^3*a*b^3+6*c^
2*b^4*d+6*c*a^3*b*d^2-12*c*a*b^3*d^2-4*a^4*d^3+6*a^2*b^2*d^3)*1/2/(a^2*b^3-
b^5)/sqrt(-a^2+b^2)*(atan((a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(1))/2))/sqr
t(-a^2+b^2))+pi*sign(2*a-2*b)*floor((f*x+exp(1))/2/pi+1/2))+(tan((f*x+exp(1
))/2)^3*c^3*b^3-3*tan((f*x+exp(1))/2)^3*c^2*a*b^2*d+3*tan((f*x+exp(1))/2)^3
*c*a^2*b*d^2-2*tan((f*x+exp(1))/2)^3*a^3*d^3+tan((f*x+exp(1))/2)^3*a^2*b*d^
3+tan((f*x+exp(1))/2)^3*a*b^2*d^3-tan((f*x+exp(1))/2)^3*b^3*d^3-tan((f*x+ex
p(1))/2)*c^3*b^3+3*tan((f*x+exp(1))/2)*c^2*a*b^2*d-3*tan((f*x+exp(1))/2)*c*
a^2*b*d^2+2*tan((f*x+exp(1))/2)*a^3*d^3+tan((f*x+exp(1))/2)*a^2*b*d^3-tan((
f*x+exp(1))/2)*a*b^2*d^3-tan((f*x+exp(1))/2)*b^3*d^3)/(a^2*b^2-b^4)/(tan((f
*x+exp(1))/2)^4*a-tan((f*x+exp(1))/2)^4*b-2*tan((f*x+exp(1))/2)^2*a+a+b))
```

maple [B] time = 0.60, size = 790, normalized size = 3.46

$$-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)a^{3}d^{3}}{f\,b^{2}\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)b-a-b\right)}+\frac{6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)a^{2}c\,d^{2}}{fb\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\left(\tan^{2}\left(\frac{e}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x)
```

```
[Out] -2/f/b^2/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2
*f*x)^2*b-a-b)*a^3*d^3+6/f/b/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*
f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a^2*c*d^2-6/f/(a^2-b^2)*tan(1/2*e+1/2*f*
x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a*c^2*d+2/f*b/(a^2-b
^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*
c^3+4/f/b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x
)/((a-b)*(a+b))^(1/2))*a^4*d^3-6/f/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arct
anh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^3*c*d^2-6/f/b/(a-b)/(a+
b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2)
)*a^2*d^3+2/f/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f
```

```
*x)/((a-b)*(a+b))^(1/2))*c^3*a+12/f/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh
((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a*c*d^2-6/f*b/(a-b)/(a+b)/((
a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*c^2
*d-1/f*d^3/b^2/(tan(1/2*e+1/2*f*x)-1)+2/f*d^3/b^3*ln(tan(1/2*e+1/2*f*x)-1)*
a-3/f*d^2/b^2*ln(tan(1/2*e+1/2*f*x)-1)*c-1/f*d^3/b^2/(tan(1/2*e+1/2*f*x)+1)
-2/f*d^3/b^3*ln(tan(1/2*e+1/2*f*x)+1)*a+3/f*d^2/b^2*ln(tan(1/2*e+1/2*f*x)+1
)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 11.30, size = 7958, normalized size = 34.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*
c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a
^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^
3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3
*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^
3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2
 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c
*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) +
 (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b
^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^
4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2
*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 +
 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*tan(e/2 + (f*
x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^
5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c
))/b^3)*(2*a*d - 3*b*c)*1i)/b^3 + (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 -
 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6
```

+ 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c)*1i)/b^3)/((64*(8*a^8*d^9 - 4*a^7*b*d^9 + 12*a^4*b^4*d^9 + 6*a^5*b^3*d^9 - 20*a^6*b^2*d^9 + 27*b^8*c^4*d^5 - 27*b^8*c^5*d^4 - 90*a*b^7*c^3*d^6 + 99*a*b^7*c^4*d^5 - 9*a*b^7*c^5*d^4 + 18*a*b^7*c^6*d^3 - 60*a^3*b^5*c*d^8 - 39*a^4*b^4*c*d^8 + 96*a^5*b^3*c*d^8 + 24*a^6*b^2*c*d^8 + 111*a^2*b^6*c^2*d^7 - 144*a^2*b^6*c^3*d^6 - 15*a^2*b^6*c^4*d^5 - 39*a^2*b^6*c^5*d^4 - 3*a^2*b^6*c^7*d^2 + 105*a^3*b^5*c^2*d^7 + 113*a^3*b^5*c^3*d^6 + 3*a^3*b^5*c^4*d^5 + 9*a^3*b^5*c^5*d^4 + 2*a^3*b^5*c^6*d^3 - 165*a^4*b^4*c^2*d^7 + 55*a^4*b^4*c^3*d^6 - 12*a^4*b^4*c^4*d^5 + 9*a^4*b^4*c^5*d^4 - 57*a^5*b^3*c^2*d^7 - 23*a^5*b^3*c^3*d^6 - 12*a^5*b^3*c^4*d^5 + 54*a^6*b^2*c^2*d^7 + 4*a^6*b^2*c^3*d^6 - 36*a^7*b*c*d^8))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c))/b^3 - (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3

+ 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a *b^6 + b^7 - a^2*b^5 - a^3*b^4) - (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3* b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^ 2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3 *a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^ 10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^ 2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c))/b^3))*(2*a*d - 3* b*c)*2i)/(b^3*f) - ((2*tan(e/2 + (f*x)/2)*(b^3*c^3 - 2*a^3*d^3 + b^3*d^3 + a*b^2*d^3 - a^2*b*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2*(a + b)*(a - b)) - (2*tan(e/2 + (f*x)/2)^3*(b^3*c^3 - 2*a^3*d^3 - b^3*d^3 + a*b^2*d^3 + a ^2*b*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2*(a + b)*(a - b)))/(f*(a + b + tan(e/2 + (f*x)/2)^4*(a - b) - 2*a*tan(e/2 + (f*x)/2)^2)) + (atan(((((32 *tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c ^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c *d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2* c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a ^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^ 4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3 *a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a + b) ^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4* a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3 *b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a* b*c)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (((32*tan(e/2 + (f*x)/2) *(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5 *a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4* d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c *d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6* c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57 *a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d ^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5)) /(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b ^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2 *b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3 *a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3* a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 -

```
a^3*b^6) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)
*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8
 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*
b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a
^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(a*d - b*
c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*1i)/(b^9 - 3*a
^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*a^8*d^9 - 4*a^7*b*d^9 + 12*a^4*b^4*d
^9 + 6*a^5*b^3*d^9 - 20*a^6*b^2*d^9 + 27*b^8*c^4*d^5 - 27*b^8*c^5*d^4 - 90*
a*b^7*c^3*d^6 + 99*a*b^7*c^4*d^5 - 9*a*b^7*c^5*d^4 + 18*a*b^7*c^6*d^3 - 60*
a^3*b^5*c*d^8 - 39*a^4*b^4*c*d^8 + 96*a^5*b^3*c*d^8 + 24*a^6*b^2*c*d^8 + 11
1*a^2*b^6*c^2*d^7 - 144*a^2*b^6*c^3*d^6 - 15*a^2*b^6*c^4*d^5 - 39*a^2*b^6*c
^5*d^4 - 3*a^2*b^6*c^7*d^2 + 105*a^3*b^5*c^2*d^7 + 113*a^3*b^5*c^3*d^6 + 3*
a^3*b^5*c^4*d^5 + 9*a^3*b^5*c^5*d^4 + 2*a^3*b^5*c^6*d^3 - 165*a^4*b^4*c^2*d
^7 + 55*a^4*b^4*c^3*d^6 - 12*a^4*b^4*c^4*d^5 + 9*a^4*b^4*c^5*d^4 - 57*a^5*b
^3*c^2*d^7 - 23*a^5*b^3*c^3*d^6 - 12*a^5*b^3*c^4*d^5 + 54*a^6*b^2*c^2*d^7 +
 4*a^6*b^2*c^3*d^6 - 36*a^7*b*c*d^8))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (
((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*
d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b
^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b
^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*
b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 +
12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^
2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c
^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (((32*(a*b^11*c
^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^
3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*
d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2
 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a
*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a
+ b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10
- 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 -
 a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^
3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^
5 - a^6*b^3))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d
+ a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (((32*tan(e/2 + (f*x)/2
)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 +
5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4
*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*
c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6
*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 5
7*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*
d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5)
)/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*
b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^
2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 +
```

```
3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3
*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 -
 a^3*b^6) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2
)*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^
8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2
*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*
a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(a*d - b
*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2
*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*
a^2*d - 3*b^2*d + a*b*c)*2i)/(f*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^{3}\sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

3.261 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\,\sqrt{a+b}}-\frac{(bc-ad)^2\sin(e+fx)}{bf\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right.}{af(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] d^2*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)*tan(1/2*e+
1/2*f*x)/(a+b)^(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)/f-(-a*d+b*c)^2*sin(f*x+e)/b
/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a^2*d^2+b^2*c^2)*arctanh((a-b)^(1/2)*tan(
1/2*e+1/2*f*x)/(a+b)^(1/2))/a/b^2/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770}

$$\frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\,\sqrt{a+b}}-\frac{(bc-ad)^2\sin(e+fx)}{bf\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right.}{af(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[(Sqrt[a - b]
*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*f) + (2*(b^
2*c^2 - a^2*d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*b^2*Sqrt[a + b]*f) - ((b*c - a*d)^2*Sin[e + f*x])/(b*(a^2 - b^2)*
f*(b + a*Cos[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^2\sec(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(-\frac{(-bc+ad)^2}{ab(b+a\cos(e+fx))^2} + \frac{b^2c^2-a^2d^2}{ab^2(b+a\cos(e+fx))} + \frac{d^2\sec(e+fx)}{b^2}\right.$$

$$= \frac{d^2\int\sec(e+fx)\,dx}{b^2} - \frac{(bc-ad)^2\int\frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab} + \frac{\left(b^2c^2-a^2d^2\right)\int}{ab^2}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} - \frac{(bc-ad)^2\sin(e+fx)}{b\left(a^2-b^2\right)f(b+a\cos(e+fx))} + \frac{(bc-ad)^2}{ab}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,b^2\sqrt{a+b}\,f} -$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,b^2\sqrt{a+b}\,f} -$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}f} + \frac{2\left(b^2\right.}{}$$

Mathematica [A] time = 0.72, size = 180, normalized size = 0.91

$$\frac{\frac{2\left(a^3d^2-ab^2\left(c^2+2d^2\right)+2b^3cd\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b(bc-ad)^2\sin(e+fx)}{(b-a)(a+b)(a\cos(e+fx)+b)} + d^2\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTanh[((-a + b)*Tan[(e +
f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - d^2*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] + d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*(b*c -
 a*d)^2*Sin[e + f*x])/((-a + b)*(a + b)*(b + a*Cos[e + f*x])))/(b^2*f)
```

fricas [B] time = 9.54, size = 798, normalized size = 4.03

$$\left[-\frac{\left(ab^3c^2-2\,b^4cd-\left(a^3b-2\,ab^3\right)d^2+\left(a^2b^2c^2-2\,ab^3cd-\left(a^4-2\,a^2b^2\right)d^2\right)\cos\left(fx+e\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos\left(fx+\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*((a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c^2 - 2*a*
b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*c
os(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x +
 e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x +
e) + b^2)) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b
^3 + b^5)*d^2)*log(sin(f*x + e) + 1) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f
*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) + 2*((a^2*b
^3 - b^5)*c^2 - 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x +
e))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 +
b^7)*f), 1/2*(2*(a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c
^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + (
(a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)
*log(sin(f*x + e) + 1) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4
*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) - 2*((a^2*b^3 - b^5)*c^2
- 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x + e))/((a^5*b^2
- 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 + b^7)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(-d^2*1/2/b^2*ln(abs(tan((f*x+exp(1))/2)-1))+d^2*1/2/b^2*ln(abs
(tan((f*x+exp(1))/2)+1))+(2*c^2*a*b^2-4*c*b^3*d-2*a^3*d^2+4*a*b^2*d^2)*1/2/
(-a^2*b^2+b^4)/sqrt(-a^2+b^2)*(atan((a*tan((f*x+exp(1))/2)-b*tan((f*x+exp(1
))/2))/sqrt(-a^2+b^2))+pi*sign(2*a-2*b)*floor((f*x+exp(1))/2/pi+1/2))+(-tan
```

```
((f*x+exp(1))/2)*c^2*b^2+2*tan((f*x+exp(1))/2)*c*a*b*d-tan((f*x+exp(1))/2)*
a^2*d^2)/(-a^2*b+b^3)/(tan((f*x+exp(1))/2)^2*a-tan((f*x+exp(1))/2)^2*b-a-b)
)
```

maple [B] time = 0.84, size = 486, normalized size = 2.45

$$\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)a^2d^2}{fb\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)b-a-b\right)}-\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)acd}{f\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x)
```

```
[Out] 2/f/b/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*
x)^2*b-a-b)*a^2*d^2-4/f/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^
2-tan(1/2*e+1/2*f*x)^2*b-a-b)*a*c*d+2/f*b/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*t
an(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)*c^2-2/f/b^2/(a-b)/(a+b)/((a
-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a^3*
d^2+2/f/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((
a-b)*(a+b))^(1/2))*c^2*a+4/f/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*
tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*a*d^2-4/f*b/(a-b)/(a+b)/((a-b)*(a+b
))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2))*c*d-1/f*d^2/
b^2*ln(tan(1/2*e+1/2*f*x)-1)+1/f*d^2/b^2*ln(tan(1/2*e+1/2*f*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.75, size = 4926, normalized size = 24.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] - (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^
4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d
^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^
2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (d^
2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4
*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*
c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (
32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*
a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2)*1i)/b^
2 + (d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5
*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^
6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b
^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (d^2*((32*(a
*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2
+ a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a
^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*d^2*ta
n(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 -
 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2)*1i)/b^2)/((64*(
a^5*d^6 + 2*a*b^4*d^6 - a^4*b*d^6 - 2*b^5*c*d^5 + 2*a^2*b^3*d^6 - 3*a^3*b^2
*d^6 + 4*b^5*c^2*d^4 + a*b^4*c^2*d^4 - 4*a*b^4*c^3*d^3 + 2*a^2*b^3*c*d^5 +
2*a^3*b^2*c*d^5 - a^4*b*c^2*d^4 + 3*a^2*b^3*c^2*d^4 + a^2*b^3*c^4*d^2 - a^3
*b^2*c^2*d^4 - 6*a*b^4*c*d^5))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (d^2*((3
2*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2
*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 +
4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4
*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (d^2*((32*(a*b^8*c^2 - b
^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^
2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d -
 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*d^2*tan(e/2 + (f*x
)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))
/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2))/b^2 + (d^2*((32*tan(e/2 +
(f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3
*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*
d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d
))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (d^2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a
*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^
6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c
*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^
9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4
 + b^5 - a^2*b^3 - a^3*b^2))))/b^2))/b^2))*2i)/(b^2*f) - (2*tan(e/2 + (f*x)
/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(f*(a + b)*(a*b - b^2)*(a + b - tan(e/
2 + (f*x)/2)^2*(a - b))) - (atan(((((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6
*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*
d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 -
 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3
```

```
- a^3*b^2) + ((a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^
9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2
 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d -
2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(e/2 + (f*x)/2)*
(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9
- 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5
 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*
b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a +
b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^
4*b^4 - a^6*b^2) + (((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*
d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2
*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*
d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (
(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8
*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^
2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))
/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a
 + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4
*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^
3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))
/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)
^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2
))/((64*(a^5*d^6 + 2*a*b^4*d^6 - a^4*b*d^6 - 2*b^5*c*d^5 + 2*a^2*b^3*d^6 -
3*a^3*b^2*d^6 + 4*b^5*c^2*d^4 + a*b^4*c^2*d^4 - 4*a*b^4*c^3*d^3 + 2*a^2*b^3
*c*d^5 + 2*a^3*b^2*c*d^5 - a^4*b*c^2*d^4 + 3*a^2*b^3*c^2*d^4 + a^2*b^3*c^4*
d^2 - a^3*b^2*c^2*d^4 - 6*a*b^4*c*d^5))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) -
 (((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4
+ a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d
^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^
3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + ((a*d - b*c)*((a +
b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2
 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 -
2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^
2*b^4 - a^3*b^3) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a + b)^3*(a - b)^3)
^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b
^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^
2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6
+ 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*
b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(e/2 + (
f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*
a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d
^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d)
)/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - ((a*d - b*c)*((a + b)^3*(a - b)^3)^(1
/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a
^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^
```

```
8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) -
 (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*
b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2
*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4
- a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*
b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c))/(b
^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a*d - b*c)*((a + b)^3*(a - b)^3)^(
1/2)*(a^2*d - 2*b^2*d + a*b*c)*2i)/(f*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^2 \sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

3.262 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/2))/(a-b)^(3/2)
/(a+b)^(3/2)/f-(-a*d+b*c)*tan(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))
```

Rubi [A] time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*f) - ((b*c - a*d)*Tan[e + f*x])/((a^2 - b^2)*f*(a + b
*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))} + \frac{\int \frac{(-ac+bd)\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{-a^2+b^2}$$

$$= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))} + \frac{(ac-bd)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{a^2-b^2}$$

$$= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))} + \frac{(ac-bd)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))} + \frac{(2(ac-bd))\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x,\right.}{b\left(a^2-b^2\right)f}$$

$$= \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f} - \frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))}$$

Mathematica [A] time = 0.40, size = 97, normalized size = 0.97

$$\frac{\frac{(ad-bc)\sin(e+fx)}{(a-b)(a+b)(a\cos(e+fx)+b)} - \frac{2(ac-bd)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(b*c) + a*d)*Sin[e + f*x])/((a - b)*(a + b)*(b + a*Cos[e
 + f*x])))/f
```

fricas [A] time = 0.51, size = 394, normalized size = 3.94

$$\left[\frac{\left(abc-b^2d+\left(a^2c-abd\right)\cos\left(fx+e\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos\left(fx+e\right)-\left(a^2-2\,b^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{a^2-b^2}\left(b\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right.}{a^2\cos\left(fx+e\right)^2+2\,ab\cos\left(fx+e\right)+b^2}\right.}{2\left(\left(a^5-2\,a^3b^2+ab^4\right)f\cos\left(fx+e\right)+\left(a^4b-2\,a^2b^3+b^5\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*b*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2
*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(
f*x + e) + b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^
5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f), ((a*b
*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) - ((a^2*b - b^3)
*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x +
e) + (a^4*b - 2*a^2*b^3 + b^5)*f)]
```

giac [A] time = 1.57, size = 180, normalized size = 1.80

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-a^2+b^2}}\right)\right)\left(ac-bd\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a-b\right)\left(a^2-b^2\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(a*c - b*d)/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) - (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
```

```
*e))/((a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)*(a^2 -
b^2)))/f
```

maple [A] time = 0.63, size = 132, normalized size = 1.32

$$\frac{-\frac{2(da-cb)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)b-a-b\right)}+\frac{2(ca-bd)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(-2*(a*d-b*c)/(a^2-b^2)*tan(1/2*e+1/2*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(
1/2*e+1/2*f*x)^2*b-a-b)+2*(a*c-b*d)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh
((a-b)*tan(1/2*e+1/2*f*x)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.20, size = 106, normalized size = 1.06

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)(a\,c-b\,d)}{f\,(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(a\,d-b\,c)}{f\,(a+b)\,(a-b)\left((b-a)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (2*atanh((tan(e/2 + (f*x)/2)*(a - b)^(1/2))/(a + b)^(1/2))*(a*c - b*d))/(f*
(a + b)^(3/2)*(a - b)^(3/2)) + (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(a + b
)*(a - b)*(a + b - tan(e/2 + (f*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

3.263 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=186

$$-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}+\frac{2b\left(-2a^2d+abc+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2}+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\mathrm{ta}}{}\right.}{f\sqrt{c-d}\sqrt{c+d}\,(}$$

```
[Out] 2*b*(-2*a^2*d+a*b*c+b^2*d)*arctanh((a-b)^(1/2)*tan(1/2*e+1/2*f*x)/(a+b)^(1/
2))/(a-b)^(3/2)/(a+b)^(3/2)/(-a*d+b*c)^2/f-b^2*sin(f*x+e)/(a^2-b^2)/(-a*d+b
*c)/f/(b+a*cos(f*x+e))+2*d^2*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(
1/2))/(-a*d+b*c)^2/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3988, 3056, 3001, 2659, 208}

$$-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}+\frac{2b\left(-2a^2d+abc+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2}+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\mathrm{ta}}{}\right.}{f\sqrt{c-d}\sqrt{c+d}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[
a + b]])/((a - b)^(3/2)*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTanh[(Sq
rt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a
*d)^2*f) - (b^2*Sin[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*(b + a*Cos[e + f*x
]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))^2(d+c\cos(e+fx))}\,dx$$

$$= -\frac{b^2\sin(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(b+a\cos(e+fx))} - \frac{\int \frac{-abd-\left(abc-a^2d+b^2d\right)\cos(}{(b+a\cos(e+fx))(d+c\cos(}}{\left(a^2-b^2\right)(bc-a}$$

$$= -\frac{b^2\sin(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(b+a\cos(e+fx))} + \frac{d^2\int \frac{1}{d+c\cos(e+fx)}\,dx}{(bc-ad)^2} +$$

$$= -\frac{b^2\sin(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(b+a\cos(e+fx))} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{c+d+(-}\right.}{(b}$$

$$= \frac{2b\left(abc-2a^2d+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f} + \frac{2d^2\tanh^{-1}\left(\frac{\sqrt{}}{-}\right.}{\sqrt{c-d}\,\sqrt{c+}}$$

Mathematica [A] time = 1.06, size = 176, normalized size = 0.95

$$\frac{\frac{2d^2\left(a^2-b^2\right)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{2b\left(-2a^2d+abc+b^2d\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b^2(bc-ad)\sin(e+fx)}{a\cos(e+fx)+b}}{f(b-a)(a+b)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + (2*(a^2 - b^2)*d^2*ArcTanh[((-c + d)*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (b^2*(b*c - a*d)*Sin[e + f*x])
/(b + a*Cos[e + f*x]))/((-a + b)*(a + b)*(b*c - a*d)^2*f)
```

fricas [B] time = 108.51, size = 2852, normalized size = 15.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/2*((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^
4)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b
- a*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2
- 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x +
e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - ((a^5
- 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt
(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt
(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
 e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*
b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*sin(f*x + e))/(
((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d
+ (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^
2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*x + e) + ((a^4*b^3
- 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3
 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*((a^5 - 2*a^3*b^2 + a*b^4)
*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c^2 + d^2)*arctan(
-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (a*b^3
*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2
*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)
*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(
f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 -
b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - 2*((a^2*b^3 - b^5)*
c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d
^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^
3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6
*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*
x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)
*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3
*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*(a*b^3*
c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2*
b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)*
cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a
)/((a^2 - b^2)*sin(f*x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e)
 + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) -
 (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(
f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) -
2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 +
(a^3*b^2 - a*b^4)*d^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 -
2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3
*b^4)*d^4)*f*cos(f*x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 -
 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 +
 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)
*f), ((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4
```

```
)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b -
 a*b^3)*d^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2
*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c^2 + d^2)*arctan(-sqr
t(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - ((a^2*b^3
- b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a
*b^4)*d^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2
*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 +
 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f
*cos(f*x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 +
 a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f)]
```

giac [B] time = 0.41, size = 339, normalized size = 1.82

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-c^2+d^2}}+\frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a^2bc-b^3c-a^3d+ab^2d\right)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)}-\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + b^2*tan(1/2*f*x + 1/2*e)/((a^2*b*c -
 b^3*c - a^3*d + a*b^2*d)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e
)^2 - a - b)) - (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*
e))/sqrt(-a^2 + b^2)))/((a^2*b^2*c^2 - b^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d
+ a^4*d^2 - a^2*b^2*d^2)*sqrt(-a^2 + b^2)))/f
```

maple [A] time = 0.72, size = 210, normalized size = 1.13

$$\frac{\frac{2d^2\operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)}{(da-cb)^2\sqrt{(c+d)(c-d)}}+\frac{2b\left(-\frac{(da-cb)b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)b-a-b\right)}-\frac{\left(2a^2d-abc-b^2d\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{(da-cb)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*(2*d^2/(a*d-b*c)^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)
/((c+d)*(c-d))^(1/2))+2*b/(a*d-b*c)^2*(-(a*d-b*c)*b/(a^2-b^2)*tan(1/2*e+1/2
*f*x)/(a*tan(1/2*e+1/2*f*x)^2-tan(1/2*e+1/2*f*x)^2*b-a-b)-(2*a^2*d-a*b*c-b^
2*d)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*e+1/2*f*x)/((a-b
)*(a+b))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [B] time = 15.56, size = 20827, normalized size = 111.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

```
[Out] (d^2*atan(((d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*
d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5
*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2
- 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d -
8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 +
 a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2
*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/
(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^
2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3
*b^2*c*d) + (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7
+ 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b
^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^
5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6
*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d
+ 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8
*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 -
 a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*
```

d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*
b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 -
 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c
^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^
7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a
^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3
*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d +
3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a
*b^5*c^2*d - 3*a^5*b*c*d^2) + (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*
(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 +
4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d
^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10
*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d
^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*
d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d
 - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d -
 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5
 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c
^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*
a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*
d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b
^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 +
 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c
^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*
b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2
 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 +
a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4
*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2
 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c
^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d) + (d^2*(c^2 - d^2)^(1/2)*
((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 -
a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a
^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^
2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d
 - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*
d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5
*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a
^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*
c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) - (d^2*(c^2 - d^2)^(1/2)
((32(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^
7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d
^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*
c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c
^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6
*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 +

6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*
d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b
^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 -
21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c
^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^
6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4
 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 +
a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d
^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d
- 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) - (32*
d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*
b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*
a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7
 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10
*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^
5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^
6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6
*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^
4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c
^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a
^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d
^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^
5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 +
36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c
^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^
8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*
d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5
*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^
3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2
*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a
*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3
 + 2*a*b*c^3*d))/((64*(b^5*d^5 - a*b^4*d^5 + 2*a^4*b*d^5 - b^5*c*d^4 - 3*a^
2*b^3*d^5 + 2*a^3*b^2*d^5 - 2*a*b^4*c^2*d^3 + 2*a^2*b^3*c*d^4 - 5*a^3*b^2*c
*d^4 + 2*a^2*b^3*c^2*d^3 - a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 + 3*a*b^4*c*
d^4 - 2*a^4*b*c*d^4))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*
c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4
*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^
2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2
+ (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4
*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 +
3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^
4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*
c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*
b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*
b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^

2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b
*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) + (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*
c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b
^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*
d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a
*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*
b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*
c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^
2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b
^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 -
21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^
5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^
5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3
 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2
*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a
^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^
4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c
*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (32*d^2*tan(e/2 +
 (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b
^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 +
 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*
d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*
b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3
*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*
b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b
^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c
^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^
3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^
2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b
^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 1
4*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^
4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a
^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3
 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4
 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^
2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b
^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2
*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(
a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)
- (d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a
*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*
d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5
*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3
*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*
c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*

a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) - (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) - (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))*(c^2 - d^2)^(1/2)*2i)/(f*(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)) + (2*b^2*tan(e/2 + (f*x)/2))/(f*(a + b)*(a + b - tan(e/2 + (f*x)/2)^2*(a - b))*(a^2*d + b^2*c - a*b*c - a*b*d)) + (b*atan(((b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^

4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6 *c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4 *c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4* c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^ 4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b *c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2 *c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c* d - 2*a^3*b^2*c*d) + (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d ^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3* a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a *b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3* b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^ 4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2* d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^ 4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^ 3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4 *c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27 *a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d ^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c *d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (32*b*tan(e/2 + (f*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^ 2*d - 2*a^2*d + a*b*c)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c ^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^ 6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10* c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7* c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c *d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c* d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^ 3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7 *c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 3 0*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^ 5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5 *b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^ 3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12 *a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a* b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^ 8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b ^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5* b^3*c*d)))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2

- b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a
^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*
a^5*b^3*c*d))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*1i)/(a^
8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2
 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*
d + 6*a^5*b^3*c*d) + (b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*
b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d
^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*
c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*
c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c
^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a
^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 -
 b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2
- a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)
- (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^
2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b
^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*
b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b
^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4
*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d
^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*
c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a
^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^
3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b
^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 -
4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3
*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^
3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b
^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^
2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) -
(32*b*tan(e/2 + (f*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b
c)(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^
7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b
^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*
a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c
^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*
c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c
^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6
*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*
d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b
^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 +
 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*
c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a
^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d
^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b

^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 +
2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2
 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*
a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c
^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*
d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)
^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b
^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*
b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*((a +
 b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*1i)/(a^8*d^2 - b^8*c^2 + 3
*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 -
3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))
/((64*(b^5*d^5 - a*b^4*d^5 + 2*a^4*b*d^5 - b^5*c*d^4 - 3*a^2*b^3*d^5 + 2*a^
3*b^2*d^5 - 2*a*b^4*c^2*d^3 + 2*a^2*b^3*c*d^4 - 5*a^3*b^2*c*d^4 + 2*a^2*b^3
*c^2*d^3 - a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4 - 2*a^4*b*c*
d^4))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^
3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b
^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^
2*d - 3*a^5*b*c*d^2) + (b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*
a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4
*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^
5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^
3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4
*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4
*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2
 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^
2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d
) + (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d -
a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7
*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*
a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3
*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b
^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3
*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^
7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7
*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*
d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5
*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4
- 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c
^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*
c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4
*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*
c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2)
+ (32*b*tan(e/2 + (f*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a
*b*c)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*

c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d) - (b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) - (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 +

```
10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^
4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*
b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c
*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^
2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) - (32*b*tan(e/2 + (f*x
)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*(2*a^10*c*d^6 -
2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 -
4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d
^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^1
0*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^
4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*
d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d
^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5
 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3
*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*
b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4
- 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*
c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*
a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*
d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b
^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^
9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3
*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^
3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 +
 a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d -
2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)^3*(a - b)^3)^(1/2)*
(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c
^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*
d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*((a + b)^3*(a - b)^3)^(1/
2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^
4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7
*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)^3*(a - b)^3)
^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*2i)/(f*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2
- 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2
 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))**2*(c + d*sec(e + f*x))), x)
```

3.264 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec}{a-}}}{df(c+d)\sqrt{-\tan^2}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/
f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3969, 3832, 3973}

$$\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec}{a-}}}{df(c+d)\sqrt{-\tan^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3969

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1}{}}}{df}$$

Mathematica [A] time = 3.85, size = 183, normalized size = 0.86

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left((a-b)(c+d)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{f(c-d)(c+d)(a\cos(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/
((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

maple [A] time = 1.67, size = 355, normalized size = 1.67

$$\frac{2\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\left(1+\cos\left(fx+e\right)\right)^{2}\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\sqrt{\frac{a-b}{a+b}}\right)ac+\mathrm{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(
(b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c+EllipticF((-1+cos(f*x+e))
/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*b*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*a*d+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+
b))^(1/2))*b*c)*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f*x+e)^2/(c-d)/(c+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\,\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

3.265 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=196

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-}{(a+}\right.}{df\sqrt{\frac{a+b}{c+d}}}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/d/f/((a+b)/(c+d))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3982}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-}{(a+}\right.}{df\sqrt{\frac{a+b}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(d*Sqrt[(a + b)/(c + d)]*f)
```

Rule 3982

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(-2*(a + b*Csc[
e + f*x])*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])
)]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c
 + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(
c - d))])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1}{(c+d)(a+}}}{d\sqrt{\frac{a+b}{c+d}}f}$$

Mathematica [C] time = 32.59, size = 44216, normalized size = 225.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

maple [A] time = 2.40, size = 351, normalized size = 1.79

$$2\left(\operatorname{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)a - \operatorname{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)b + 2\operatorname{EllipticPi}\left(\frac{(-1+\right.\right.$$
$$\overline{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/
(a-b)/(c+d))^(1/2))*a-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x
+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b+2*EllipticPi((-1+cos(f*x+e))*((a-b)/
(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)
)*b)*cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(
f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*((b
+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))
/(b+a*cos(f*x+e))/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec\left(e + fx\right)} \sec\left(e + fx\right)}{\sqrt{c + d \sec\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)), x)
```

3.266 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f\sqrt{c+d}(bc-ad)}$$

```
[Out] 2*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(
1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3984}

$$\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+}{(a-}\right.}{f\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b
)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]
)))]*(c + d*Sec[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\cdots}}{\sqrt{c+d}\,(bc-\cdots}$$

Mathematica [A] time = 3.60, size = 233, normalized size = 1.21

$$\frac{4\sin^2\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)\sqrt{a+b\sec(e+fx)}\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)\csc^2\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)}{ad-bc}}\,F\left(\sin^{-1}\left(\cdots\right.\right.}{f(a+b)\sqrt{c+d\sec(e+fx)}\sqrt{\frac{(c+d)\csc^2\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx)+b)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqr
t[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]
], (2*(b*c - a*d))/((a + b)*(c - d))]*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x
)/2]^2)/((a + b)*f*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

fricas [F] time = 1.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sqrt{d\sec\left(fx+e\right)+c}\,\sec\left(fx+e\right)}{bd\sec\left(fx+e\right)^2+ac+(bc+ad)\sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/(b*
d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\,\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

maple [A] time = 2.09, size = 219, normalized size = 1.14

$$\frac{2\left(\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{a-b}{a+b}}}{\sin\left(fx+e\right)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)\sqrt{\frac{d+c\cos}{\left(1+\cos\left(f\right.\right.}}}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(b+a\cos\left(fx+e\right)\right)\sqrt{\frac{a-b}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*sin(f*x+e)^2*cos(f*x+e)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f
*x+e))/cos(f*x+e))^(1/2)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(
f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c
+d))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)/(-1+cos(f*x+e))/(d
+c*cos(f*x+e))/(b+a*cos(f*x+e))/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\,\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e + fx\right)}{\sqrt{a + b\sec\left(e + fx\right)}\sqrt{c + d\sec\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))),
x)
```

3.267 $\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx$

Optimal. Leaf size=110

$$\frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right)\middle|45\right)}{f}$$

```
[Out] 2*cot(f*x+e)*EllipticF(1/5*(2+3*sec(f*x+e))^(1/2)*5^(1/2)/(-4+5*sec(f*x+e))
^(1/2),3*5^(1/2))*(4-5*sec(f*x+e))*((1-sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)*
((1+sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)/f
```

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3984}

$$\frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right)\middle|45\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[2 + 3*Sec[e + f*x]]/(Sqrt[5]*Sqrt[-4
+ 5*Sec[e + f*x]])], 45]*(4 - 5*Sec[e + f*x])*Sqrt[(1 - Sec[e + f*x])/(4 -
5*Sec[e + f*x])]*Sqrt[(1 + Sec[e + f*x])/(4 - 5*Sec[e + f*x])])/f
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right)\middle|45\right)(4-5\sec(e+f}{f}$$

Mathematica [A] time = 1.79, size = 176, normalized size = 1.60

$$-\frac{4\sin^4\left(\frac{1}{2}(e+fx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\csc(e+fx)\sec(e+fx)\sqrt{-\left((2\cos(e+fx)+3)\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-}}{3\sqrt{5}f\sqrt{3\sec(e+fx)+2}\sqrt{5\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]])
,x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[2 + 3*Sec[e + f*x]]*Sq
rt[-4 + 5*Sec[e + f*x]])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\sec\left(fx+e\right)-4}\sqrt{3\sec\left(fx+e\right)+2}\sec\left(fx+e\right)}{15\sec\left(fx+e\right)^2-2\sec\left(fx+e\right)-8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)*sec(f*x + e)/(15
*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{5\sec\left(fx+e\right)-4}\sqrt{3\sec\left(fx+e\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

maple [C] time = 2.25, size = 177, normalized size = 1.61

$$\frac{i\left(\sin^2\left(fx+e\right)\right)\sqrt{\frac{2\cos\left(fx+e\right)+3}{\cos\left(fx+e\right)}}\,\sqrt{-\frac{4\cos\left(fx+e\right)-5}{\cos\left(fx+e\right)}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)\sqrt{5}}{5\sin\left(fx+e\right)},3\sqrt{5}\right)\cos\left(fx+e\right)\sqrt{10}\,\sqrt{\frac{2\cos\left(fx+e\right)\cdots}{1+\cos\left(fx+\cdots\right.}}}{5f\left(8\left(\cos^3\left(fx+e\right)\right)-6\left(\cos^2\left(fx+e\right)\right)-17\cos\left(fx+e\right)+15\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/5*I/f*sin(f*x+e)^2*((2*cos(f*x+e)+3)/cos(f*x+e))^(1/2)*(-(4*cos(f*x+e)-5)
/cos(f*x+e))^(1/2)*EllipticF(1/5*I*(-1+cos(f*x+e))*5^(1/2)/sin(f*x+e),3*5^(
1/2))*cos(f*x+e)*10^(1/2)*((2*cos(f*x+e)+3)/(1+cos(f*x+e)))^(1/2)*(-2*(4*co
s(f*x+e)-5)/(1+cos(f*x+e)))^(1/2)/(8*cos(f*x+e)^3-6*cos(f*x+e)^2-17*cos(f*x
+e)+15)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{5\,\sec\left(fx+e\right)-4}\,\sqrt{3\,\sec\left(fx+e\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{\frac{3}{\cos\left(e+f\,x\right)}+2}\,\sqrt{\frac{5}{\cos\left(e+f\,x\right)}-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{3\sec\left(e+fx\right)+2}\,\sqrt{5\sec\left(e+fx\right)-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))**(1/2)/(-4+5*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(3*sec(e + f*x) + 2)*sqrt(5*sec(e + f*x) - 4)),
x)
```

3.268 $\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right)|\frac{1}{45}\right)}{3\sqrt{5}f}$$

```
[Out] 2/15*I*cot(f*x+e)*EllipticF(I*5^(1/2)*(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e
))^(1/2),1/15*5^(1/2))*(2+3*sec(f*x+e))*((1-sec(f*x+e))/(2+3*sec(f*x+e)))^(
1/2)*((1+sec(f*x+e))/(2+3*sec(f*x+e)))^(1/2)/f*5^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3984}

$$\frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right)|\frac{1}{45}\right)}{3\sqrt{5}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),x]
```

```
[Out] (((2*I)/3)*Cot[e + f*x]*EllipticF[I*ArcSinh[(Sqrt[5]*Sqrt[4 - 5*Sec[e + f*x
]])/Sqrt[2 + 3*Sec[e + f*x]]], 1/45]*Sqrt[(1 - Sec[e + f*x])/(2 + 3*Sec[e +
 f*x])]*Sqrt[(1 + Sec[e + f*x])/(2 + 3*Sec[e + f*x])]*(2 + 3*Sec[e + f*x]))
/(Sqrt[5]*f)
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \frac{2i\cot(e+fx)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right)|\frac{1}{45}\right)\sqrt{\frac{1-\sec(e+fx)}{2+3\sec(e+fx)}}}{3\sqrt{5}f}$$

Mathematica [A] time = 0.47, size = 176, normalized size = 1.41

$$-\frac{4\sin^4\left(\frac{1}{2}(e+fx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\csc(e+fx)\sec(e+fx)\sqrt{-\left((2\cos(e+fx)+3)\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-}}{3\sqrt{5}f\sqrt{4-5\sec(e+fx)}\sqrt{3\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),
x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[4 - 5*Sec[e + f*x]]*Sq
rt[2 + 3*Sec[e + f*x]])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{3\sec\left(fx+e\right)+2}\sqrt{-5\sec\left(fx+e\right)+4}\sec\left(fx+e\right)}{15\sec\left(fx+e\right)^2-2\sec\left(fx+e\right)-8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)*sec(f*x + e)/(
15*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{3\sec\left(fx+e\right)+2}\sqrt{-5\sec\left(fx+e\right)+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

maple [A] time = 2.07, size = 170, normalized size = 1.36

$$-\frac{i\sqrt{-\frac{2\left(4\cos\left(fx+e\right)-5\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{10}\,\sqrt{\frac{2\cos\left(fx+e\right)+3}{1+\cos\left(fx+e\right)}}\,\left(\sin^{2}\left(fx+e\right)\right)\mathrm{EllipticF}\left(\frac{3i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\frac{\sqrt{5}}{15}\right)\cos\left(fx+e\right)\sqrt{\frac{4\cos\left(fx+e\right)\cdots}{\cos\left(fx+e\right)}}}{15f\left(8\left(\cos^{3}\left(fx+e\right)\right)-6\left(\cos^{2}\left(fx+e\right)\right)-17\cos\left(fx+e\right)+15\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/15*I/f*(-2*(4*cos(f*x+e)-5)/(1+cos(f*x+e)))^(1/2)*10^(1/2)*((2*cos(f*x+e
)+3)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)^2*EllipticF(3*I*(-1+cos(f*x+e))/sin(f
*x+e),1/15*5^(1/2))*cos(f*x+e)*((4*cos(f*x+e)-5)/cos(f*x+e))^(1/2)*((2*cos(
f*x+e)+3)/cos(f*x+e))^(1/2)/(8*cos(f*x+e)^3-6*cos(f*x+e)^2-17*cos(f*x+e)+15
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{3\,\sec\left(fx+e\right)+2}\,\sqrt{-5\,\sec\left(fx+e\right)+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{\frac{3}{\cos\left(e+f\,x\right)}+2}\,\sqrt{4-\frac{5}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{4-5\sec\left(e+fx\right)}\,\sqrt{3\sec\left(e+fx\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))**(1/2)/(2+3*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(4 - 5*sec(e + f*x))*sqrt(3*sec(e + f*x) + 2)),
x)
```

3.269 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=396

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)\middle|\frac{(a-\cdots}{(a+\cdots}\right.}{bdf\sqrt{\frac{a+b}{c+d}}}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/b/d/f/((a+b)/(c+d))^(1/2)-2*a
*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*s
ec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1
/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(
1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/b/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3985, 3984, 3982}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)\middle|\frac{(a-\cdots}{(a+\cdots}\right.}{bdf\sqrt{\frac{a+b}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(b*d*Sqrt[(a + b)/(c + d)]*f) - (2*a*S
qrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]
*(c + d*Sec[e + f*x]))/(b*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3982

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(-2*(a + b*Csc[
e + f*x])*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])
)]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c
 + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(
c - d))])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3985

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> -Dist[a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/Sqrt[c + d*Csc[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx}{b}$$

$$= \frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-}}{bd\sqrt{\frac{a+b}{c+d}}}$$

Mathematica [C] time = 32.48, size = 39039, normalized size = 98.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

maple [A] time = 2.17, size = 291, normalized size = 0.73

$$-\frac{2\left(\mathrm{EllipticF}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{a-b}{a+b}}}{\sin\left(fx+e\right)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)-2\,\mathrm{EllipticPi}\left(\frac{\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{a-b}{a+b}}}{\sin\left(fx+e\right)},\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\cos\left(fx+e\right)\sqrt{\frac{b+a\,c}{co}}}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(b+a\,co\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f
*x+e),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))*cos(f*x+e)*((b+
a*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x
```

```
+e)^2*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+co
s(f*x+e))/(c+d))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))/((
a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\,\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\,\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))
), x)
```

3.270 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx\right.}{bf\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*d*g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+
1/2*f*x),2,2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f
*x+e))^(1/2)/b/f/(c+d*sec(f*x+e))^(1/2)+2*(-a*d+b*c)*g*(cos(1/2*e+1/2*f*x)^
2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/2*f*x),2*a/(a+b),2^(1/2)
*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x+e))^(1/2)/b/(a+
b)/f/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3971, 3859, 2807, 2805, 3975}

$$\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx\right.}{bf\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e + f*x]),
x]
```

```
[Out] (2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/
(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*(a + b)*f*Sqrt[c + d*Sec[e + f
*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{(-bc+ad)\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx}{b}$$

$$= \frac{\left(dg\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}\,dx}{b\sqrt{c+d\sec(e+fx)}} - \frac{((-bc}{}$$

$$= \frac{\left(dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sec(e+fx)}} - \frac{((-bc+a}{}$$

$$= \frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{bf\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-ad)}{}$$

Mathematica [C] time = 4.02, size = 223, normalized size = 1.31

$$-\frac{2ig\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{-\frac{c(\cos(e+fx)-1)}{c+d}}\sqrt{\frac{c(\cos(e+fx)+1)}{c-d}}\sqrt{c+d\sec(e+fx)}\left(\Pi\left(1-\frac{c}{d};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d}\right.\right.\right.}{bf\sqrt{\frac{1}{c-d}}\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x
]))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]
*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(a*(-c + d))/(-(
b*c) + a*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c +
d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(b*Sqrt[(c - d)
^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

maple [C] time = 2.24, size = 465, normalized size = 2.74

$$-\frac{2i\sqrt{\frac{d+c\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(c+d\right)}}\,\left(2\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},-\frac{a-b}{a+b},i\sqrt{\frac{c-d}{c+d}}\right)a^{2}d-2\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},-\frac{a-b}{a+b},i\sqrt{\frac{c-}{c+}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x)
```

```
[Out] -2*I/f*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*(2*EllipticPi(I*(-1+co
s(f*x+e))/sin(f*x+e),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a^2*d-2*EllipticPi
(I*(-1+cos(f*x+e))/sin(f*x+e),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c+Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*d+EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*b^2*c-EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*b^2*d-2*EllipticPi(I*(-1+cos(f*x+e))/sin(
f*x+e),-1,I*((c-d)/(c+d))^(1/2))*a^2*d+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f
*x+e),-1,I*((c-d)/(c+d))^(1/2))*b^2*d)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*
(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(d+c*cos(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)
/b/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{a+\frac{b}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x)
),x)
```

```
[Out] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec{\left(e + f x \right)}\right)^{\frac{3}{2}} \sqrt{c + d \sec{\left(e + f x \right)}}}{a + b \sec{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(c + d*sec(e + f*x))/(a + b*sec(e + f*
x)), x)
```

3.271 $\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/
2*f*x),2*a/(a+b),2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g
*sec(f*x+e))^(1/2)/(a+b)/f/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3975, 2807, 2805}

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\left(g\sqrt{d+c\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{d+c\cos(e+fx)}}}{\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

Mathematica [A] time = 0.23, size = 83, normalized size = 1.00

$$\frac{2g\sqrt{g\sec(e+fx)}\,\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f
*x]]),x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

maple [C] time = 2.15, size = 236, normalized size = 2.84

$$-\frac{2i\left(a\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{c-d}{c+d}}\right)+b\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{c-d}{c+d}}\right)-2a\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\right.\right.}{f\left(d+c\cos(fx+e)\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\ (a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*I/f*(a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))+b*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))-2*a*EllipticPi(I
*(-1+cos(f*x+e))/sin(f*x+e),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2)))*((d+c*cos(
f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*((d+c
*cos(f*x+e))/cos(f*x+e))^(1/2)/(d+c*cos(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)/(a
-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(e+f\,x)}\right)\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b\sec\left(e+fx\right)\right)\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/((a + b*sec(e + f*x))*sqrt(c + d*sec(e + f
*x))), x)
```

3.272 $\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=168

$$\frac{2(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2d\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)\right.}{af\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*d*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/
2*f*x),2,2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x
+e))^(1/2)/a/f/(c+d*sec(f*x+e))^(1/2)+2*(a*c-b*d)*(cos(1/2*e+1/2*f*x)^2)^(1
/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/2*f*x),2*b/(a+b),2^(1/2)*(c/(
c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x+e))^(1/2)/a/(a+b)/f/
(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2962, 3971, 3859, 2807, 2805, 3975}

$$\frac{2(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2d\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)\right.}{af\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*x]),x]
```

```
[Out] (2*d*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/(c
 + d)]*Sqrt[g*Sec[e + f*x]])/(a*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(a*c - b*d
)*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*b)/(a + b), (e + f*x)/2,
 (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*(a + b)*f*Sqrt[c + d*Sec[e + f*x]]
)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
(c_))^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist
[g^m, Int[(g*Csc[e + f*x])^(p - m)*(b + a*Csc[e + f*x])^m*(c + d*Csc[e + f*
x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[m]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \frac{\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{b+a\sec(e+fx)}\,dx}{g}$$
$$= \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{ag} + \frac{(ac-bd)\int \frac{(g\sec(e+fx))^{3/2}}{(b+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx}{ag}$$
$$= \frac{\left(d\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}\,dx}{a\sqrt{c+d\sec(e+fx)}} + \frac{((ac-$$
$$= \frac{\left(d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sec(e+fx)}} + \frac{\left((ac-bd)\right.}{}$$
$$= \frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}} + \frac{2(ac-bd)}{}$$

Mathematica [C] time = 3.89, size = 222, normalized size = 1.32

$$-\frac{2i\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{-\frac{c(\cos(e+fx)-1)}{c+d}}\sqrt{\frac{c(\cos(e+fx)+1)}{c-d}}\sqrt{c+d\sec(e+fx)}\left(\Pi\left(1-\frac{c}{d};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d}\right.\right.\right.}{af\sqrt{\frac{1}{c-d}}\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*
x]),x]
```

```
[Out] ((-2*I)*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x])
)/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]*S
qrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(b*(-c + d))/(-(a*
c) + b*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c + d)
/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a*Sqrt[(c - d)^(
-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sqrt{g\sec\left(fx+e\right)}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

maple [C] time = 2.23, size = 479, normalized size = 2.85

$$-\frac{2i\left(2\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},-1,i\sqrt{\frac{c-d}{c+d}}\right)a^{2}d-2\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},-1,i\sqrt{\frac{c-d}{c+d}}\right)b^{2}d+\mathrm{EllipticF}\left(\frac{i(-1+c}{\sin}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x)
```

```
[Out] -2*I/f*(2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))
*a^2*d-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*
b^2*d+EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a^2*c-El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a^2*d+EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*d-2*EllipticPi(I*(-1+cos(f*x
+e))/sin(f*x+e),(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c+2*EllipticPi(I*(-1
+cos(f*x+e))/sin(f*x+e),(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*b^2*d)*cos(f*x+e
)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*(g
/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^2/(-1+cos
(f*x+e))/(d+c*cos(f*x+e))/a/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sqrt{g\sec\left(fx+e\right)}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{g}{\cos\left(e+f\,x\right)}}}{a+b\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x)
),x)
```

```
[Out] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}{a+b\cos\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*cos(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sec(e + f*x))*sqrt(c + d*sec(e + f*x))/(a + b*cos(e + f*x))
, x)
```

3.273 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

```
[Out] EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/(1+sec(f*x+e)))
^(1/2)*(a+b*sec(f*x+e))^(1/2)/c/f/((a+b*sec(f*x+e))/(a+b)/(1+sec(f*x+e)))^(
1/2)
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3968}

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[e + f*x])^(-1)]*Sqrt[a + b*Sec[e + f*x]])/(c*f*Sqrt[(a + b*Sec[e +
f*x])/((a + b)*(1 + Sec[e + f*x]))])
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\,\sqrt{a+b\sec(e+fx)}}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

Mathematica [B] time = 5.75, size = 264, normalized size = 2.78

$$\frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\,\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\,\sqrt{\sec(e+fx)+1}\,\sec^4\left(\frac{1}{2}(e+fx)\right)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)}{\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}}+\cdots}{4cf(\sec(e+fx)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/
(a + b)]*Sec[(e + f*x)/2]^4*Sqrt[1 + Sec[e + f*x]])/(((1 + Cos[e + f*x])^(-
1))^(3/2)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]) + (Sec[(
e + f*x)/2]^5*Sqrt[1 + Sec[e + f*x]]*(-Sin[(e + f*x)/2] + Sin[(3*(e + f*x))
/2]))/((1 + Cos[e + f*x])^(-1))^(3/2) - 8*Sqrt[Sec[e + f*x]]*(Sin[e + f*x]
- Tan[(e + f*x)/2])))/(4*c*f*(1 + Sec[e + f*x]))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

maple [A] time = 1.97, size = 153, normalized size = 1.61

$$-\frac{\mathrm{EllipticE}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}, \sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\left(1+\cos\left(f\right.\right.}{cf\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*((b+a*cos(
f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-1+c
os(f*x+e))*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(1+cos(f*x+e))^2/(b+a*cos(f*
x+e))/sin(f*x+e)^2*(-a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x)/c
```

3.274 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=295

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g}}{}$$

```
[Out] -g*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/f/(c+c*cos(f*x+e))/(a+b
*sec(f*x+e))^(1/2)+g*(b+a*cos(f*x+e))*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*
e+1/2*f*x)*EllipticE(sin(1/2*e+1/2*f*x),2^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x
+e))^(1/2)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^(1/2)+(a-b)*
g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticF(sin(1/2*e+1/2*f
*x),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^
(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*b*g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2
*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/2*f*x),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.282, Rules used = {3971, 3859, 2807, 2805, 3975, 2768, 2752, 2663, 2661, 2655, 2653}

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/(c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e + f*x]])
+ ((a - b)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2*b*g*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a + b)]*
Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a*Cos[e + f*
x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/(f*(c + c*Cos[e + f*x])*Sqrt[a + b*S
ec[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\left((-a+b)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx\right) + \frac{b\int \frac{(g}{\sqrt{}}}{}$$

$$= -\frac{\left((-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}\,(c}}{\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(a(-a+b)g\sqrt{}\right.}{}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\,c}{f(c}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\,c}{f(c}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{(a-b)g}{}$$

Mathematica [F] time = 20.05, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]),x]
```

```
[Out] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

maple [C] time = 2.22, size = 292, normalized size = 0.99

$$\frac{i\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\left(2a\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-2b\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] I/c/f*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(2*a*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-2*b*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(
-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b)
)^(1/2))+4*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2)
)*b)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/
(b+a*cos(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c+\frac{c}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x)
),x)
```

```
[Out] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(g \sec{\left(e + f x \right)}\right)^{\frac{3}{2}} \sqrt{a + b \sec{\left(e + f x \right)}}}{\sec{\left(e + f x \right)} + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(sec(e + f*x) + 1
), x)/c
```

3.275 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}}{cf(a-b)}$$

```
[Out] -2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(
1/2)/(a-b)/c/f+EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/
(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e))/(a
+b)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3972, 3832, 3968}

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}}{cf(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[e + f*x]))/(a - b))])/((a - b)*c*f) + (EllipticE[ArcSin[Tan[e + f*x
]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[
a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1 + S
ec[e + f*x]))])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3972

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[(Cs
c[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = -\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} - \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx}{-ac+bc}$$

$$= -\frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx}{a+b}}}{(a-b)cf}$$

Mathematica [A] time = 14.70, size = 375, normalized size = 1.79

$$\frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2(e+fx)(a\cos(e+fx)+b)\left(\frac{2\sin(e+fx)}{b-a}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{b-a}\right)}{f(c\sec(e+fx)+c)\sqrt{a+b\sec(e+fx)}}-\frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]

[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*Sec[e + f*x]^2*((2*Sin[e + f*x])
/(-a + b) - (2*Tan[(e + f*x)/2])/(-a + b)))/(f*Sqrt[a + b*Sec[e + f*x]]*(c
```

```
+ c*Sec[e + f*x])) - (2*Cos[e/2 + (f*x)/2]^2*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e
 + f*x)/2]^2*Sec[e + f*x]]*((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*
Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x
)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos
[e + f*x])]*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2
))/(((a - b)/(a + b))^(3/2)*(a + b)*f*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]
*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)}{bc\sec\left(fx+e\right)^{2}+(a+b)c\sec\left(fx+e\right)+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(b*c*sec(f*x + e)^2 + (a + b
)*c*sec(f*x + e) + a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

maple [A] time = 2.01, size = 225, normalized size = 1.08

$$-\frac{\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\,\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)\left(2\,\text{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right.\right.}{cf\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/c/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*
x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b-a*Elli
pticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)))/(b+a*cos(f*x+e))/sin(f*x+e)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a + b*s
ec(e + f*x))), x)/c
```

3.276 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=214

$$\frac{2a\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+}}{cf(a-b)\sqrt{}}$$

```
[Out] 2*a*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/(a-b)/b/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*
(1/(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e))
/(a+b)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3976, 3832, 3968}

$$\frac{2a\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+}}{cf(a-b)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/((a - b)*b*c*f) - (EllipticE[ArcSin[Tan[e +
f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sq
rt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1
+ Sec[e + f*x]))])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3976

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx}{-ac+bc}$$

$$= \frac{2a\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx}{a+b}}}{(a-b)bcf}$$

Mathematica [A] time = 5.31, size = 156, normalized size = 0.73

$$\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left((a+b)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2aF\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a}{a}\right.\right.}{cf(b-a)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}(\cos(e+fx)+1)^2\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^4*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x])
)]*((a + b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*a*Elli
pticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/((-a + b)*c*f*Sqrt[Cos[e
 + f*x]/(1 + Cos[e + f*x])]*(1 + Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^{2}}{bc\sec\left(fx+e\right)^{2}+(a+b)c\sec\left(fx+e\right)+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)^2/(b*c*sec(f*x + e)^2 + (a +
 b)*c*sec(f*x + e) + a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

maple [A] time = 1.90, size = 224, normalized size = 1.05

$$\frac{\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\,\left(1+\cos\left(fx+e\right)\right)^{2}\left(2\,\text{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\sqrt{\frac{a-b}{a+b}}\right)a-a\,\text{Ellipt}\right.}{cf\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx+e\right)^{2}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/c/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(2*Elliptic
F((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-a*EllipticE((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f*x+e)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^{2}\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a +
b*sec(e + f*x))), x)/c
```

3.277 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=229

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+}}{cf(a-}$$

```
[Out] -g*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/(a-b)/f/(c+c*cos(f*x+e)
)/(a+b*sec(f*x+e))^(1/2)+g*(b+a*cos(f*x+e))*(cos(1/2*e+1/2*f*x)^2)^(1/2)/co
s(1/2*e+1/2*f*x)*EllipticE(sin(1/2*e+1/2*f*x),2^(1/2)*(a/(a+b))^(1/2))*(g*s
ec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^
(1/2)+g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticF(sin(1/2*e
+1/2*f*x),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*
x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {3975, 2768, 2752, 2663, 2661, 2655, 2653}

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+}}{cf(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e +
 f*x]]) + (g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a
*Cos[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e +
 f*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = \frac{\left(g\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}\,(c+c\cos(e+fx))}}{\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} - \frac{\left(ag\sqrt{b+a\cos\right.}{}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g\sqrt{b+a\cos\right.}{}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g(b+a\cos(\right.}{2(a}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{a+b\sec(e+fx)}} + \frac{g\sqrt{\frac{b+a\cos}{a}}}{}$$

Mathematica [C] time = 8.46, size = 1019, normalized size = 4.45

$$\frac{(b+a\cos(e+fx))(g\sec(e+fx))^{3/2}\left(\frac{2\csc(e)}{(b-a)f}+\frac{2\sec\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{(b-a)f}\right)\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{a+b\sec(e+fx)}\,(\sec(e+fx)c+c)} + \frac{a\sqrt{b+a\cos(e+fx)}\,\csc\left(\frac{e}{2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*(g*Sec[e + f*x])^(3/2)*((2*Csc[e
])/((-a + b)*f) + (2*Sec[e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/((-a + b)*f)
))/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])) + (AppellF1[1/2, 1/2, 1/
2, 3/2, (Csc[e]*(b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]])
)/(a*Sqrt[1 + Cot[e]^2]*(1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2]))), (Csc[e]*(
```

```
b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]))/(a*Sqrt[1 + Cot
[e]^2]*(-1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2]))))]*Cos[e/2 + (f*x)/2]^2*Sqrt
[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(g*Sec[e + f*x])^(3/2)*Sec[f*x - Arc
Tan[Cot[e]]]*Sqrt[(a*Sqrt[1 + Cot[e]^2] - a*Sqrt[1 + Cot[e]^2]*Sin[f*x - Ar
cTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] - b*Csc[e])]*Sqrt[(a*Sqrt[1 + Cot[e]^2
] + a*Sqrt[1 + Cot[e]^2]*Sin[f*x - ArcTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] +
 b*Csc[e])]*Sqrt[b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]]
)/((-a + b)*f*Sqrt[1 + Cot[e]^2]*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*
x])) + (a*Cos[e/2 + (f*x)/2]^2*Sqrt[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(
g*Sec[e + f*x])^(3/2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[e]*(b + a*Co
s[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(
1 - (b*Sec[e])/(a*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(b + a*Cos[e]*Cos[f*x +
 ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(-1 - (b*Sec[e]
)/(a*Sqrt[1 + Tan[e]^2]))))]*Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/(Sqrt[1 + Ta
n[e]^2]*Sqrt[(a*Sqrt[1 + Tan[e]^2] - a*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + T
an[e]^2])/(b*Sec[e] + a*Sqrt[1 + Tan[e]^2])]*Sqrt[(a*Sqrt[1 + Tan[e]^2] + a
*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(b*Sec[e]) + a*Sqrt[1 + Ta
n[e]^2])]*Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])
- ((Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/Sqrt[1 + Tan[e]^2] + (2*a*Cos[e]*(b +
 a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a^2*Cos[e]^2 + a^
2*Sin[e]^2))/Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
]))/(2*(-a + b)*f*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\,\sqrt{g\sec\left(fx+e\right)}\,g\sec\left(fx+e\right)}{bc\sec\left(fx+e\right)^{2}+(a+b)c\sec\left(fx+e\right)+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(g*sec(f*x + e))*g*sec(f*x + e)/(b*c*
sec(f*x + e)^2 + (a + b)*c*sec(f*x + e) + a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

maple [C] time = 2.31, size = 222, normalized size = 0.97

$$\frac{i\left(2a\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-b\,\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-}{a+}}\right.\right.}{cf\left(b+a\cos\left(fx+e\right)\right)\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\;\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] I/c/f*(2*a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2)))*((b+a*cos(f*x+e))/(1+cos(f
*x+e))/(a+b))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*((b+a*cos(f*x+e))/cos
(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a+b\sec(e+fx)}\sec(e+fx)+\sqrt{a+b\sec(e+fx)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + s
qrt(a + b*sec(e + f*x))), x)/c
```

3.278 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=312

$$\frac{g^2\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\,\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e}{cf(a}$$

```
[Out] g^2*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/(a-b)/f/(c+c*cos(f*x+e
))/(a+b*sec(f*x+e))^(1/2)-g^2*(b+a*cos(f*x+e))*(cos(1/2*e+1/2*f*x)^2)^(1/2)
/cos(1/2*e+1/2*f*x)*EllipticE(sin(1/2*e+1/2*f*x),2^(1/2)*(a/(a+b))^(1/2))*(
g*sec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e
))^(1/2)-g^2*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticF(sin(
1/2*e+1/2*f*x),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*s
ec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*g^2*(cos(1/2*e+1/2*f*x)^2)^(1
/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/2*f*x),2,2^(1/2)*(a/(a+b))^(1
/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e
))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.282, Rules used = {3979, 3859, 2807, 2805, 3975, 2768, 2752, 2663, 2661, 2655, 2653}

$$\frac{g^2\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\,\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e}{cf(a}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Se
c[e + f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec
[e + f*x]])) - (g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/
2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2
*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a
 + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (g^2*(b + a*C
os[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f
*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(c*g)/d,
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = -\left(g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx\right) + \frac{g\int \frac{(g\sec(e+\ldots}{\sqrt{a+b\sec(\ldots}}}{c}$$

$$= -\frac{\left(g^2\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}\,(c+c\cos(e\ldots}}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(ag^2\sqrt{b+a\,\mathrm{c}\ldots}\right.}{\ldots}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} + \frac{g^2(b+\ldots}{(a-b\ldots}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} + \frac{g^2(b+\ldots}{(a-b\ldots}$$

$$= -\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{a+b\sec(e+fx)}} - \frac{g^2\sqrt{\ldots}}{\ldots}$$

Mathematica [F] time = 15.55, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

```
[Out] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b \sec\left(fx+e\right)+a}\left(c \sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

maple [C] time = 2.50, size = 355, normalized size = 1.14

$$i\left(4a\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}, \sqrt{-\frac{a-b}{a+b}}\right)-2b\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}, \sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}, \sqrt{-\frac{a}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] I/c/f*(4*a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-2*b
*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-4*EllipticPi(I*(-1+cos(f*x+e))/sin(f*
x+e),-1,I*((a-b)/(a+b))^(1/2))*a+4*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),
-1,I*((a-b)/(a+b))^(1/2))*b)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*
((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(g/cos(f*x+e))^(5/2)*co
s(f*x+e)^3/(b+a*cos(f*x+e))/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^2/(a-b)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.279 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec}{a-}}}{df(c+d)\sqrt{-\tan^2}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/
f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3969, 3832, 3973}

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec}{a-}}}{df(c+d)\sqrt{-\tan^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3969

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(1}{}}}{df}$$

Mathematica [A] time = 0.40, size = 183, normalized size = 0.86

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left((a-b)(c+d)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{f(c-d)(c+d)(a\cos(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/
((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec\left(fx+e\right)+a} \sec\left(fx+e\right)}{d \sec\left(fx+e\right)+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

maple [A] time = 1.62, size = 355, normalized size = 1.67

$$\frac{2\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\left(1+\cos\left(fx+e\right)\right)^{2}\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}, \sqrt{\frac{a-b}{a+b}}\right)ac+\mathrm{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(
(b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c+EllipticF((-1+cos(f*x+e))
/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*b*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*a*d+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+
b))^(1/2))*b*c)*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f*x+e)^2/(c-d)/(c+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec\left(fx+e\right)+a} \sec\left(fx+e\right)}{d \sec\left(fx+e\right)+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

3.280 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx\right.}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*b*g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+
1/2*f*x),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f
*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*(-a*d+b*c)*g*(cos(1/2*e+1/2*f*x)^
2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/2*f*x),2*c/(c+d),2^(1/2)
*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/d/(c+
d)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3971, 3859, 2807, 2805, 3975}

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx\right.}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*(b*c - a
*d)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{\left(bg\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{((bc-$$

$$= \frac{\left(bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{((bc-ad)}{}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2(bc-ad)}{}$$

Mathematica [C] time = 3.98, size = 223, normalized size = 1.31

$$-\frac{2ig\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{-\frac{a(\cos(e+fx)-1)}{a+b}}\sqrt{\frac{a(\cos(e+fx)+1)}{a-b}}\sqrt{a+b\sec(e+fx)}\left(\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b}\right.\right.\right.}{df\sqrt{\frac{1}{a-b}}\sqrt{a\cos(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Cot[e + f*x]*(EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)] - EllipticPi[((a - b)*c)/(-(b
*c) + a*d), I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)])*Sqrt[g*Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[(a - b)^(-
1)]*d*f*Sqrt[b + a*Cos[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

maple [C] time = 2.03, size = 465, normalized size = 2.74

$$-\frac{2i\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\left(\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)acd+\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)a\,d^{2}-\mathrm{Ellip}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -2*I/f*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*a*c*d+EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),(-(a-b)/(a+b))^(1/2))*a*d^2-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),(-(a-b)/(a+b))^(1/2))*b*c*d-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b
)/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d
),I*((a-b)/(a+b))^(1/2))*a*c*d+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(
c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*b*c^2-2*EllipticPi(I*(-1+cos(f*x+e))/sin(
f*x+e),-1,I*((a-b)/(a+b))^(1/2))*b*c^2+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f
*x+e),-1,I*((a-b)/(a+b))^(1/2))*b*d^2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*
(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(b+a*cos(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)
/d/(c+d)/(c-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\,\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c+\frac{d}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
),x)
```

```
[Out] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec{\left(e + f x \right)}\right)^{\frac{3}{2}} \sqrt{a + b \sec{\left(e + f x \right)}}}{c + d \sec{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*
x)), x)
```

3.281 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=102

$$\frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\,\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(
1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/(c+d)/f/(a+b*sec(f*x+e))^(1
/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3973}

$$\frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\,\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/
(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/((c + d)*f*Sqrt[a
 + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{2\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\tan(e+fx)}{(c+d)f\sqrt{a+b\sec(e+fx)}\,\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] time = 4.21, size = 187, normalized size = 1.83

$$\frac{2\sqrt{\sec(e+fx)}\sqrt{\sec(e+fx)+1}\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left((c+d)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+f\right.\right.\right.\right.}{f(c-d)(c+d)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*Ellipti
cF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*d*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[Cos[e + f*x]*Sec[(e +
 f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/((c - d)*(c + d)*f*S
qrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

maple [B] time = 1.81, size = 238, normalized size = 2.33

$$-\frac{2\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\left(1+\cos\left(fx+e\right)\right)^2\left(2\,\mathrm{EllipticPi}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\frac{c-d}{c+d},\sqrt{\frac{a-b}{a+b}}\right)d-\right.}{f\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(2*EllipticP
i((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d-EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c-EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*d)*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f*x
+e)^2/(c-d)/(c+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

3.282 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bdf}-\frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi}{df(c+d)\sqrt{-\tan^2(e+}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/b/d/f-2*c*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)
*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/f/(a+b*
sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3977, 3832, 3973}

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bdf}-\frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi}{df(c+d)\sqrt{-\tan^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(b*d*f) - (2*c*EllipticPi[(2*d)/(c + d), ArcSi
n[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e +
 f*x]^2])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3977

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[1/d, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c/d, Int[Csc[e + f*x]/(Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx &= \frac{\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx}{d}\\ &= \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{bdf}\end{aligned}$$

Mathematica [A] time = 3.06, size = 165, normalized size = 0.79

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\,\sec(e+fx)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\,\left((c+d)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2c\Pi\right.}{f(c-d)(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] - 2*c*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)])*Sec[e + f*x])/((c - d)*(c + d)*f*Sqrt[a + b
*Sec[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

maple [A] time = 1.79, size = 236, normalized size = 1.13

$$-\frac{2\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\left(1+\cos\left(fx+e\right)\right)^2\left(-1+\cos\left(fx+e\right)\right)\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(fx+\right.}{\sin\left(fx+e\right)}\right.\right.}{f\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx+e\right)^2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x+
e))*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c+EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*c*EllipticPi((-1+cos(f*
x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2)))/(b+a*cos(f*x+e))/sin(f*x
+e)^2/(c-d)/(c+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\,\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x
)
```

3.283 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/
2*f*x),2*c/(c+d),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g
*sec(f*x+e))^(1/2)/(c+d)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3975, 2807, 2805}

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\left(g\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}\,(d+c\cos(e+f}}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}\,(d+c\cos(e+fx))}\,d}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [A] time = 0.29, size = 83, normalized size = 1.00

$$\frac{2g\sqrt{g\sec(e+fx)}\,\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

maple [C] time = 2.08, size = 238, normalized size = 2.87

$$\frac{2i\left(2c\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},-\frac{c-d}{c+d},i\sqrt{\frac{a-b}{a+b}}\right)-c\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-d\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}\right.}{f\left(b+a\cos\left(fx+e\right)\right)\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\,\left(c+d\right)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2*I/f*(2*c*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/(a
+b))^(1/2))-c*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-
d*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2)))*((b+a*cos(f
*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*((b+a*
cos(f*x+e))/cos(f*x+e))^(1/2)/(b+a*cos(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)/(c+
d)/(c-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)

[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a+b\sec\left(e+fx\right)}\,\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)

3.284 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=166

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*g^2*(cos(1/2*e+1/2*f*x)^2)^(1/2)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+
1/2*f*x),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f
*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*c*g^2*(cos(1/2*e+1/2*f*x)^2)^(1/2
)/cos(1/2*e+1/2*f*x)*EllipticPi(sin(1/2*e+1/2*f*x),2*c/(c+d),2^(1/2)*(a/(a+
b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/d/(c+d)/f/(a
+b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3979, 3859, 2807, 2805, 3975}

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*c*g^2*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)/2, (2*
a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(c*g)/d,
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(cg)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{\left(g^2\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{(cg^2}{}$$

$$= \frac{\left(g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left(cg^2\sqrt{\frac{b+}{}}\right.}{}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2cg^2\sqrt{\frac{b+}{}}}{}$$

Mathematica [C] time = 4.12, size = 246, normalized size = 1.48

$$-\frac{2ig\cot(e+fx)(g\sec(e+fx))^{3/2}\sqrt{-\frac{a(\cos(e+fx)-1)}{a+b}}\sqrt{\frac{a(\cos(e+fx)+1)}{a-b}}\sqrt{a\cos(e+fx)+b}\left((ad-bc)\Pi\left(1-\frac{a}{b};i\sinh^{-1}\right.\right.}{bdf\sqrt{\frac{1}{a-b}}(ad-bc)\sqrt{a+b\sec}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Sqrt[b + a*Cos[e + f*x]]*Cot[e + f*x]*((-(b*c) + a*d)*Elliptic
Pi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)] + b*c*EllipticPi[((a - b)*c)/(-(b*c) + a*d), I*ArcSinh[Sqrt[(a -
 b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)])*(g*Sec[e + f*x])^(3
/2))/(Sqrt[(a - b)^(-1)]*b*d*(-(b*c) + a*d)*f*Sqrt[a + b*Sec[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

maple [C] time = 2.14, size = 346, normalized size = 2.08

$$2i\left(2c^2\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},-\frac{c-d}{c+d},i\sqrt{\frac{a-b}{a+b}}\right)-\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)dc-\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2*I/f*(2*c^2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/
(a+b))^(1/2))-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*
d*c-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*d^2-2*c^2*
EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))+2*Ellipti
cPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))*d^2)*((b+a*cos(f
*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+
cos(f*x+e))*(g/cos(f*x+e))^(5/2)*cos(f*x+e)^3/(b+a*cos(f*x+e))/(1/(1+cos(f*
x+e)))^(3/2)/sin(f*x+e)^2/d/(c+d)/(c-d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)

[Out] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)

[Out] Timed out

3.285 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$

Optimal. Leaf size=67

$$\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}$$

```
[Out] 1/20*cot(1/2*e+1/2*f*x)^5/c^7/f-1/14*cot(1/2*e+1/2*f*x)^7/c^7/f+1/36*cot(1/
2*e+1/2*f*x)^9/c^7/f
```

Rubi [A] time = 0.30, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {12, 270}

$$\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(20*c^7*f) - Cot[(e + f*x)/2]^7/(14*c^7*f) + Cot[(e + f*
x)/2]^9/(36*c^7*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{2\,\text{Subst}\left(\int -\frac{(1-x^2)^2}{8c^7x^{10}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^{10}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{10}}-\frac{2}{x^8}+\frac{1}{x^6}\right)dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}$$

$$= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f} - \frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}$$

Mathematica [B] time = 0.96, size = 151, normalized size = 2.25

$$\frac{\csc\left(\frac{e}{2}\right)\left(-718830\sin\left(e+\frac{fx}{2}\right)+467208\sin\left(e+\frac{3fx}{2}\right)+659400\sin\left(2e+\frac{3fx}{2}\right)-303192\sin\left(2e+\frac{5fx}{2}\right)-179640\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^9*(-971082*Sin[(f*x)/2] - 718830*Sin[e + (f*x)/2
] + 467208*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 303192*Sin[2*
e + (5*f*x)/2] - 179640*Sin[3*e + (5*f*x)/2] + 30753*Sin[3*e + (7*f*x)/2] +
 89955*Sin[4*e + (7*f*x)/2] - 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e + (9*
f*x)/2]))/(23063040*c^7*f)
```

fricas [B] time = 1.18, size = 121, normalized size = 1.81

$$\frac{47\cos\left(fx+e\right)^5+127\cos\left(fx+e\right)^4+101\cos\left(fx+e\right)^3+11\cos\left(fx+e\right)^2-8\cos\left(fx+e\right)+2}{315\left(c^7f\cos\left(fx+e\right)^4-4\,c^7f\cos\left(fx+e\right)^3+6\,c^7f\cos\left(fx+e\right)^2-4\,c^7f\cos\left(fx+e\right)+c^7f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] 1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 + 101*cos(f*x + e)^3 + 11*cos
(f*x + e)^2 - 8*cos(f*x + e) + 2)/((c^7*f*cos(f*x + e)^4 - 4*c^7*f*cos(f*x
```

```
+ e)^3 + 6*c^7*f*cos(f*x + e)^2 - 4*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e
))
```

giac [A] time = 3.49, size = 50, normalized size = 0.75

$$\frac{63\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+35}{1260\,c^{7}f\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="giac")
```

```
[Out] 1/1260*(63*tan(1/2*f*x + 1/2*e)^4 - 90*tan(1/2*f*x + 1/2*e)^2 + 35)/(c^7*f*
tan(1/2*f*x + 1/2*e)^9)
```

maple [A] time = 1.04, size = 49, normalized size = 0.73

$$\frac{-\frac{2}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{9\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}}{4f\,c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x)
```

```
[Out] 1/4/f/c^7*(-2/7/tan(1/2*e+1/2*f*x)^7+1/9/tan(1/2*e+1/2*f*x)^9+1/5/tan(1/2*e
+1/2*f*x)^5)
```

maxima [A] time = 0.33, size = 68, normalized size = 1.01

$$-\frac{\left(\frac{90\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{63\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-35\right)\left(\cos\left(fx+e\right)+1\right)^{9}}{1260\,c^{7}f\,\sin\left(fx+e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -1/1260*(90*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 63*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 35)*(cos(f*x + e) + 1)^9/(c^7*f*sin(f*x + e)^9)
```

mupad [B] time = 1.93, size = 47, normalized size = 0.70

$$\frac{63\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-90\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+35}{1260\,c^{7}\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

```
[Out] (63*tan(e/2 + (f*x)/2)^4 - 90*tan(e/2 + (f*x)/2)^2 + 35)/(1260*c^7*f*tan(e/
2 + (f*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6
 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e
+ f*x)**2 + 7*sec(e + f*x) - 1), x)/c**7
```

3.286 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx$

Optimal. Leaf size=89

$$-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}$$

```
[Out] 1/40*cot(1/2*e+1/2*f*x)^5/c^8/f-3/56*cot(1/2*e+1/2*f*x)^7/c^8/f+1/24*cot(1/
2*e+1/2*f*x)^9/c^8/f-1/88*cot(1/2*e+1/2*f*x)^11/c^8/f
```

Rubi [A] time = 0.34, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {12, 270}

$$-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(40*c^8*f) - (3*Cot[(e + f*x)/2]^7)/(56*c^8*f) + Cot[(e
+ f*x)/2]^9/(24*c^8*f) - Cot[(e + f*x)/2]^11/(88*c^8*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{2\,\text{Subst}\left(\int \frac{(1-x^2)^3}{16c^8x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^{12}}-\frac{3}{x^{10}}+\frac{3}{x^8}-\frac{1}{x^6}\right)dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f}$$
$$= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}$$

Mathematica [A] time = 1.12, size = 175, normalized size = 1.97

$$-\frac{\csc\left(\frac{e}{2}\right)\left(486024\sin\left(e+\frac{fx}{2}\right)-351450\sin\left(e+\frac{3fx}{2}\right)-299970\sin\left(2e+\frac{3fx}{2}\right)+145695\sin\left(2e+\frac{5fx}{2}\right)+180015\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] -1/15375360*(Csc[e/2]*Csc[(e + f*x)/2]^11*(425964*Sin[(f*x)/2] + 486024*Sin
[e + (f*x)/2] - 351450*Sin[e + (3*f*x)/2] - 299970*Sin[2*e + (3*f*x)/2] + 1
45695*Sin[2*e + (5*f*x)/2] + 180015*Sin[3*e + (5*f*x)/2] - 63580*Sin[3*e +
(7*f*x)/2] - 44990*Sin[4*e + (7*f*x)/2] + 6710*Sin[4*e + (9*f*x)/2] + 15004
*Sin[5*e + (9*f*x)/2] - 1975*Sin[5*e + (11*f*x)/2] + Sin[6*e + (11*f*x)/2])
)/(c^8*f)
```

fricas [A] time = 0.89, size = 146, normalized size = 1.64

$$\frac{152\cos\left(fx+e\right)^6+395\cos\left(fx+e\right)^5+289\cos\left(fx+e\right)^4+15\cos\left(fx+e\right)^3-19\cos\left(fx+e\right)^2+10\cos}{1155\left(c^8f\cos\left(fx+e\right)^5-5c^8f\cos\left(fx+e\right)^4+10c^8f\cos\left(fx+e\right)^3-10c^8f\cos\left(fx+e\right)^2+5c^8f\cos\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 + 395*cos(f*x + e)^5 + 289*cos(f*x + e)^4 + 15*c
os(f*x + e)^3 - 19*cos(f*x + e)^2 + 10*cos(f*x + e) - 2)/((c^8*f*cos(f*x +
```

```
e)^5 - 5*c^8*f*cos(f*x + e)^4 + 10*c^8*f*cos(f*x + e)^3 - 10*c^8*f*cos(f*x
+ e)^2 + 5*c^8*f*cos(f*x + e) - c^8*f)*sin(f*x + e))
```

giac [A] time = 10.61, size = 64, normalized size = 0.72

$$\frac{231\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-495\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+385\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-105}{9240\,c^{8}f\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="giac")
```

```
[Out] 1/9240*(231*tan(1/2*f*x + 1/2*e)^6 - 495*tan(1/2*f*x + 1/2*e)^4 + 385*tan(1
/2*f*x + 1/2*e)^2 - 105)/(c^8*f*tan(1/2*f*x + 1/2*e)^11)
```

maple [A] time = 1.20, size = 62, normalized size = 0.70

$$\frac{-\frac{1}{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}-\frac{3}{7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}+\frac{1}{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}+\frac{1}{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}}}{8f\,c^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x)
```

```
[Out] 1/8/f/c^8*(-1/11/tan(1/2*e+1/2*f*x)^11-3/7/tan(1/2*e+1/2*f*x)^7+1/3/tan(1/2
*e+1/2*f*x)^9+1/5/tan(1/2*e+1/2*f*x)^5)
```

maxima [A] time = 0.34, size = 88, normalized size = 0.99

$$\frac{\left(\frac{385\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{495\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{231\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-105\right)\left(\cos\left(fx+e\right)+1\right)^{11}}{9240\,c^{8}f\sin\left(fx+e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 495*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 105)*(cos(f*x +
e) + 1)^11/(c^8*f*sin(f*x + e)^11)
```

mupad [B] time = 2.43, size = 60, normalized size = 0.67

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{5}-\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{7}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}-\frac{1}{11}}{8\,c^8\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^8),x)
```

```
[Out] (tan(e/2 + (f*x)/2)^2/3 - (3*tan(e/2 + (f*x)/2)^4)/7 + tan(e/2 + (f*x)/2)^6
/5 - 1/11)/(8*c^8*f*tan(e/2 + (f*x)/2)^11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4\left(e+fx\right)\sec\left(e+fx\right)}{\sec^8\left(e+fx\right)-8\sec^7\left(e+fx\right)+28\sec^6\left(e+fx\right)-56\sec^5\left(e+fx\right)+70\sec^4\left(e+fx\right)-56\sec^3\left(e+fx\right)+28\sec^2\left(e+fx\right)-8\sec\left(e+fx\right)+1}\,dx}{c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**8,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**8 - 8*sec(e + f*x)**7
+ 28*sec(e + f*x)**6 - 56*sec(e + f*x)**5 + 70*sec(e + f*x)**4 - 56*sec(e +
 f*x)**3 + 28*sec(e + f*x)**2 - 8*sec(e + f*x) + 1), x)/c**8
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
          'sin','cos','tan','cot','sec','csc',
          'arcsin','arccos','arctan','arccot','arcsec','arccsc',
          'sinh','cosh','tanh','coth','sech','csch',
          'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
              'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
              'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
              'polylog','lambert_w','elliptic_f','elliptic_e',
              'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```